



Causal Network Analysis and Fault Root Point Detection Based on Symbolic Transfer Entropy

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Abstract. Transfer entropy (TE) is a model-free method based on data-driven information theory. It can obtain causal relationships between variables. It has been used for modeling, monitoring and fault diagnosis of complex industrial processes. It can detect the causal relationship between variables without the need to assume any underlying model, but its calculation process is complicated and the calculation time is long. In order to overcome this limitation, symbol transfer entropy is proposed. The symbol transfer entropy is robust and fast to calculate. It can also quantify the dominant direction of information flow between time series with identical and non-identical coupling systems, thereby improving the accuracy of causal paths. Sex. Through the symbolic transfer of entropy, a causal network diagram can be obtained, and the root cause of the fault can be found. The effectiveness and accuracy of the method are verified by simulation and actual industrial cases (Tennessee-Eastman process)

Keywords: Symbolic transfer entropy · Causal network · Root cause of failure

1 Introduction

Fault diagnosis is the detection and root cause identification of abnormal events, which is a complex and time-consuming task. Reis and Gins [1] emphasized improvements in the speed and accuracy of fault diagnosis as the most immediate potential benefit of industrial process monitoring. Causal analysis techniques can be used for data-based fault diagnosis. In these techniques, a causal relationship between the measured variables is inferred. Because the symptoms of a fault propagate throughout the process along these causal relationships, the inferred causality can indicate the path of the fault throughout the process. People have studied many changes in causality analysis in fault diagnosis, including: transfer entropy [2–10]; Granger causality [11–13]; cross-correlation [14]; partially directed coherence; and convergence cross-mapping [15].

In recent years, with the development of scientific information technology, information transmission in industrial information systems has become the focus of attention.

Generally speaking, the mutual transmission of information in the system will change the complexity of variables. This change Trends can help us analyze and study the causality between variables. In order to quantify the complexity between variables, Claude Eshannon, the founder of information theory, first proposed the concept of information entropy. The concept of entropy summarizes the calculation method of information entropy and uses information entropy to quantify the complexity between signals. On this basis, Schreiber [17] further proposed the concept of Transfer Entropy (TE), which unifies the complexity of the signal and the transfer of information, and quantifies the exchange of information between systems. Transitive entropy can essentially describe the causal relationship caused by the flow of information, and has been widely used in disciplines such as neurology [18] and economics [19]. In the field of process industry, Bauer et al. [20] applied the method of transfer entropy to the study of disturbance propagation path maps of chemical processes to find the root cause of the disturbance and proposed a method to optimize the transfer entropy parameters based on historical data. Taking the causality process between some known variables as the goal, using historical data to optimize the parameters of transfer entropy, and then applying the optimized parameters to the detection example of the causality of unknown variables, the transfer entropy can detect the variables. Cause and effect relationship, construct causality diagram through causality and then analyze the root cause of disturbance. Based on this, scholars have proposed many improved transfer entropy algorithms. Vakorin et al. [21] combined the multivariate transfer entropy algorithm with the Granger causality algorithm and proposed Partial Transfer Entropy (PTE) to detect multivariate variables. The indirect coupling relationship between them improves the sensitivity of the algorithm to detect causality between variables, and proves that the indirect coupling relationship is the main interference in the process of detecting causality. Dks et al. [22] combined Partial Symbolic Transfer Entropy (PSTE) with STE and PTE. Partial symbolic transfer entropy improves the anti-interference ability. Duan et al. [23] proposed Direct Transfer Entropy (DTE) on this basis. Direct transfer entropy is used to detect the direct causality between variables. This method can detect whether there is a direct cause and effect between variables in the system. This paper proposes discrete direct transfer entropy and differential direct transfer entropy for two discrete and continuous random systems, and analyzes the conversion relationship between them. Yu et al. [24] combined the chemical industry process with numerical alarm sequence data to perform the transfer entropy calculation, avoiding the use of kernel density to estimate joint probability density and conditional probability density, which can greatly reduce the complexity of the algorithm.

Mladenovic et al. [25] proposed the use of symbolic processing to reduce the number of computation operations in iterative-based simulation methods and accelerate their computation. The validity of the algorithm is verified by two examples. And estimates of second-order statistics in wireless channels. This method can be easily extrapolated to any other application that requires calculations in a single simulation run.

In recent years, symbolic data analysis has received widespread attention and has been applied in many research fields, including astrophysics and geophysics, biology and medicine, fluid flow, chemistry, mechanical systems, artificial intelligence, communication systems, and more recently Data mining and big data [26–28]. The basic step of

this method is to quantize the original data into the corresponding symbol sequence. The resulting time series is then treated as a transformed version of the original data in order to highlight its time information. In fact, this symbolization process has been shown to significantly improve the signal-to-noise ratio of some noisy time series [29]. In addition, compared with continuous value time series [30] processing, symbolic data analysis also makes communication and numerical calculations more efficient and effective.

2 Method

2.1 Transfer Entropy

Transfer entropy was proposed by Schreiber in 2000. It provides an information theory method to detect causality by measuring the reduction of uncertainty [22]. According to information theory, the formula for the transfer entropy from $X = [x_1, x_2, x_3, \dots, x_t, \dots, x_n]'$ to $Y = [y_1, y_2, y_3, \dots, y_t, \dots, y_n]$ is:

$$TE_{X \rightarrow Y} = \iint f(y_{t+h}, y_t^{(k)}, x_t^{(l)}) \log_2 \left(\frac{f(y_{t+h} | y_t^{(k)}, x_t^{(l)})}{f(y_{t+h} | y_t^{(k)})} \right) d\omega \quad (1)$$

Among them, x_t and y_t respectively represent the values of the variables x and y at the moment t ; and k l represent the order of the cause variable and the result variable, respectively, and x_t and the length of the l segment before it are defined as $x_t^{(l)} = [x_t, x_{t-\tau}, \dots, x_{t-(l-1)\tau}]$. Similarly, y_t and the k before it The segment length is defined as $y_t^{(k)} = [y_t, y_{t-\tau}, \dots, y_{t-(k-1)\tau}]$, τ is the sampling period, h is the prediction range, ω is a random variable, and f is a complete or conditional probability density function (PDF). In this TE method, a kernel function method or a histogram can be used to estimate the probability density function, which is a non-parametric method that can be used to fit a distribution of any shape.

In (1), x and y are two consecutive time series. Therefore, Eq. (1) does not apply to discrete time series. For discrete time series, however, the discrete transfer entropy of x to y is:

$$TE_{X \rightarrow Y} = \sum p(y_{t+h}, y_t^{(k)}, x_t^{(l)}) \log_2 \left(\frac{p(y_{t+h} | y_t^{(k)}, x_t^{(l)})}{p(y_{t+h} | y_t^{(k)})} \right) \quad (2)$$

Similarly, the discrete transfer entropy of y to x is:

$$TE_{Y \rightarrow X} = \sum p(x_{t+h}, x_t^{(k)}, y_t^{(l)}) \log_2 \left(\frac{p(x_{t+h} | x_t^{(k)}, y_t^{(l)})}{p(x_{t+h} | x_t^{(k)})} \right) \quad (3)$$

The meaning of the symbols is the same as in (1). y and x are discrete time series, $y_t^{(k)} = [y_t, y_{t-\tau}, \dots, y_{t-(k-1)\tau}]$ and $x_t^{(l)} = [x_t, x_{t-\tau}, \dots, x_{t-(l-1)\tau}]$ are also discrete time series, h is the prediction range, P is the complete or conditional probability density function, and k and l are the cause variables and The order of the resulting variable, τ is the sampling period.

2.2 Symbolic Transfer Entropy

We use a symbolic technique to estimate the transfer entropy. This technique has been introduced in the concept of permutation entropy. The symbol is defined by reordering the amplitude values of the time series x_i and y_i . For a given time series $X_i = \{x(i), x(i + l), \dots, x(i + (m - 1)l)\}$, sort the time series in ascending order of amplitude $\{x(i + (k_{i1} - 1)l) \leq x(i + (k_{i2} - 1)l) \leq \dots \leq x(i + (k_{im} - 1)l)\}$, if the amplitudes are equal, $x(i + (k_{i1} - 1)l) = x(i + (k_{i2} - 1)l)$, And $k_{i1} < k_{i2}$ is written as $x(i + (k_{i1} - 1)l) \leq x(i + (k_{i2} - 1)l)$ to ensure that each X_i uniquely maps to one of the possible arrangements of $m!$. Therefore, the sequence can be represented by a symbol: $\hat{x}_i \equiv (k_{i1}, k_{i2}, \dots, k_{im})$, and the relative frequency of the symbol is used to estimate the joint probability and conditional probability of the permutation index sequence. This reduces the calculation speed of transfer entropy. Given the symbol sequences $\{\hat{x}_i\}$ and $\{\hat{y}_i\}$, we define the symbol transfer entropy (STE) as:

$$T_{Y \rightarrow X}^S = \sum p(\hat{x}_{i+\delta}, \hat{x}_i, \hat{y}_i) \log \frac{p(\hat{x}_{i+\delta} | \hat{x}_i, \hat{y}_i)}{p(\hat{x}_{i+\delta} | \hat{x}_i)} \quad (4)$$

$$T_{X \rightarrow Y}^S = \sum p(\hat{y}_{i+\delta}, \hat{y}_i, \hat{x}_i) \log \frac{p(\hat{y}_{i+\delta} | \hat{y}_i, \hat{x}_i)}{p(\hat{y}_{i+\delta} | \hat{y}_i)} \quad (5)$$

The other symbols have the same meaning as in (1).

2.3 Significance Test

The value of the transfer entropy represents the causal relationship between the variables. If x is the cause variable of y , then the transfer entropy value between x and y must be greater than zero. If x is not the cause variable of y , it indicates that there is no relationship between them. There is a causal relationship, and the transfer entropy between x and y must be equal to zero. However, in the actual situation, the system is affected by various interferences and noises, so that the value of the transfer entropy will not be exactly equal to zero. Therefore, a threshold needs to be set as a standard to determine whether the obtained transfer entropy is significant. That is, the significance level of the transfer entropy result is judged by this threshold value. The transfer entropy result above the threshold value is significant, and the transfer entropy result below the threshold value is not significant. Kante et al. Reconstructed the sequence method by Monte Carlo It becomes a hypothesis test problem. The original hypothesis of the problem is that the value of the transfer entropy is not obvious. When reconstructing the sequence, it must be guaranteed that its statistical characteristics are unchanged. However, the chronological order of the values needs to be completely disrupted. The correlation between the original variables will be destroyed, so that the value of the new transfer entropy calculated by the reconstructed sequence meets the original hypothesis, and the threshold is constructed by the result of the new transfer entropy. If the transfer entropy between the variables is greater than the threshold, it is rejected. Null hypothesis.

In this paper, we choose the method of Duan et al. To scramble the sequence of x and y at the same time, and then construct a new sequence as follows:

$$\begin{cases} X^s = [X_i, X_{i+1}, \dots, X_{i+M-1}] \\ Y^s = [Y_j, Y_{j+1}, \dots, Y_{j+M-1}] \end{cases} \quad (6)$$

In the above formula, M is the number of samples in the new sequence, and the total number of samples in the original sequence is N . Therefore, the range of the values of i and j is $[1, N - M + 1]$, because the new sequence is a part directly selected from the original sequence. So if the original sequence is stationary, the statistical characteristics of the new sequence are basically the same as the original sequence. At the same time, in order to ensure that there is no correlation between the original sequence and the new sequence, i and j should satisfy $\|i - j\| \geq e$, and e is a sufficiently large integer, and its value should be much larger than the prediction range h of the transfer entropy. It is guaranteed that the sequences of two variables differ by a sufficient length, and the new sequence has two variables with a long time interval. It can be considered that the correlation between the two variables is eliminated by the long time interval between the variables. Thus, two variable sequences without causality are obtained, so that many groups of such sequences can be obtained repeatedly and the transfer entropy value of the sequence is calculated as te_s , which is put into NTE. $NTE = [te_1, te_2, \dots, te_s]$, the significance threshold is calculated as follows:

$$S_{Y \rightarrow X} = \mu_{NTE} + 3\sigma_{NTE} \quad (7)$$

Where μ_{NTE} is the mean of NTE and σ_{NTE} is the standard deviation of NTE.

3 Case Study

This section uses the Tennessee Eastman (TE) process as an example to conduct an experimental study to illustrate the specific modeling process of a causal network modeling method based on symbolic transfer entropy. A causal network model in the TE process is established. To verify the effectiveness of the proposed method in fault diagnosis.

The duration of each TE process simulation is 48 h. In the initial stage, the system runs normally, and then the fault is introduced into the simulation after the simulation has reached the 8th hour. Therefore, each simulation will generate 960 observation samples, of which the first 160 samples are data running in the normal state, and the last 800 samples are faulty data.

In the simulation, the fault 7 in the TE process is due to the pressure loss in C in stream 4. When this fault occurs, the flow ($\times 4$) in stream 4 will decrease instantly due to the pressure loss. Subsequently, when the controller detects this change, it will compensate for the drop in flow due to pressure loss by controlling the valve-related variable ($\times 45$) of the feed in flow 4 to increase its opening. After a period of oscillating adjustment, the flow in stream 4 will return to the original stable value, and the valve feeding the control stream 4 will undergo a step change and then stabilize to a new stable value. Therefore, both variables can be used as the source of fault 7. During the time period that the controller adjusts after the fault occurs, the fault will propagate between the variables, causing a smear effect, causing many other variables not related to the fault to oscillate and be detected as fault variables by the system. For example, product stripper pressure ($\times 16$), separator pressure ($\times 13$), reactor cooling water outlet temperature ($\times 21$), reactor pressure ($\times 7$), separator cooling water outlet temperature ($\times 22$), and two cooling units. Water flow ($\times 51$ and $\times 52$) and so on.

Firstly, the variables that need to be subjected to the transfer entropy causality analysis are selected based on the contribution map of the PCA. The relevant literature has also selected this, so we directly use the variables in the previous literature here, and find that our selected variables are the same as before. The analysis above is basically the same. Next, select the data segment. There are two places in this article to select the data segment. One is to select the data segment when calculating the transfer entropy value, and the other is to select the data segment when using the Monte Carlo method for significance test. In many experiments, 959 sample points were selected when calculating the transfer entropy value, and 400 sample points were selected for the significance test. The results of the method using symbolic transfer entropy are analyzed as follows (Table 1):

Table 1. Transfer entropy results for fault 7

| | To 4 | To 21 | To 22 | To 45 | To 51 |
|---------|------------------|------------------|------------------|------------------|------------------|
| From 4 | | 2.6467 2.7384 | 2.7881 2.7399 | 3.0239 2.7355 | 2.7425 2.7395 |
| From 21 | 2.6200 2.7503 | | 2.5654 2.7315 | 2.8937 2.7400 | 2.7531 2.7394 |
| From 22 | 2.6815 2.7480 | 2.4767 2.7346 | | 2.4948 2.7491 | 2.7769 2.7383 |
| From 45 | 3.1306 2.7313 | 3.0605 2.7255 | 2.7993 2.7476 | | 2.7894 2.7370 |
| From 51 | 2.7153 2.7290 | 2.7022 2.7399 | 2.8067 2.7425 | 2.4988 2.7397 | |

The above table calculates the transfer entropy value between the two variables selected by the PCA contribution map. The two rows of data in each cell in the table above, the first row of data represents the transfer entropy value, and the second row of data represents the significance of Monte Carlo. The threshold of the transfer entropy between two variables calculated by the qualitative method, only when the value of the transfer entropy is greater than the Monte Carlo threshold, it indicates that there is a causal relationship between the two variables. The figure is shown as follows (Fig. 1):

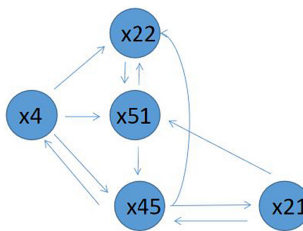


Fig. 1. Path propagation diagram of fault 7

As can be seen from the path diagram, when fault 7 occurs, C in stream 4 experiences a pressure loss, which causes the flow ($\times 4$) in stream 4 to decrease instantaneously. When the controller detects this change, it will control the correlation The valve variable ($\times 45$) of the feedstock and the flow rate of the incoming cooling water ($\times 51$) will also change. The change in the feed amount ($\times 45$) of the relevant valve will affect the change of the reactor cooling water temperature ($\times 21$). In turn, The change of the reactor cooling water temperature ($\times 21$) will also affect the change of the feed amount ($\times 45$). Therefore, $\times 45$ and $\times 21$ affect each other. And the change of the reactor cooling water temperature ($\times 21$) can also affect the cooling water flow rate ($\times 51$) of the subsequent separator system. It can be seen that the fault propagation path diagram is basically consistent with the propagation in the actual process. It also proves the validity of the symbol transfer entropy and saves a lot of calculation time. Since the original signal is symbolized, the anti-interference ability is more Well, the effect of noise is effectively reduced, and the accuracy of the resulting propagation path diagram is improved. It can be clearly seen from the diagram that the root cause of the fault 7 is $\times 4$. This is in sign with the actual situation, and the above illustrates the effectiveness and efficiency of the symbolic entropy.

4 Conclusion

This paper proposes a model-free approach to transfer entropy based on data-driven information theory, capable of calculating causal relationships between variables, and has been used in the fields of modeling, monitoring, and fault diagnosis of complex industrial processes. The symbolized signal is used to find the value of the transfer entropy between two variables. It is also necessary to calculate a threshold between the two variables by using the Monte Carlo significance test method. When the transfer entropy value between the variables is greater than the threshold, it indicates that there is a causal relationship between the two variables, so that the causal relationship can be obtained. Through the simulation and the actual industrial case TE (Tennessee-Eastman) process, the simulated fault 7 is simulated, and the causality diagram is drawn by using the method of symbolic transfer entropy, and the correct root cause of the fault is found from the causality diagram.

References

1. Reis, M., Gins, G.: Industrial process monitoring in the big data/industry 4.0 era: from detection, to diagnosis, to prognosis. *Processes* **5**(3), 35 (2017)
2. Bauer, M., Cox, J.W., Caveness, M.H., Downs, J.J., Thornhill, N.F.: Finding the direction of disturbance propagation in a chemical process using transfer entropy. *IEEE Trans. Control Syst. Technol.* **15**(1), 12–21 (2007)
3. Wakefield, B.J., Lindner, B.S., McCoy, J.T., Auret, L.: Monitoring of a simulated milling circuit: fault diagnosis and economic impact. *Miner. Eng.* **120**, 132–151 (2018)
4. Shu, Y., Zhao, J.: Data-driven causal inference based on a modified transfer entropy. *Comput. Aided Chem. Eng.* **31**, 1256–1260 (2012)

5. Landman, R., Jamsa-Jounela, S.L.: Hybrid approach to casual analysis on a complex industrial system based on transfer entropy in conjunction with process connectivity information. *Control Eng. Pract.* **53**, 14–23 (2016)
6. Naghoosi, E., Huang, B., Domlan, E., Kadali, R.: Information transfer methods in causality analysis of process variables with an industrial application. *J. Process Control* **23**(9), 1296–1305 (2013)
7. Hajihosseini, P., Salahshoor, K., Moshiri, B.: Process fault isolation based on transfer entropy algorithm. *ISA Trans.* **53**(2), 230–240 (2014)
8. Duan, P., Yang, F., Chen, T., Shah, S.L.: Direct causality detection via the transfer entropy approach. *IEEE Trans. Control Syst. Technol.* **21**(6), 2052–2066 (2013)
9. Duan, P., Yang, F., Shah, S., Chen, T.: Transfer zero-entropy and its application for capturing cause and effect relationship between variables. *IEEE Trans. Control Syst. Technol.* **23**(3), 855–867 (2015)
10. Lindner, B., Chioua, M., Groenewald, J.W.D., Auret, L., Bauer, M.: Diagnosis of oscillations in an industrial mineral process using transfer entropy and nonlinearity index. In: *Proceedings of the 10th IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes*, Warsaw, Poland (2018)
11. Landman, R., Kortela, J., Sun, Q., Jamsa-Jounela, S.L.: Fault propagation analysis of oscillations in control loops using data-driven causality and plant connectivity. *Comput. Chem. Eng.* **71**, 446–456 (2014)
12. Yuan, T., Qin, S.J.: Root cause diagnosis of plant-wide oscillations using Granger causality. *J. Process Control* **24**(2), 450–459 (2014)
13. Zhang, L., Zheng, J., Xia, C.: Propagation analysis of plant-wide oscillations using partial directed coherence. *J. Chem. Eng. Jpn* **48**(9), 766–773 (2015)
14. Bauer, M., Thornhill, N.F.: A practical method for identifying the propagation path of plant-wide disturbances. *J. Process Control* **18**(7–8), 707–719 (2008)
15. Luo, L., Cheng, F., Qiu, T., Zhao, J.: Refined convergent cross-mapping for disturbance propagation analysis of chemical processes. *Comput. Chem. Eng.* **106**, 1–16 (2017)
16. Shannon, C.E.: A mathematical theory of communication. *Bell Syst. Tech. J.* **27**(4), 623–656 (1948)
17. Schreiber, T.: Measuring information transfer. *Phys. Rev. Lett.* **85**(2), 461–464 (2000)
18. Choi, H.: Localization and regularization of normalized transfer entropy. *Neurocomputing* **139**, 408–414 (2014)
19. Sensoy, A., Sobaci, C., Sensoy, S., et al.: Effective transfer entropy approach to information flow between exchange rates and stock markets. *Chaos, Solitons Fractals* **68**, 180–185 (2014)
20. Bauer, M., Cox, J.W., Caveness, M.H., et al.: Finding the direction of disturbance propagation in a chemical process using transfer entropy. *IEEE Trans. Control Syst. Technol.* **15**(1), 12–21 (2007)
21. Vakorin, V.A., Krakovska, O.A.: Confounding effects of indirect connections on causality estimation. *J. Neurosci. Methods* **1844**(1), 152–160 (2009)
22. Diks, C.G.H., Papan, A., Kyrsov, K., et al.: Partial Symbolic Transfer Entropy. *CeNDEF Working Papers*, vol. 13–16 (2013)
23. Duan, P., Yang, F., Chen, T., et al.: Direct causality detection via the transfer entropy approach. *IEEE Trans. Control Syst. Technol.* **21**(6), 2052–2066 (2013)
24. Yu, W., Yang, F.: Detection of causality between process variables based on industrial alarm data using transfer entropy. *Entropy* **17**(8), 5868–5887 (2015)
25. Mladenovic, V., Milosevic, D., Lutovac, M., Cen, Y., Debevc, M.: An operation reduction using fast computation of an iteration-based simulation method with microsimulation-semisymbolic analysis. *Entropy* **20**, 62 (2018)
26. Daw, C., Finney, C., Tracy, E.: A review of symbolic analysis of experimental data. *Rev. Sci. Instrum.* **74**, 915–930 (2003)

27. Amigó, J.M., Keller, K., Unakafova, V.A.: Ordinal symbolic analysis and its application to biomedical recordings. *Philos. Trans. A Math. Phys. Eng. Sci.* **373**(2034), 20140091 (2015)
28. Susto, G.A., Cenedese, A., Terzi, M.: Time-series classification methods: review and applications to power systems data. In: *Big Data Application in Power Systems*. Elsevier, Amsterdam, The Netherlands (2017)
29. Graben, P.: Estimating and improving the signal-to-noise ratio of time series by symbolic dynamics. *Phys. Rev. E Stat. Nonlin. Soft Matter Phys.* **64**(5), 051104 (2001)
30. Mukherjee, K., Ray, A.: State splitting and merging in probabilistic finite state automata for signal representation and analysis. *Signal Process.* **104**, 105–119 (2014)