Research on Source Detection and Its Performance Analysis in Sensor Array

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Abstract. In this paper, the application of information theory to describe the existing problem of signal source in sensor array is investigated in the presence of complex additive white Gaussian noise (CAWGN). Firstly, We derive the theoretical formula of constant modulus scattering signal detection information under the condition of target matching in a single source scenario based on the information theory approach (ITA) and the theoretical expressions between detection probability and false alarm probability. Then, according to the Neyman-Pearson (N-P) criterion, the detection probability and false alarm probability of the constant modulus scattering signal are derived separately and derive the corresponding detection information through existing methods. Finally, the simulations of detection information and receiver operating characteristics (ROC), according to the presented expressions, are carried out to compare the detection performance based on information theory and N-P criterion. Our analysis indicates that the theoretical detection performance of ITA can be obviously better than that of N-P criterion, which also verifies the reliability and effectiveness of ITA.

Keywords: Sensor array · Source detection · Information theory · Detection information · N-P criterion · ROC

1 Introduction

Since Shannon founded the information theory [1], great achievements have been made in the field of communication, which laid a solid foundation for the rapid development of communication. Kondo has studied relationship between the optimum threshold and mutual information for radar return signal [2]. Some researchers have studied the relationship between information theory and parameter estimation in sensor array [3]. The Sensor Array system may make measurements of a source in order to determine its unknown characteristics.

In the work of Woodward and Davies [4–6], information theory was considered applicable to radar measurement problems for the first time. Woodward and Davies studied the range mutual information with respect to SNR in the single target radar detection. Shi and Xu [7] have investigated the application of

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information theory to describe radar measurement problems and the theoretical expressions and an asymptotic upper bound for the scattering information and the range-DOA information are presented in a single target scenario.

Single source detection is the basis of single source research, and is the precondition of resolution and tracking. It should be noted that detection here refers to the statistical judgment of the existence of single source. There are a lot of researches on the source detection at home and abroad. It includes multi-target detection under the condition of complex Gaussian clutter by using the maximum likelihood ratio test method [8], exploiting results from detection theory for deriving fundamental limitations on resolution and obtaining general resolution technology not based on any specific [9], proposing a polarization optimization method based on glowworm swarm optimization (GSO) algorithm to select the polarization waveforms of the transmit array to maximize detection probability [10], proposing new algorithms to achieve superior network throughput performance over existing schemes [11], proposing IRS-assisted secure strategy to significantly boost the secrecy rate performance [12], presenting an improved beamforming method to overcome this shortcoming [13] and proposing channel simulator and emulator that can get more accurate and realistic Doppler frequency than those of the existing models [14].

N-P criterion, a special case of Bayesian criterion, is to maximize the detection probability P_D under the condition of the false alarm probability P_{FA} limited within a specified constant range. Ma and Wang [15] have developed an efficient Bayesian approach to target detection in Cognitive radar. The approach, which uses the N-P criterion with priori probability, can improve the target detection effectively.

Few papers involve relating information theory with source detection, let alone comparing the detection performance of ITA with N-P criterion, which are the core content of this paper. In the remainder of this paper, we first construct the receiving signal model of sensor array in Sect. 2. In Sect. 3, based on the previous research of sensor array detection information, we further discuss the detection information in constant modulus scattering signal. In Sect. 4, the corresponding expressions of detection probability and false alarm probability in constant modulus scattering signal are derived based on ITA, and the relationship between detection probability and false alarm probability is obtained through different approximation methods. In Sect. 5, on the basis of reference [2,16], the relationship between detection probability and false alarm probability is deduced by using N-P criterion in constant modulus scattering signal, and we got the corresponding detection information. In Sect. 6, the numerical results are presented, the detection performance of ITA and N-P criterion is analyzed, which shows the superiority of each method. Finally, in Sect. 7, the main results of this paper are discussed and concluded.

2 System Model

Assuming that the uniform linear array model is shown in Fig. 1., a narrowband far-field source impinging on the antenna array and elements. The received signals at m-th (m = 0, 1, . . . , M) array elements are expressed as

$$x_m(t) = s(t)ve^{j\omega_0\tau_m(\theta)} + w_m(t). \tag{1}$$

In (1), $s(t)$ is the source signal, ω_0 is the angular frequency of the carrier signal. v equal to 0 or 1 indicates that the target does not exist or exists. $\tau_m(\theta) = md\sin\theta/c$ represents the time delay of the source signal at the direction angle with the m-th array element, where d is the distance between any two adjacent elements, c is the propagation speed of the signal, and $w_m(t)$ represents the additive Gaussian white noise with noise power N_0 at the m-th array element.

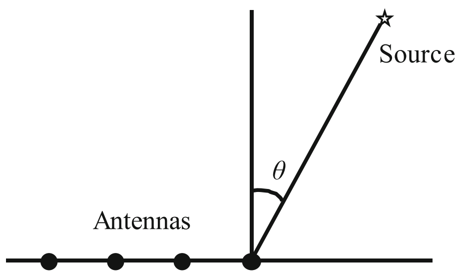

Fig. 1. System model.

Based on (1), a matrix equation is constructed as

$$\mathbf{x}(t) = v\mathbf{A}(\theta)s(t) + \mathbf{w}(t) \tag{2}$$

where $\mathbf{A}(\theta)$ is the transfer matrix between the source signal and the received signal

$$\mathbf{A}(\theta) = [\mathbf{a}(\theta)] = \begin{bmatrix} \exp(j\omega_0\tau_0(\theta)) \\ \exp(j\omega_0\tau_1(\theta)) \\ \vdots \\ \exp(j\omega_0\tau_{M-1}(\theta)) \end{bmatrix}. \tag{3}$$

Concisely, omitting time t, we rewrite (2) as

$$\mathbf{x} = v\mathbf{A}(\theta)s + \mathbf{w} \tag{4}$$

The source signal is given by $s = \alpha e^{j\varphi}$, where the scattering coefficient α takes one distribution, namely, constant distribution. And the spatial phase φ is uniformly distributed in the interval $[0, 2\pi]$, then the prior probability density is $p(\varphi) = 1/2\pi$. Suppose the scattering coefficient α is a constant, $s = \alpha e^{j\varphi}$ is a constant modulus scattering signal.

Resultantly, we will provide the proof of the above case in the following sections.

Here, we define the SNR as

$$\rho^2 = \frac{E\left(\alpha^2\right)}{N_0}. \tag{5}$$

Note that the second moment of noise is given by $E\left[\mathbf{w}\mathbf{w}^H\right] = N_0 I$.

Next, we introduce a basic assumption on the system model about the direction of angle (DOA) Θ. The source is uniformly distributed in the observation interval$\left[-\frac{|\Theta|}{2}, \frac{|\Theta|}{2}\right]$, where $|\Theta|$ denotes the observation range, the priori probability of Θ is given by

$$p(\theta) = 1/|\Theta|\,. \tag{6}$$

3 Detection Information of Source

We can define the existence information as the mutual information between the spatial existence status and the received signal, i.e. $I(\mathbf{x}; v)$. According to information theory [1,17,18], the quantity of information obtainable from a sensor array observation is the difference of the entropies of the a priori and a posteriori probability distribution, that is

$$I(\mathbf{x}; v) = H(v) - H(v|\mathbf{x}) \tag{7}$$

where

$$\begin{aligned} H(v) &= -\mathrm{E}_v\left[\log_2 p\left(v\right)\right] \\ &= -p\left(0\right)\log_2 p\left(0\right) - p\left(1\right)\log_2 p\left(1\right) \end{aligned} \tag{8}$$

and

$$H(v|\mathbf{x}) = -\mathrm{E}_{\mathbf{x}}\left[\sum p(v|\mathbf{x})\log_2 p(v|\mathbf{x})\right]. \tag{9}$$

Note that $p\left(v\right)$ and $p(v|\mathbf{x})$ is the a priori and the a posteriori probability density function of, respectively.

Next, we will discuss existence information in constant modulus scattering signal.

3.1 Constant Modulus Scattering Signal

Constant modulus scattering means that the amplitude α of the scattering coefficient of the source is a constant, the phase φ of the scattering coefficient of the source is uniformly distributed over the interval $[0, 2\pi]$.

Under the given parameters Θ, V and Φ, the probability density function of the received signal $\mathbf{X}$ is expressed as

$$p\left(\mathbf{x}|\theta, \varphi, v\right) = \left(\frac{1}{\pi N_0}\right)^M \exp\left(-\frac{1}{N_0}\left(\mathbf{x} - v\mathbf{a}\left(\theta\right)s\right)^{\mathrm{H}}\left(\mathbf{x} - v\mathbf{a}\left(\theta\right)s\right)\right). \tag{10}$$

The joint probability density of V and $\mathbf{X}$ is

$$\begin{aligned} p(\mathbf{x}, v) &= \iint p(\mathbf{x}, \theta, \varphi, v)\, d\varphi d\theta \\ &= \iint p(\mathbf{x} | \theta, \varphi, v)\, p(\theta)\, p(\varphi)\, p(v)\, d\varphi d\theta \\ &= \frac{p(v)}{2\pi |\Theta|} \left(\frac{1}{\pi N_0}\right)^M \exp\left(-\frac{\left(\mathbf{x}^{\mathrm{H}}\mathbf{x} + \alpha^2 v M\right)}{N_0}\right) \int_{-|\Theta|/2}^{|\Theta|/2} \int_0^{2\pi} \exp\left(\frac{2\alpha v}{N_0} \Re\left(e^{-j\varphi} \mathbf{a}^{\mathrm{H}}(\theta)\, \mathbf{x}\right)\right) d\varphi d\theta \end{aligned} \tag{11}$$

where $\Re(\bullet)$ denotes taking the real part of a complex number.

The conditional probability distribution $p(v|\mathbf{x})$ can be obtained by Bayes formula,

$$\begin{aligned} p(v|\mathbf{x}) &= \frac{p(\mathbf{x}, v)}{p(\mathbf{x})} \\ &= \frac{p(\mathbf{x}, v)}{p(\mathbf{x}, 0) + p(\mathbf{x}, 1)} \\ &= \frac{p(v) \frac{1}{2\pi|\Theta|} \exp\left(-\frac{1}{N_0}\alpha^2 v M\right) \int_{-|\Theta|/2}^{|\Theta|/2} \int_0^{2\pi} \exp\left(\frac{2\alpha v}{N_0} \Re\left(e^{-j\varphi} \mathbf{a}^{\mathrm{H}}(\theta)\, \mathbf{x}\right)\right) d\varphi d\theta}{p(0) + p(1) \frac{1}{2\pi|\Theta|} \exp\left(-\frac{1}{N_0}\alpha^2 M\right) \int_{-|\Theta|/2}^{|\Theta|/2} \int_0^{2\pi} \exp\left(\frac{2\alpha}{N_0} \Re\left(e^{-j\varphi} \mathbf{a}^{\mathrm{H}}(\theta)\, \mathbf{x}\right)\right) d\varphi d\theta} \end{aligned} \tag{12}$$

where (12) is a posterior probability distribution of the existence of the source when the received signal is known.

As shown in Fig. 1, substituting (12) into (7), we can simulate the detection information curve of constant modulus scattering signal.

4 The Relationship Between Detection Probability and False Alarm Probability Based on ITA

In this section, we derive the theoretical expressions between detection probability and false alarm probability in a single source scenario respectively based on ITA. For simplicity, following $\mathbf{x}_1$ and $\mathbf{x}_0$ represent the presence and absence of the source in the actual received signal, respectively.

Theorem 1. *In constant modulus scattering signal model, the false alarm probability P_{FA} can be approximated as*

$$P_{FA} \approx p(1). \tag{13}$$

Proof. In constant modulus scattering signal model, according to (12) and the definition of false alarm probability, we get false alarm probability as

$$
\begin{aligned}
P_{FA} &= p(1|\mathbf{x}_0) = \frac{p(\mathbf{x}_0, 1)}{p(\mathbf{x}_0)} \\
&= \frac{p(1)\frac{1}{2\pi|\Theta|}\exp\left(-\frac{M\alpha^2}{N_0}\right)\int_{-|\Theta|/2}^{|\Theta|/2}\int_0^{2\pi}\exp\left(\frac{2\alpha}{N_0}\Re\left(e^{-j\varphi}\mathbf{a}^{\mathrm{H}}(\theta)\,\mathbf{w}\right)\right)d\varphi d\theta}{p(0)+p(1)\frac{1}{2\pi|\Theta|}\exp\left(-\frac{M\alpha^2}{N_0}\right)\int_{-|\Theta|/2}^{|\Theta|/2}\int_0^{2\pi}\exp\left(\frac{2\alpha}{N_0}\Re\left(e^{-j\varphi}\mathbf{a}^{\mathrm{H}}(\theta)\,\mathbf{w}\right)\right)d\varphi d\theta} \\
&= \frac{p(1)\frac{1}{2\pi|\Theta|}\exp\left(-\frac{M\alpha^2}{N_0}\right)\int_0^{2\pi}\int_{-|\Theta|/2}^{|\Theta|/2}\exp\left(\frac{2\alpha}{N_0}\Re\left(e^{-j\varphi}\mathbf{w}(\theta)\right)\right)d\theta d\varphi}{p(0)+p(1)\frac{1}{2\pi|\Theta|}\exp\left(-\frac{M\alpha^2}{N_0}\right)\int_0^{2\pi}\int_{-|\Theta|/2}^{|\Theta|/2}\exp\left(\frac{2\alpha}{N_0}\Re\left(e^{-j\varphi}\mathbf{w}(\theta)\right)\right)d\theta d\varphi}
\end{aligned} \tag{14}
$$

where $\mathbf{w}(\theta)$ equals $\mathbf{a}^{\mathrm{H}}(\theta)\,\mathbf{w}$, $\mathbf{w}(\theta)$ is a scalar. And there is no source signal in received signal.

Then we can approximate the main part of (14) to

$$
\begin{aligned}
&\frac{1}{|\Theta|}\int_{-|\Theta|/2}^{|\Theta|/2}\exp\left(\frac{2\alpha}{N_0}\Re\left(e^{-j\varphi}\mathbf{w}(\theta)\right)\right)d\theta \\
&= \frac{1}{|\Theta|}\int_{-|\Theta|/2}^{|\Theta|/2}\exp\left(\frac{2\alpha}{N_0}\left(\mathbf{w}_R(\theta)\cos\varphi + \mathbf{w}_I(\theta)\sin\varphi\right)\right)d\theta \\
&\approx \frac{1}{|\Theta|}\sum_{-|\Theta|/2}^{|\Theta|/2-1}\exp\left(\frac{2\alpha}{N_0}\left(\mathbf{w}_R(\theta)\cos\varphi + \mathbf{w}_I(\theta)\sin\varphi\right)\right) \\
&\approx E\left[\exp\left(\frac{2\alpha}{N_0}\left(\mathbf{w}_R(\theta)\cos\varphi + \mathbf{w}_I(\theta)\sin\varphi\right)\right)\right]
\end{aligned} \tag{15}
$$

where $\mathbf{w}_R(\theta)$ and $\mathbf{w}_I(\theta)$ denote the real and imaginary parts of $\mathbf{w}(\theta)$, respectively. $\mathbf{w}(\theta)$ obeys complex Gaussian distribution with zero mean and variance MN_0, so $\mathbf{w}_R(\theta)\cos\varphi + \mathbf{w}_I(\theta)\sin\varphi$ obeys complex Gaussian distribution with zero mean and variance $\frac{MN_0}{2}$. Then (15) can be further derived as

$$
\begin{aligned}
&E\left[\exp\left(\frac{2\alpha}{N_0}\left(\mathbf{w}_R(\theta)\cos\varphi + \mathbf{w}_I(\theta)\sin\varphi\right)\right)\right]^{\mathbf{w}_R(\theta)\cos\varphi+\mathbf{w}_I(\theta)\sin\varphi=\lambda} \\
&= \int_{-\infty}^{+\infty} e^{\frac{2\alpha}{N_0}\lambda}\frac{1}{\sqrt{\pi M N_0}}e^{-\frac{\lambda^2}{MN_0}}d\lambda \\
&= e^{\frac{M\alpha^2}{N_0}}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{\pi M N_0}}e^{-\frac{(\lambda-M\alpha)^2}{MN_0}}d\lambda \\
&= e^{\frac{M\alpha^2}{N_0}}.
\end{aligned} \tag{16}
$$

By substituting (16) into (14), we can easily obtain the result of (13).

In conclusion, we can obtain the expression of (13). And according to (13), we can find that the derivation of P_{FA} has nothing to do with whether the source signal exists or not in received signal $\mathbf{x}$.

Next, we will derive the theoretical expressions of detection probability and the approximate expressions of detection probability and false alarm probability in constant modulus scattering signal model.

According to above (12) and the definition of detection probability, we get detection probability as

$$P_D = p(1|\mathbf{x}_1) = \frac{p\left(1\right)\frac{1}{2\pi|\varTheta|}\exp\left(-\frac{M\alpha^2}{N_0}\right)\int_{-|\varTheta|/2}^{|\varTheta|/2}\int_0^{2\pi}\exp\left(\frac{2\alpha}{N_0}\Re\left(e^{-j\varphi}\mathbf{a}^{\mathrm{H}}\left(\theta\right)\mathbf{x}_1\right)\right)d\varphi d\theta}{1-p\left(1\right)+p\left(1\right)\frac{1}{2\pi|\varTheta|}\exp\left(-\frac{M\alpha^2}{N_0}\right)\int_{-|\varTheta|/2}^{|\varTheta|/2}\int_0^{2\pi}\exp\left(\frac{2\alpha}{N_0}\Re\left(e^{-j\varphi}\mathbf{a}^{\mathrm{H}}\left(\theta\right)\mathbf{x}_1\right)\right)d\varphi d\theta}. \tag{17}$$

Substituting (13) into (17), we can obtain the relationship between detection probability P_D and false alarm probability P_{FA} as

$$P_D = \frac{P_{FA}\frac{1}{2\pi|\varTheta|}\exp\left(-\frac{M\alpha^2}{N_0}\right)\int_{-|\varTheta|/2}^{|\varTheta|/2}\int_0^{2\pi}\exp\left(\frac{2\alpha}{N_0}\Re\left(e^{-j\varphi}\mathbf{a}^{\mathrm{H}}\left(\theta\right)\mathbf{x}_1\right)\right)d\varphi d\theta}{1-P_{FA}+P_{FA}\frac{1}{2\pi|\varTheta|}\exp\left(-\frac{M\alpha^2}{N_0}\right)\int_{-|\varTheta|/2}^{|\varTheta|/2}\int_0^{2\pi}\exp\left(\frac{2\alpha}{N_0}\Re\left(e^{-j\varphi}\mathbf{a}^{\mathrm{H}}\left(\theta\right)\mathbf{x}_1\right)\right)d\varphi d\theta}. \tag{18}$$

Next, we will proceed to the derivation of N-P criterion in constant modulus scattering signal model.

5 The Relationship Between Detection Probability and False Alarm Probability Under N-P Criterion

According to reference [16] and system model, we have the following two hypotheses

$$\begin{aligned} H_0 &: \mathbf{x} = \mathbf{w} \\ H_1 &: \mathbf{x} = \mathbf{A}\left(\theta\right)s + \mathbf{w} \end{aligned} \tag{19}$$

where H_0 represents the hypothesis that the source does not exist and H_1 represents the hypothesis that the source does exist.

Next, we will derive the theoretical expressions of detection probability and false alarm probability of the two scattering coefficient signals under the N-P criterion.

According to reference [16] and (17), in the constant modulus model under the N-P criterion, we have false alarm probability as

$$P_{FA} = \int_{T'}^{+\infty} p(\varUpsilon|H_0)d\varUpsilon = \exp\left(-\frac{{T'}^2}{PN_0}\right). \tag{20}$$

From the above (20), we can get the detection threshold $T' = \sqrt{-PN_0 \ln P_{FA}}$. Then we have detection probability

$$P_D = \int_{T'}^{+\infty} p(\varUpsilon|H_1)d\varUpsilon = Q_M\left(\sqrt{2M\rho^2}, \sqrt{-2\ln P_{FA}}\right) \tag{21}$$

where $Q_M(\cdot)$ represents the Q function of Marcum.

Equation (21) indicates the relationship between the detection probability and the false alarm probability of single source under the N-P criterion.

6 Detection Information Under N-P Criterion

When the source position is known, this paper adopts the relationship between the detection information of the NP detector and the false alarm probability and the prior probability proposed in the reference [2],

$$\begin{aligned}
&I\left(V;\hat{V}\right)=H\left(V\right)-H\left(V\left|\hat{V}\right.\right)\\
&=-p\left(1\right)\log p\left(1\right)-\left(1-p\left(1\right)\right)\log\left(1-p\left(1\right)\right)\\
&-\left(p\left(1\right)P_D-p\left(1\right)P_{FA}+P_{FA}\right)\left(-A\log A-\left(1-A\right)\log\left(1-A\right)\right)\\
&-\left(1-p\left(1\right)P_D+p\left(1\right)P_{FA}-P_{FA}\right)\left(-D\log D-\left(1-D\right)\log\left(1-D\right)\right)
\end{aligned}\tag{22}$$

where V indicates whether a source is really exist or not and $\hat{V}$ indicates a array receiving signal expressed in stochastic process,

$$A=\frac{p\left(1\right)P_D}{p\left(1\right)P_D-p\left(1\right)P_{FA}+P_{FA}}\tag{23}$$

indicates source real existence probability under the decision signal "source existence" and

$$D=\frac{\left(1-p\left(1\right)\right)\left(1-P_{FA}\right)}{1-p\left(1\right)P_D+p\left(1\right)P_{FA}-P_{FA}}\tag{24}$$

indicates source non-existence probability under the decision signal "source non-existence".

In the next section, we will simulate the ROC curves and detection information curves under N-P criterion.

7 Numerical Results

The simulation parameters are set as follows: the actual value of DOA in a single source scenario is $\theta_0=0^{\circ}$ (that is from the normal direction of the sensor arrays), and the observation internal is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. The number of antenna M is set as 16 in the CAWGN environment. The value of the reflection coefficient is 1 when α is a constant.

7.1 Detection Information of Constant Modulus Scattering Signal

In Fig. 2, the prior probability of v is set equal and the detection information in a single source detection is illustrated, where s is a constant modulus scattering signal.

The blue solid curve represents the detection information curve of ITA. And the orange virtual curve represents the reachable region of detection information and SNR for any false alarm probability according to the detection information expression of N-P criterion. We can clearly observe the change trend of detection information in different SNR intervals obtained by sensor array under ITA and N-P criterion.

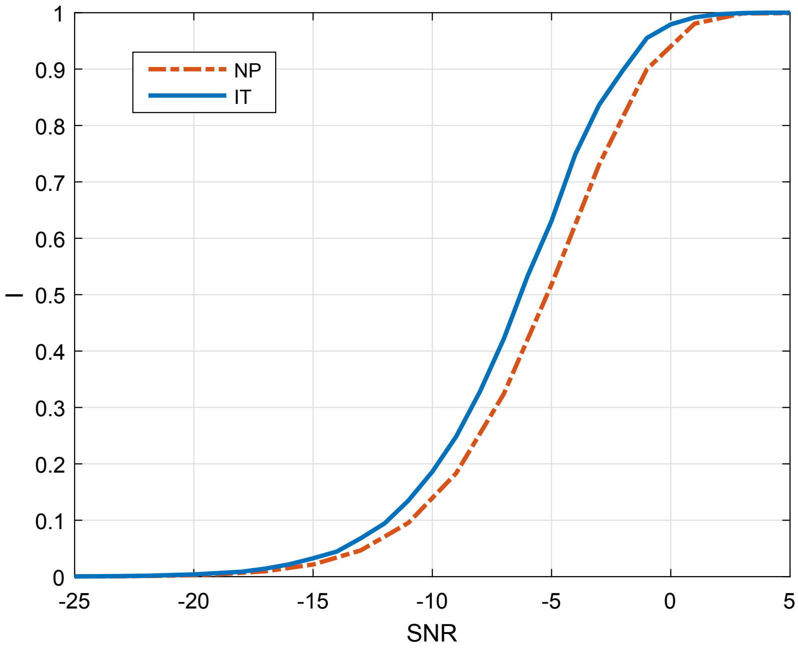

Fig. 2. Relationship between detection information of source and SNR

In the case of ITA, detection information quickly reaches the theoretical maximum value 1 from 0. And The blue curve is higher than the orange curve in the whole process of SNR change, which indicates that the detection performance of ITA is better than that of N-P criterion with the change of SNR.

7.2 The ROC Curves Under ITA and N-P Criterion for Constant Modulus Scattering Signal

According to the detection information curve of source in Fig. 2 and ROC simulation results, we take −20 dB to 0 dB as the observation interval, and select some representative curves to reflect the general trend of ROC curve.

– 20 dB and −10 dB represent low and medium SNR interval respectively.

Whether it is a blue curve or an orange curve, under the same false alarm probability P_{FA}, the value P_D of the dotted curve is higher than that of the solid curve under the same SNR, which indicates that the performance of ITA is worse than that of N-P criterion in terms of receiver operating characteristics in the midsole SNR range (Fig. 3).

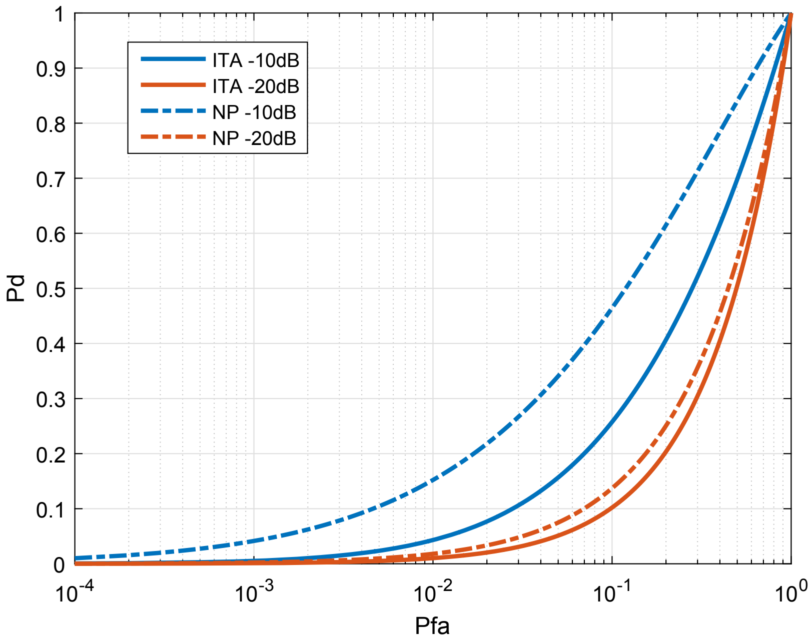

Fig. 3. ROC curves under ITA and N-P criterion for constant modulus scattering signal

7.3 The Relationship Between Detection Information and Prior Probability for Constant Modulus Scattering Signal

According to our previous research progress, combined with the research results of other researchers in related fields, we believe that the relationship between detection information and prior probability is more important than ROC.

In Fig. 2, the prior probability of v is set equal and the detection information in a single source detection is illustrated, where s is a constant modulus scattering signal.

According to the detection information curve of source in Fig. 2, we take -20 dB to 0 dB as the observation interval, and select some representative curves to reflect the general trend of detection information. Note that the prior probability is used as an independent variable instead of an equal probability distribution. Here, -20 dB and -10 dB represent low and medium SNR interval respectively.

The blue solid curve represents the detection information curve of ITA. And the orange virtual curve represents the reachable region of detection information and prior probability $p(1)$ for any false alarm probability P_{FA} according to the detection information expression of N-P criterion.

In Fig. 4 and Fig. 5, we can observe that the detection information value I of the blue solid curve is significantly greater than that of the orange dotted curve under the same prior probability $p(1)$ condition, which indicates that the detection performance of ITA is better than that of N-P criterion in the middle and low SNR range. This is predictable because the detection information is the theoretical value given by the information theory method.

Theoretical analysis shows that, even for small and weak sources, ITA has the possibility of detection, which is more beneficial to the detection of small and weak sources.

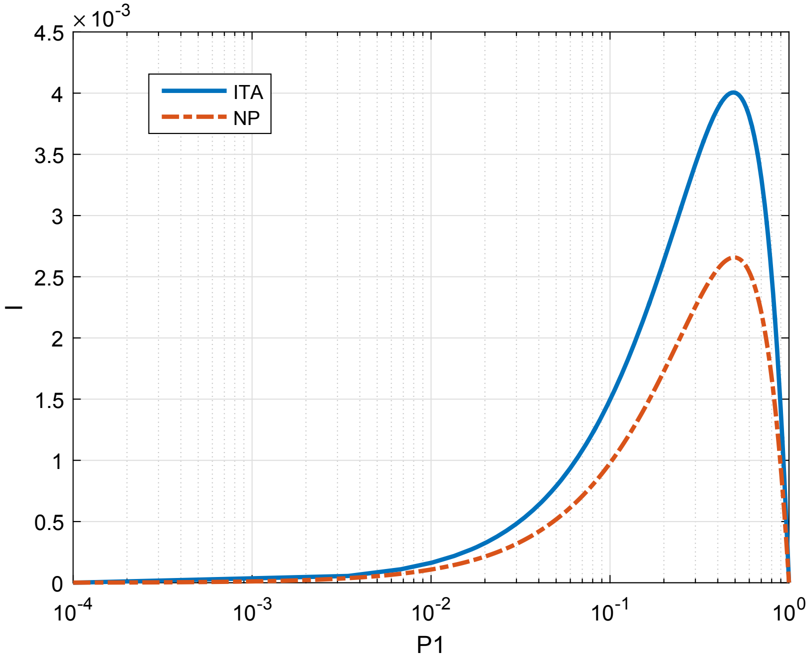

Fig. 4. SNR $= -20$ dB, the relationship between detection information and prior probability

The above comparative analyses further confirm the reliability and effectiveness of ITA in single source detection, and also provide important guiding significance for the selection of appropriate method for the detection of signals.

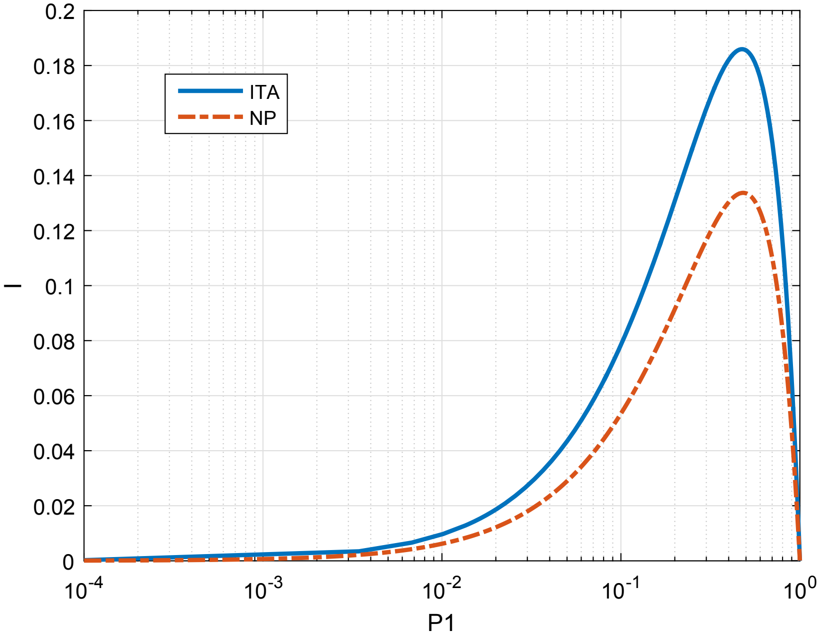

Fig. 5. SNR $= -10\,$dB, the relationship between detection information and prior probability

8 Conclusion

In this paper, we apply information theory to sensor array system in single source scenario, introduce the boolean variable v is to indicate whether the signal exists or not in the system model and unify source detection with parameter estimation. The closed form expressions between detection probability and false alarm probability have been derived respectively for constant modulus scattering signal. Theoretical analysis and numerical results are provided to corroborate that the detection information of ITA is better than N-P criterion, and the detection probability of N-P criterion is better than detection information criterion. At present, there is no final conclusion on which of the two criteria is the best, but our research work breaks the situation that N-P criterion dominates the whole country and opens up a new direction for the field of source detection. In addition, some issues such multi-sources detection with interference are worthy of further investigations.

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