



# Resource Allocation in Massive Non-Orthogonal Multiple Access System

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**Abstract.** With the popularity of large-scale communication scenarios, massive multiple access technology has attracted academic attention. In this paper, a resource allocation problem in massive Non-Orthogonal Multiple Access (NOMA) network is studied. The optimization goal is the energy efficiency (EE) of the system. The resource allocation problem includes two parts: subcarrier allocation and power allocation. Firstly, a many-to-one matching algorithm is proposed to assign subcarriers. Then, a power allocation algorithm is designed to optimize the objective function using successive convex approximation. Further, an iterative algorithm for power allocation is studied and the suboptimal solution of the function is obtained. Simulation results show that the proposed resource allocation scheme can effectively improve the EE of the system.

**Keywords:** Massive NOMA · Resource allocation · Energy efficiency

## 1 Introduction

With the rapidly development of the Internet, indicators such as EE, delay and reliability of the network have attracted more attention. With the popularization of B5G, massive multiple access is becoming more important due to its ability to improve spectrum efficiency [1, 2]. Compared with Code Division Multiple Access (CDMA) used in 3G [3], Orthogonal Frequency Division Multiple Access (OFDMA) used in 4G [4], NOMA, which greatly improves the spectrum efficiency and capacity of the system, can cope with the explosive growth of IIoT applications.

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In NOMA systems, Successive Interference Cancellation (SIC) applied to the receiving end can achieve high spectral efficiency at the cost of increasing the complexity of the receiver [5, 6]. Literature [7] uses the inherent characteristics of NOMA to solve the problems of non-orthogonal coordination direct transmission and relay transmission. In order to minimize the interruption probability of relay selection schemes, a two-stage relay selection strategy is proposed [8]. In order to maximize the throughput of the single-carrier MIMO-NOMA system, a suboptimal joint power allocation and precoding design are proposed [9]. In addition, two relay-assisted NOMA schemes were proposed to achieve low-delay and high-reliability for Vehicle-to-Everything (V2X) services [10]. Resource allocation was decentralized in a multi-antenna system with Full-Duplex (FD) base stations, to achieve simultaneous uplink and downlink transmissions [11].

In recent years, massive multiply access has been paid more attention. Literature [1] gives a general overview of massive access wireless communication, and describes the research results and progress of it at the current stage. Literature [12] studies unlicensed Massive-Device Multiple Access (MaDMA) Multiuser Detection (MUD) problem. Literature [13] explains requirements and challenges about massive Machine-Type Communications (MTC) application, and an overview of key technologies to overcome these.

Literature [14] proposes a massive NOMA technology, which is expected to support a large number of IoT devices in cellular networks. In literature [15], the information theoretical upper bound of the overall transmission rate is derived under the massive communication scenario in uplink. It proved the relationship between the optimal number of active devices and coherent time slots, and designed a two-stage practical communication framework. Literature [16] studies the capacity range of multi-input multi-output massive multiple channel, and used the information theory method based on Gallager error index analysis to characterize the finite dimensional region of the channels.

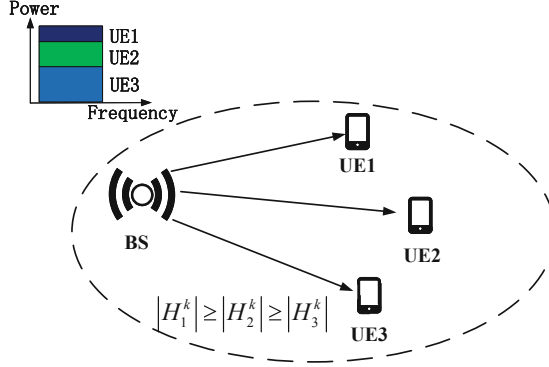
Most of the existing researches are focused on the limitations of massive NOMA systems. So we choose its EE as the optimization objective and optimize the objective function through the proposed resource allocation algorithm. Finally, the effectiveness of the proposed optimization method is proved by simulation.

The remainder of this paper is outlined as follows. In Sect. 2, we describe the system model. Section 3 addresses Resource Allocation Algorithm, while Sect. 4 demonstrates and analyze the simulation results. Finally, we summarize the conclusions of research in Sect. 5.

## 2 System Model

In this section, we describe the new downlink massive NOMA network setting consisting of a BS and  $N$  UEs, as shown in Fig. 1. Each of the nodes is equipped with a transmit antenna and a receive antenna. The system frequency band is divided into  $K$  subcarriers. The signals of different UEs or different packets of an UE can be superposed in one subcarrier to transmit simultaneously. Besides,

all the UEs are connected to the BS. The SIC commonly used in NOMA systems is used in the receiver continuation. We assume that the UEs follow independent Poisson point processes (PPPs) with the density of  $\lambda_u$ . The channels of the downlink stage are independent Rayleigh fading channels, and the path loss exponent is  $\alpha$ .



**Fig. 1.** A massive multiple access downlink system model

## 2.1 Signaling Model

At the downlink system, we take the  $n^{\text{th}}$  UE ( $n \in 1, 2, \dots, N$ ) as an example to analyze the signals. Suppose that the signal transmitted from the BS to the  $n^{\text{th}}$  UE on the  $k^{\text{th}}$  subcarrier is  $x_n^k$ , and the power of the signal is  $p_n^k$ . Additionally,  $h_n$  and  $\varpi_n$  denote the small-scale and the large-scale channel coefficients from the BS to the  $n^{\text{th}}$  UE. Then, the received signal on the  $n^{\text{th}}$  UE as

$$y_n^k = \sqrt{\varpi_n p_n^k} h_n^k x_n^k + \underbrace{\sum_{s \neq n}^N \sqrt{\varpi_s p_s^k} h_s^k x_s^k}_{\text{Interference from other UEs}} + z_n^k, \quad (1)$$

where  $z_n^k$  denotes the Additive White Gaussian Noise (AWGN) with mean zero and variance  $\sigma^2$  at the receiver of the  $n^{\text{th}}$  UE.

The SIC decoding sequence is determined by the channel coefficient. At the system, we assume that the channel coefficients follow  $|H_1^k| \geq |H_2^k| \geq \dots \geq |H_N^k|$ , where  $H_n^k = |h_n^k|^2 \varpi_n$ . UEs with poor channel conditions adopt higher transmission power to meet the Quality of Service (QoS), so the decoding sequence is arranged in ascending order according to the size of the channel coefficient. The Signal-to-Interference-plus-Noise Ratio (SINR) of the  $n^{\text{th}}$  UE is given by

$$\text{SINR}_n^k = \frac{|h_n^k|^2 \varpi_n p_n^k}{\sigma^2 + \sum_{s=1}^{n-1} |h_s^k|^2 \varpi_s p_s^k}. \quad (2)$$

Thus, the throughput of the  $n^{th}$  UE is

$$R = \sum_{k=1}^K \sum_{n=1}^N W c_n^k \log_2(1 + SINR_n), \quad (3)$$

where  $c_n^k$  indicates whether the  $n^{th}$  UE is served by the  $k^{th}$  subcarrier. Total power of the system is

$$P_t = P_m + P_o, \quad (4)$$

where  $P_m = \sum_{k=1}^K \sum_{n=1}^N p_n^k$ , which is the sum of the transmitted power,  $P_o$  is the power required for normal operation of the system, and it is determined. Therefore, the EE of the system can be expressed as the ratio of the total throughput to the overall energy consumption as follows

$$\max EE(p_1^k, p_2^k, \dots, p_n^k) = \frac{R}{P_t}. \quad (5)$$

## 2.2 Energy Efficiency Model

In this paper, the optimization objective is to maximize the EE of UEs in the system. The formula for the optimization function can be expressed as follows

$$\max EE(p_1^k, p_2^k, \dots, p_n^k) = \frac{\sum_{k=1}^K \sum_{n=1}^N W c_n^k \log_2 \left( 1 + \frac{|h_n^k|^2 \sigma_n p_n^k}{\sigma^2 + \sum_{s=1}^{n-1} |h_s^k|^2 \varpi_s p_s^k} \right)}{\sum_{k=1}^K \sum_{n=1}^N p_n^k + P_o}. \quad (6)$$

Due to the need for overall power constraints and QoS of UE in missive multiple access systems, constraints C1, C2 and C3 are set as

$$C1 : P_m \leq P_{max}, \quad (7)$$

$$C2 : R_n \geq R_{min}, \forall n, \quad (8)$$

$$C3 : c_n^k \in \{0, 1\}, \forall n, k$$

$$\sum_{n=1}^N c_n^k \leq \delta, \forall n, \quad (9)$$

where  $P_{max}$  is the upper limit of system transmission power,  $R_{min}$  is the minimum value of each UE under the condition that QoS of UE is guaranteed.  $\delta$  is the upper limit of the number of UE that can be served by a subcarrier. At the same time, in order to ensure the correct decoding of the receiver, the system also needs to meet the SIC decoding threshold, so the constraint C4 is set as

$$C4 : |h_n^k|^2 \varpi_n p_n^k - \sum_{s=1}^{n-1} |h_s^k|^2 \varpi_s p_s^k \geq p_{thr}, \forall n, k, \quad (10)$$

where  $p_{thr}$  is the decoding threshold of SIC.

### 3 Recourse Allocation Algorithm

The optimization objective is a non-convex function, so it is difficult to obtain the global optimal solution by conventional methods. In this paper, the optimization objective is divided into two sub-problems, namely, subcarrier allocation and power allocation. Finally, the suboptimal solution of the objective function is obtained by using iterative algorithm.

#### 3.1 Subcarrier Allocation

The priority of subcarriers is determined according to the channel coefficient, and the priority list of UE is determined according to the EE, their priority list is

$$\begin{aligned} \{SC\} &= \{SC_1, SC_2, \dots, SC_k, \dots, SC_K\}, \\ \{UE\} &= \{UE_1, UE_2, \dots, UE_n, \dots, UE_N\}, \end{aligned} \quad (11)$$

where  $\{SC\}$ ,  $\{UE\}$  represent the total priority list of the subcarriers and the total priority list of the UEs, respectively.  $\{SC_k\}$  represents the priority list of all UE corresponding to the  $k^{th}$  subcarrier,  $\{UE_n\}$  represents the priority list of all subcarriers corresponding to the  $n^{th}$  UE.  $\{UE^u\}$  represents UEs which have been unmatched. According to the literature [17], we propose a many-to-one matching algorithm to match the subcarriers and the UEs.

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#### Algorithm 1. Many-to-one Matching Algorithm

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- 1: Initializes the priority list of the subcarriers and UEs, initializes the unmatched list of UEs
  - 2: **while**  $\{UE^u\} \neq 0$  **do**
  - 3:   Each UE (represented by  $n$ ) sends matching request to the subcarriers (represented by  $k$ ) in order of priority from the highest to the lowest in  $\{UE^u\}$
  - 4:   **if** The number of UE serviced by  $k$  is less than  $\delta$  **then**
  - 5:     Match  $n$  with  $k$ , and take  $n$  out of  $\{UE^u\}$
  - 6:   **else**
  - 7:     Compare  $n$  with the UEs which subcarrier  $k$  have been matched according to  $\{SC_k\}$
  - 8:     **if** There is an UE whose priority is less than  $n$  **then**
  - 9:       Replace the lowest priority UE that subcarrier  $k$  has matched with  $n$ , put the replaced UE into  $\{UE^u\}$ , removing UE  $n$  from  $\{UE^u\}$
  - 10:    **else**
  - 11:     Subcarrier rejects the request from  $n$
  - 12:    **end if**
  - 13: **end if**
  - 14: **end while**
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### 3.2 Power Allocation

Since we have matched the subcarriers to the UEs in the last part, we can treat  $c_n^k$  as a definite value. In this paper, the following equation is always true

$$\log_2 \left( 1 + SINR_n^k \right) = \log_2 \left( \sigma^2 + \sum_{s=1}^n |h_s^k|^2 \varpi_s p_s^k \right) - \log_2 \left( \sigma^2 + \sum_{s=1}^{n-1} |h_s^k|^2 \varpi_s p_s^k \right). \tag{12}$$

We can define it as  $\log_2 \left( 1 + SINR_n^k \right) \triangleq f_n^k(p) - g_n^k(p)$ , in the same way, the formula of the throughput can be converted to

$$R = \sum_{k=1}^K \sum_{n=1}^N W c_n^k \log_2 \left( 1 + SINR_n \right) \triangleq F(p) - G(p). \tag{13}$$

Since  $f_n^k(p)$  and  $g_n^k(p)$  is convex,  $F(p)$ ,  $G(p)$  and  $P_t$  are all convex functions according to the properties of convex functions. The objective function can be expressed as

$$\max_p EE \left( p_1^k, p_2^k, \dots, p_n^k \right) = \frac{F(p) - G(p)}{P_t}. \tag{14}$$

Among all the constraints,  $C1$ ,  $C3$  and  $C4$  satisfy the definition of a convex function,  $C2$  can be represented as  $R_{\min} + W g_n^k(p) - W f_n^k(p) \leq 0$ , it is the difference form of two convex functions. At this time, the optimization objective is still A non-convex function. We can assume that a maximum EE is  $\varepsilon^*$ , and the optimization objective can be transformed into B, forming A differential convex function. Therefore, successive convex approximation can be used to calculate the suboptimal solution of the objective function.

$f_n^k(p)$  is a convex function, for any feasible point  $p^*$ , we have

$$f_n^k(p) \geq f_n^k(p^*) + f_n^k(p)'|_{p=p^*} (p - p^*) \triangleq f_n^k(p)^*, \tag{15}$$

where

$$f_n^k(p)'|_{p=p^*} (p - p^*) = \frac{\sum_{s=1}^n |h_s^k|^2 \varpi_s (p - p^*)}{\left( \sigma^2 + \sum_{s=1}^n |h_s^k|^2 \varpi_s p^* \right) \ln 2}. \tag{16}$$

So we can get

$$F(p) \geq \sum_{k=1}^K \sum_{n=1}^N W c_n^k f_n^k(p)^* \triangleq F(p)^*. \tag{17}$$

The optimization goal can be translated as

$$\begin{aligned} & \min_p \varepsilon^* P_t + G(p) - F(p)^* \\ & \text{s.t. } C1, C3, C4 \\ & C2 : R_{\min} + W g_n^k(p) - W f_n^k(p)^* \leq 0. \end{aligned} \tag{18}$$

The suboptimal solution of the function can be obtained by using the convex function solver. We propose an iterative algorithm to obtain the suboptimal solution of the function.

**Algorithm 2.** Iterative Algorithm for Power Allocation

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- 1: Set iteration index  $\alpha = 1$ , Set the maximum number of iterations is  $\alpha_{max}$
  - 2: Initializing  $\varepsilon_1^*$ , randomly taking a viable solution to  $\varepsilon_1^*$
  - 3: **while**  $\varepsilon_n^* < \varepsilon_{n+1}^*$  and  $\alpha < \alpha_{max}$  **do**
  - 4:   Use the standard convex function solver to solve equation (18) and obtain the value of  $p_n^*$
  - 5:   Using the obtained value of  $p_n^*$  and equation (6), update the value of  $\varepsilon_n^*$
  - 6:   Update iteration index, let  $\alpha = \alpha + 1$
  - 7: **end while**
  - 8: Get and output the suboptimal solution of the function.
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## 4 Simulation Results

**Table 1.** parameter setting

Parameters	Value
The system bandwidth	3 MHz
Carrier frequency	2 GHz
The subcarrier bandwidth	15 KHz
Path loss exponent $\alpha$	3
Noise power spectral density	-174 dBm/Hz
The density of UEs $\lambda_u$	200 per/Km <sup>2</sup>
The maximum power of BS	46 dBm
The target data rate of UEs	2 bps/Hz
The transmit power $P_m$	20 dBm
The system operating power $P_o$	20 dBm

In this section, the proposed resource allocation algorithm in the massive NOMA system is simulated and analyzed, and it is compared with other algorithms. We deploy a BS in an area with a radius of 500 m. The UE model is an independent PPP model with a density of  $\lambda_u$  which is randomly generated in the area, and the other simulation parameters are shown in Table 1.

In Fig. 2, we research EE versus the density of UEs when the subcarrier match with different maximum numbers of superposed signals. It can be clearly seen that with the increase of the density of UEs, the greater the value of  $\delta$  is, the greater EE will be. The rate of increase in EE is also proportional to the value of  $\delta$ . However, when the value of  $\delta$  increases to a certain extent, EE will decrease with the increase of the density of UEs. It is worth mentioning that when the value of  $\delta$  is different, the corresponding maximum EE will also be different, which is 8.7, 16.5, 19.3, and 23.9, respectively.

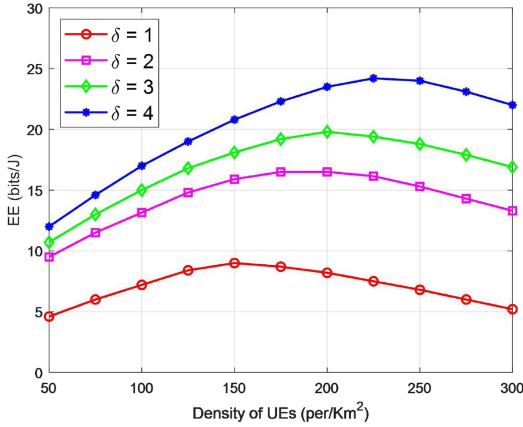


Fig. 2. EE versus the density of UEs with different value of  $\delta$

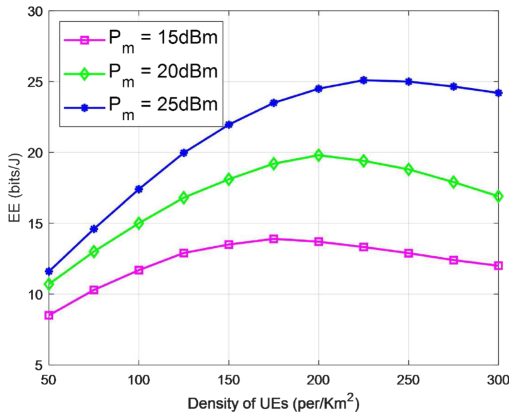


Fig. 3. EE versus the density of UEs with different maximum of the transmit power

In Fig. 3, we compared the relationship between EE and the density of UEs under different maximum transmission power. It can be seen that, with the increase of the density of UEs, the value of EE increases first and then decreases, and the increase speed is proportional to the size of the maximum transmission power. However, when the density of UEs is between 200 and 300, the value of EE corresponding to  $P_{m2} = 20$  dBm decreases the fastest.

In Fig. 4 and Fig. 5, we compare different algorithms under different systems. Among them, the subcarrier of OFDMA system can serve three UEs, and there is interference between the UEs, and the optimization goal is to maximize the EE of the system. The subcarriers in the NOMA system can also serve three UEs, but the optimization goal is to maximize the throughput of the system.

Figure 4 studies the EE versus the density if UEs under different algorithms. With the increase of the density of UEs, EE in NOMA system grows at a rate

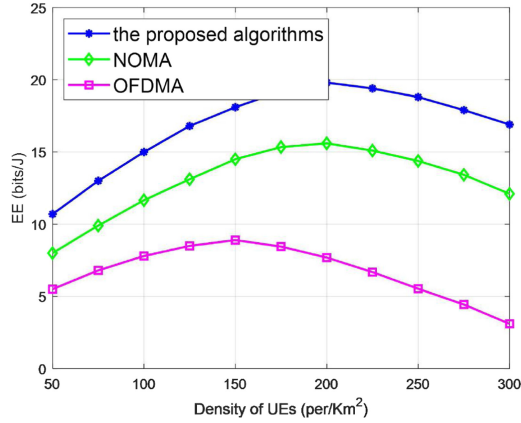


Fig. 4. EE versus the density of UEs in different algorithms

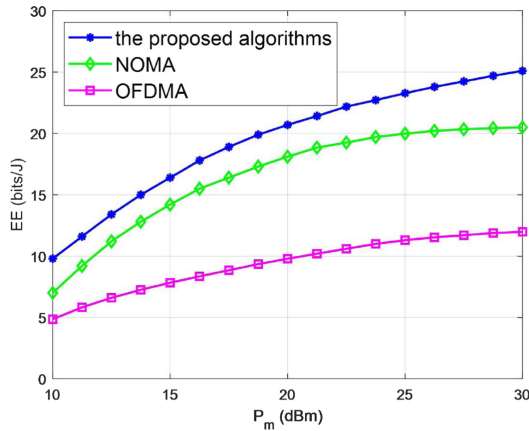


Fig. 5. EE versus the maximum of the transmit power in different algorithms

almost equal to that of the proposed algorithm, but EE in OFDMA system grows at the slowest rate and declines at the fastest. As shown in Fig. 5, we research the EE versus the maximum transmitted power under different algorithms, which shows that EE of three algorithms increase monotonically with the increase of the maximum transmitted power in the range of  $P_m$  is 10 dBm to 30 dBm. When  $P_m$  is less than 16.25 dBm, the growth rate of EE in NOMA system is the fastest; when  $P_m$  is more than 20 dBm, the growth rate of EE in the proposed algorithm is the fastest. We can see that the proposed algorithm greatly improves the EE of the system.

## 5 Conclusion

In this article, we have taken the massive NOMA as the background, the optimization goal is the EE of the system. The objective function is optimized through the proposed resource allocation algorithm. The resource allocation algorithm includes two parts, the first part is the subcarrier allocation, we have proposed a many-to-one matching algorithm for subcarrier allocation which is user-centered. The second part is power allocation which is carried out on the basis of subcarrier allocation. The power allocation algorithm uses successive convex approximation, transformed the optimization goal to a convex function. And then a iterative algorithm is designed to obtain the suboptimal solution of the function. Finally, simulation results show that the proposed optimization method can effectively improve EE in the system by comparing with other algorithms and systems.

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