Game Theoretic Analysis of Resource Allocation in Multi-tiered Networks

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Abstract. We study resource allocation in wireless multi-tiered OFDMA cellular systems, like the current 5G networks. We compare coordinated versus distributed methods of assignment of resource blocks to maximize signal strength and throughput to ensure quality of service. The distributed assignment, where every wireless client optimizes its utility, is modeled by a strategic game with the cellular users as players. We study the existence of Nash equilibrium in this game and provide bounds for the inefficiency of this strategic game via analysis of the Price of Anarchy (PoA). Our social objective is chosen to provide fair throughput by measuring the minimum data rate. We show that approximating the social optimum, i.e. maximizing the minimum data rate, to within any given factor is NP-Hard and provide an algorithm that is illustrated to perform well in practice.

Keywords: Multi-tiered networks · Price of anarchy · Nash equilibrium

1 Introduction

Next generation cellular networks, that include 5G networks, provide enhanced performance with high speeds for downlinks and uplinks via a multi-tiered architecture. The architecture utilizes macrocells, licensed small cells and device-to-device networks [3] integrating use of multiple radio access technologies amongst others.

In the context of a multi-tiered architecture, with increased number of users and connected devices [2], distributed methods that pair user equipment to base stations are becoming increasingly important keeping in view enhanced quality of service. In this paper, the resource allocation problem is investigated in the context of a multi-tier *orthogonal frequency division multiple access* (OFDMA) network. User equipment connect to base stations utilizing resource blocks corresponding to frequency and time partitions. We consider the performance of distributed systems that pair cellular users to resource blocks in order to optimize their own data rate.

A number of distributed methods for the resource allocation method have been proposed including the use of matching theory, factor graphs and optimization using auctions [3]. The approach of distributed allocation presents itself

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naturally as a game where users adopt strategies to maximize their benefit. Our goal in this paper is to analyze this game and the inefficiency that result from the strategic behavior of the players at Nash equilibrium. Our social objective is to maximize the minimum throughput, measured as a function of signal-to-interference-plus-noise ratio (SINR) that the players are able to achieve, so as to ensure a fair distributions of resources. We utilize the *price of anarchy* as a measure of the inefficiency that results from the strategic equilibrium choices in the game.

Prior work includes multiple papers that attempt to optimize resource allocation. A stable matching based algorithm is designed in [6] to allocate the resources to the uplink transmitters in a 5G cellular networks. Optimizing the power utilization has also been considered [7] especially in the context of device-to-device (D2D) networks, aimed at minimizing the transmission power of all the users in the network. The research does not account for multi-tiered architectures.

A resource allocation problem focused on D2D communications and formulated in the context of 5G Radio Access Networks has been considered in [9] and shown to be NP-hard. The optimization criterion is the sum of the capacities of all the communication links and a fair allocation problem has not been considered. The paper also proposed a many-to-one matching coalition sub-game to model the problem and determine stable solutions. Maximizing resource allocation in the context of Mobile Edge Computing (MEC) has been considered in [8].

Game theoretic modeling of resource allocation has also been considered in [4] where a Stackelberg game has been considered for power allocation in the context of soft frequency reuse (SFR), with an interference coordination method for decreasing inter-cell interference. Utilization of network slicing techniques [1] for virtualization and dynamically changing the resource allocation to optimize the quality of service and experience has also been considered.

1.1 Our Contributions

1. We formulate the resource allocation problem as a strategic game, the *Wireless Resource Game* with small cell users as players. Our work differs from previous work in considering mobile clients that are allowed to switch base stations.
2. We show that Nash equilibrium is not always possible and strategic play can result in an unstable system, when mobile clients are allowed to optimize their power levels and choice of base stations.
3. We provide bounds on the inefficiency of the strategic game as measured by the *price of anarchy* (PoA). The price of anarchy is dependent on the number of wireless clients and the ratio of the maximum to minimum signal strength. We illustrate the average behavior of the price of anarchy via a simulation that utilizes a geographically located system of small cell users and base stations.

4. We show that the optimum social objective problem is NP-Hard to approximate to within a factor of L where, theoretically, L can be arbitrary. We also show the performance of a simple swap based algorithm for determining a local optimum of the social objective function.

2 Network Model and Game Formulation

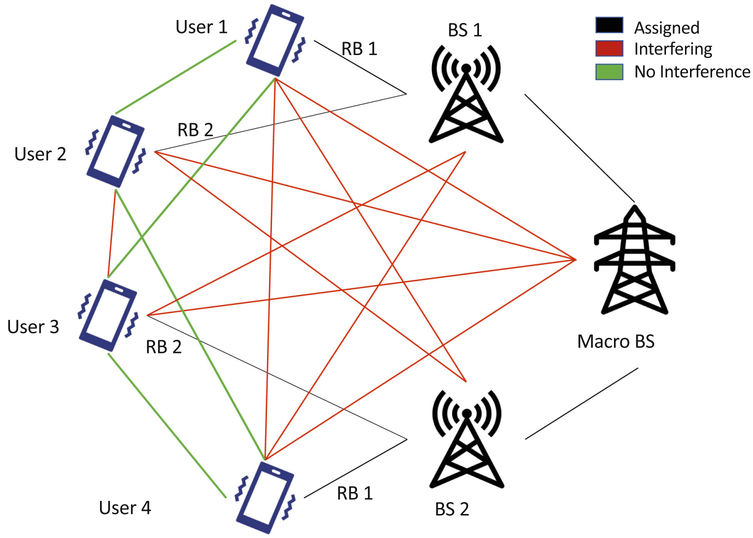


Fig. 1. Illustration of a multi-tiered cellular network

Our model for a 5G cellular wireless system includes a macro base station (MBS) and a set of small cell base stations (SBS) used by users' equipment (UEs). There is a set of fixed power levels and resource blocks (RBs) available to base stations and users. The objective of the resource allocation problem is to allocate resource blocks to small cell base station users to optimize signal quality (SINR), and hence the throughput capacity, while accounting for interference from other users and base stations.

In a distributed allocation system, each user equipment is allowed to choose a SBS for communication. This gives rise to local optimization (and selfish behavior) by the users which we model via a strategic game. Users select a resource block each, from among those available, in an effort to maximize their own spectral efficiency measured as a function of SINR, accounting for the possible interference from other users and MBS's. We study properties and characteristics of Nash equilibrium in these games, and compare the efficiency with a socially optimum fair assignment of users.

In our model, we let K be the set of small cell user equipments, where each small cell user is a transmitter belonging to set K^T and receiver belonging to K^R. The set of BSs is denoted by B. Also, small cell users are able to select the transmission power from a finite set of power levels and also choose RBs $R = \{1, 2, .., |R|\}$. The possible power over the RB for each transmitter k can be marked as the vector $P_k = [P_k^1, P_k^2, .., P_k^{|R|}]^T$ where $P_k^r = \{p_{k_1}^r, p_{k_2}^r..p_{k_l}^r\} \geq 0$ represents the power level of transmitter k over RB r which was selected from the set of power levels. And $p_k^r = 0$ if the RB r is not allocated to the transmitter. Based on the description of SBS utilization in [5] we assume that each RB in each SBS can be occupied by only one user.

As an example, consider the physical model illustrated in Fig. 1. The network consists of two SBSs (BS1 and BS2) and four small cell users. The available frequencies are divided over time periods into resource blocks. Users 1 and 2 select $RB1$ and $RB2$ on $BS1$ and users 3 and 4 select $RB2$ and $RB1$ on $BS2$ respectively. Thus, User 2 and 3 interfere with each other since they use the same resource block on two different base stations, as do users 1 and 4. That is, there is no interference between BSs or users when different RBs are being used, and interference arises due to simultaneous use of resource blocks between users or BSs.

The terms used in this paper are described in Table 1.

The SINR, denoted by $\gamma_{b,k}^r$, and the utility $u_{b,k}^r$ of a small cell user k occupying BS b and RB r are respectively defined as:

$$\gamma_{b,k}^r = \frac{\Gamma_{b,k}^r}{G_k^r - \Gamma_{b,k}^r} \text{ and } u_{b,k}^r = \log_2 (1 + \gamma_{b,k}^r) \tag{1}$$

A *Wireless Resource Game* is defined as $\mathcal{W}(K, B, R, U)$, where K is set of users, R is set of RBs, B is set of BSs, and U is the set of user utilities for a given allocation of users to resource blocks and base stations. We assume that the user utilizes her maximum power level, termed p_k^r, since there is no penalty incorporated for power usage in our current model. The strategies of the players are given by strategy profile $S = (S_k), \forall k$ where S_k is the strategy set of user k, given by the choice of the BS b and RB r, i.e., $S_k = \{(r_i, b_j)_{i \in R, j \in B}\}$. The utility function for each user k, $U_k : S \to \mathcal{R}^+$ represents the utility $u_{b,k}^r$ of the user when utilizing RB r on BS b, given the strategies of the other players. The user selects a BS and RB to maximize her utility based on the strategies of other small cell users. This gives rise to the following optimization problem for small cell user $k \in K$:

Optimization Problem 1:

$$\begin{aligned} &\max \sum_r \sum_b x_{k,b}^r \log_2 (1 + \gamma_{b,k}^r) \\ &s.t. \;\; \forall k : \sum_r \sum_b x_{k,b}^r \leq 1 \\ &\qquad \forall k, b, r : x_{k,b}^r \in \{0, 1\} \end{aligned} \tag{2}$$

Table 1. Table of parameters

Symb	Description
σ^2	noise
$g^r_{k,b}$	Channel gain between transmitter k and receiver b over RB r
p^r_k	Transmission power level of user k over RB r
p^r_b	Transmission power level of BS b over RB r
$\Gamma^r_{b,k}$	Signal strength at receiver k over RB r from transmitter b
$\Gamma^r_{M,k}$	Signal strength of interference at receiver k over RB r from MBS M
$\Gamma^r_{k',k}$	Signal strength of interference at receiver k over RB r from transmitter k' (on a separate BS)
G^r_k	Cumulative (sum of desired signal strength, interference, and noise) signal strength of received by user k over RB r
Γ_{min}	Lowest value of the signal strength of all users
Γ_{max}	Highest value of the signal strength of all users
R	Finite set of RBs $R = \{1, 2, \cdots, \|R\|\}$
B	Finite set of BSs $B = \{1, 2, \cdots, \|B\|\}$
b_k	BS that user k occupies
K_b	Users K that occupy BS b
I^r_{max}	Maximum interference in RB r
C^r_b	Capacity on BS b in RB r

$$\Gamma^r_{b,k} = g^r_{b,k} p^r_b; \quad \Gamma^r_{M,k} = g^r_{M,k} p^r_M$$

$$G^r_k = \sum_{k' \neq k} \Gamma^r_{k',k} + \sum_{b_{k'} \neq b} \Gamma^r_{b_{k'},k} + \Gamma^r_{b,k} + \Gamma^r_{M,k} + \sigma^2$$

$$\Gamma_{min} = \min_{k \in K^T, b \in B, r \in R, s \in S} \Gamma^r_{b,k}; \quad \Gamma_{max} = \max_{k \in K^T, b \in B, r \in R, s \in S} \Gamma^r_{b,k}$$

where $x^r_{k,b} = 1$ if small cell user k utilizes resource block r on the SBS b, while $x^r_{k,b} = 0$ otherwise. In this non-cooperative game for small cell users, the stable solution is a Nash equilibrium, NE (if it exists). A strategy profile S^{NE} is a Nash equilibrium strategy if:

$$U_k(S^{NE}_k, S^{NE}_{-k}) \geq U_k(S'_k, S^{NE}_{-k})$$

where S^{NE}_k is the resource utilization strategy for player k, $S^{'}_k$ is any other strategies besides Nash for user k and S^{NE}_{-k} is the strategy for the rest of the players at Nash equilibrium, $U_k = u^r_{b,k}$ when $x^r_{k,b}$ denotes users k's utility. Additional constraints on bandwidth capacity of the base stations and interference can be expressed as:

$$\begin{aligned} &\forall r, b : \sum_{k \in K_b} x^r_{k,b} \log_2 (1 + \gamma^r_{b,k}) \leq C^r_b \\ &\sum_{k' \in K^T; k' \neq k} g^r_{k',k} p^r_{k'} + \sum_{b' \neq b} g^r_{b',k} p^r_{b'} + g^r_{M,k} p^r_M \leq I^r_{max} \end{aligned} \tag{3}$$

We use Shannon's formula, $C = B * \log_2 (1 + \frac{S}{N})$, to relate SINR and link capacity.

We analyze the solution obtained from the Nash equilibrium via the social objective function that measures the minimum utility achieved by the users, i.e., $U^{soc} = \min_k \sum_b \sum_r x^r_{k,b} \log_2 (1 + \gamma^r_{b,k})$, where $x^r_{k,b}$ is the indicator function described above and U^{soc} is the social utility. This is termed the *fair* metric. The social optimum can be found using the following problem:

Optimization Problem 2: $\max \min_k \sum_b \sum_r x^r_{k,b} \log_2 (1 + \gamma^r_{b,k})$ with additional constraints as in Optimization Problem 1 and in (3)

3 Nash Equilibrium and Price of Anarchy

In this section we consider existence of Nash equilibrium. Unfortunately, the game can be unstable as we show below an instance where Nash equilibrium may not exist.

Claim. For the Wireless Resource Game, $\mathcal{W}(K, B, R, U)$ Nash Equilibrium does not always exist.

Proof. An example where Nash equilibrium does not exist is created using a random sampling Monte Carlo method. The example has 3 users and 2 RBs with only 3 BSs. We assume that there is only one MBS. The following values of interference signal strengths provide an example where no strategy is stable.

$$\begin{bmatrix} & k_1(r_1) & k_1(r_2) & k_2(r_1) & k_2(r_2) & k_3(r_1) & k_3(r_2) \\ k_1 & & & 22.92 & 28.24 & 21.94 & 16.50 \\ k_2 & 22.92 & 28.24 & & & 22.39 & 26.77 \\ k_3 & 21.94 & 16.50 & 22.39 & 26.77 & & \end{bmatrix}.$$

The value of noise is 0.30698 mW and assuming there is no interference from MBS.

In this example, there is no configuration that is a Nash equilibrium. For instance, if user k_1 occupies b_1 and r_1, k_2 occupies b_1 and r_2, k_3 occupies b_2 and r_1, then k_1 is willing to move to b_2 with r_2. Similarly, we can check that for any configuration, at least one user is willing to change its strategy in any of the configuration. □

Though the Nash equilibrium does not always exist in the general Wireless Resource Game, it does exist for special game classes as illustrated below:

Uniform Interference Property: In this case we assume that each client has the same signal strength for all RBs and BS, i.e. $\Gamma^r_{k,b} = a_k$ for all k, b, r, the same interference signal strength from other users, i.e. $\Gamma^r_{k',k} = b_k$ for all $k, k^{'}, r$ and the same interference signal strength from any BS, i.e. $\Gamma^r_{b',k} = c_k$ for all b', k, r. Also assume that the interference signal strength from the MBS is the same for all users, i.e. for all k, r, $\Gamma^r_{M,k} = a_M$.

Lemma 1. *A Nash equilibrium always exists for a Wireless Resource Game with Uniform interference property.*

Proof. Suppose a specific user is fine with a particular signal strength, and she is willing to move to any RB and BS as long as such condition has been satisfied. In this situation, for each user k, its signal strength is the same among all RBs and BSs $\Gamma^r_{k,b} = a_k$. We can claim that the user will switch to another RB and BS if and only if the load on other RBs is less than that of the user is currently in. In this game, $\frac{a_k}{g_k^{r_1} - a_k} \leq \frac{a_k}{g_k^{r_2} - a_k} \implies g_k^{r_1} \geq g_k^{r_2}$. Since the interference from all the other users and BSs are the same, $g_k^{r_1} = (n_{r_1} - 1) * (b_k + c_k) + a_m$ where n_{r_1} is the number of users on RB r_1. If user k is willing to move to r_2 from r_1, $n_{r_1} - 1 > n_{r_2}$. So let n as the list of the number of users on each RB in order from highest to lowest, i.e. $n = (n_1, n_2, ..., n_r)$ where n_i means i_{th} highest number of users among all RBs, we can find that after the users switch RBs the new list $n^{'} = (n^{'}_1, n^{'}_2, ..., n^{'}_r)$ is in a decreasing lexicographic order, which implies convergence to Nash equilibrium. □

3.1 Inefficiency of Equilibrium

We provide a bound on the inefficiency of the *Wireless Resource Game* using the *Price of Anarchy.* The PoA is defined as follows:

$$PoA = \inf_G \frac{\min_{NE(G)} \min\limits_k U_k(S_k^{NE(G)}, S_{-k}^{NE(G)})}{\min\limits_k U_k(S_k^*(G), S_{-k}^*(G))}$$

where G is a game with strategy profile S, $S_k^{NE}(G)$ is a Nash strategy for user k in game G, $S_k^*(G)$ is the social optimal strategy for user k in game G, $U_k(S)$ the utility of user k under the strategy profile S. We are using the fair metric, the social utility being defined as the minimum utility among all users.

The Theorem below illustrates the dependency on the ratio of the signal strength of the users and indicates that it should be kept bounded. It is also dependent on the number of users in the model.

Theorem 1. *1. The PoA of the Wireless Resource Game using the fair metric and without capacity and interference constraints (without Eq. 3) is lower bounded as*

$$PoA \geq \frac{1}{2|K|(\mathit{\Gamma}^m)^2(1+\log_2(\mathit{\Gamma}^m))} \text{ where } \mathit{\Gamma}^m = \frac{\mathit{\Gamma}_{max}}{\mathit{\Gamma}_{min}}.$$

2. The PoA of the Fair-Wireless Resource Game with capacity and interference constraints (Eq. 3) is lower bounded as

$$PoA \geq \frac{1}{C_{b_{max}}} \log_2(1 + \frac{\mathit{\Gamma}_{max}}{\mathit{\Gamma}^{m2} I_{max}})$$

where $C_{b_{max}} = \max_{r \in R} C_b^r$ *and* $I_{max} = \max_{r \in R} I_{max}^r$.

Proof. We first consider the case where interference and capacity constraints do not exist. Let r_k^{NE} denote the RB user k uses at a Nash equilibrium, r_k^* denote the RB at a social optimum, and G_k^r and $G_k^{r'}$ denote the total signal strength received by user k in RB r and r', respectively, which is the sum of interference signals plus the signal strength from its own BS. Then, for any RB r, we have

$$\frac{\mathit{\Gamma}_{b,k}^{r_{NE}}}{G_k^{r_{NE}} - \mathit{\Gamma}_{b,k}^{r_{NE}}} \geq \frac{\mathit{\Gamma}_{b',k}^{r'_{NE}}}{G_k^{r'_{NE}} - \mathit{\Gamma}_{b',k}^{r'_{NE}}} \implies G_k^{r_{NE}} - \mathit{\Gamma}_{b,k}^{r_{NE}} \leq \mathit{\Gamma}^m (G_k^{r'_{NE}} - \mathit{\Gamma}_{b',k}^{r'_{NE}})$$

$$G_k^{r_{NE}} - \mathit{\Gamma}_{b,k}^{r_{NE}} \leq \mathit{\Gamma}^m (\max(\sum_{k' \neq k} \mathit{\Gamma}_{b_{k'},k}^{r'} + \mathit{\Gamma}_{k',k}^{r'} + \mathit{\Gamma}_{M,k}^{r}) + \sigma^2)$$

where $\mathit{\Gamma}^m = \mathit{\Gamma}_{max}/\mathit{\Gamma}_{min}$

For the user with the least signal strength using the RB r_{NE}, the lower and upper bounds on SINR (Eq. 1) are expressed as

$$\gamma_k^{NE} = \frac{\mathit{\Gamma}_{min}}{\mathit{\Gamma}^m (\max(\sum_{k' \neq k} \mathit{\Gamma}_{b_{k'},k}^{r'} + \mathit{\Gamma}_{k',k}^{r'} + \mathit{\Gamma}_{M,k}^{r}) + \sigma^2)}$$

$$\text{and } \gamma_k^* = \frac{\mathit{\Gamma}_{max}}{\mathit{\Gamma}_{M,k}^r + \sigma^2} \leq \frac{\mathit{\Gamma}_{max}}{\mathit{\Gamma}_{min} + \sigma^2}$$

Thus, the PoA is given by

$$
\begin{aligned}
PoA &\geq \frac{\log_2\left(1+\frac{\Gamma_{\min}}{\Gamma^m\left(\max\left(\sum\limits_{k'\neq k}\Gamma^{r'}_{b'_k,k}+\Gamma^{r'}_{k',k}+\Gamma^{r}_{M,k}\right)+\sigma^2\right)}\right)}{\log_2\left(1+\frac{\Gamma_{\max}}{\Gamma_{\min}+\sigma^2}\right)} \\
&\geq \frac{\frac{\Gamma_{\min}}{\Gamma^m\left(\max\left(\sum\limits_{k'\neq k}\Gamma^{r'}_{b'_k,k}+\Gamma^{r'}_{k',k}+\Gamma^{r}_{M,k}\right)+\sigma^2\right)}}{\log_2\left(\frac{2\Gamma_{\max}}{\Gamma_{\min}+\sigma^2}\right)} \\
&\geq \frac{\frac{\Gamma_{\min}}{\Gamma^m(2(|K|-1)\Gamma_{\max}+\Gamma_{\max}+\sigma^2)}}{\log_2\left(\frac{2\Gamma_{\max}}{\Gamma_{\min}+\sigma^2}\right)} \geq \frac{\frac{\Gamma^2_{\min}}{\Gamma_{\max}((2|K|-1)\Gamma_{\max}+\sigma^2)}}{1+\log_2(\Gamma^m)} \\
&\geq \frac{\frac{\Gamma^2_{\min}}{\Gamma_{\max}((2|K|-1)\Gamma_{\max}+\Gamma_{\max})}}{1+\log_2(\Gamma^m)} \geq \frac{\frac{\Gamma^2_{\min}}{2|K|\Gamma^2_{\max}}}{1+\log_2(\Gamma^m)} \\
&\geq \frac{\frac{1}{2|K|\Gamma^{m2}}}{1+\log_2(\Gamma^m)} \geq \frac{1}{2|K|\Gamma^{m2}(1+\log_2(\Gamma^m))}
\end{aligned}
$$

The second inequality holds because the lower bound of γ_k^{NE} can be very small, also we assume that the noise σ is negligible.

PoA with Capacity and Interference Constraints: We start with the previous analysis of $G_k^{r_{NE}}$. The lower bound for user k in r_{NE} is when the user has the least signal strength while experiencing the largest interference I_{max}:

$$\gamma_k^{NE} = \frac{\Gamma_{min}}{\Gamma^m(I_{max} - \Gamma_{min} + \sigma^2)}$$

The upper bound for this user in RB r^* can be calculated based on the capacity constraint: $\gamma_k^* \leq 2^{C_{b_{max}}} - 1$ where $C_{b_{max}} = \max_{r\in R} C_b^r$. The result follows.

$$
\begin{aligned}
PoA &\geq \frac{1}{C_{b_{max}}}\log_2(1+\frac{\Gamma_{min}}{\Gamma^m(I_{max}-\Gamma_{min}+\sigma^2)}) \\
&\geq \frac{1}{C_{b_{max}}}\log_2(1+\frac{\Gamma_{min}}{\Gamma^m(I_{max}+\sigma^2)}) \\
&\geq \frac{1}{C_{b_{max}}}\log_2(1+\frac{\Gamma^2_{min}}{\Gamma_{max}(I_{max}+\sigma^2)})
\end{aligned}
$$

□

We now show that the lower bound presented in the above theorem can be arbitrarily small. This requires a wide variation in signal strength, which in practice should be kept bounded.

Claim. There exists an instance of the the Wireless Resource Game, when using the fair metric, where $PoA = \frac{1}{\log(1+M)}$ where M is arbitrarily large.

Proof. We provide an example with 2 users and 2 RBs. We ignore interference from all BSs including MBS, and assign signal values to achieve an arbitrarily small PoA. Consider values $\Gamma_{11}^1 = \Gamma_{21}^2 = M$, and $\Gamma_{21}^1 = 1, \Gamma_{11}^2 = M$. We assume that interference from noise, σ^2 is 1. The optimum assignment is when user 1 is assigned to RB 1 and user 2 to RB 2; the minimum utility and thus the social objective function is $\log(1 + \frac{M}{\sigma^2})$. It can be verified that a Nash equilibrium assignment exists when user 1 is assigned to RB 2 and user 2 is assigned to RB 1. The social objective function at this Nash equilibrium evaluates to $\log(1+\frac{1}{\sigma^2})$. The result follows. □

4 Determining Social Optimum

In this section we show that determining the social optimum, even approximately, is NP-Hard. However, we also present an efficient heuristic to discover social optimum.

Theorem 2. *Determining an approximate social optimum in the wireless resource allocation problem using the max-min measure is NP-Hard.*

Proof. We reduce from the known NP-Hard problem of the *chromatic number problem* where we must determine if a graph G is q-colorable. From a given instance $I_c = (G(V,E), q)$ of the chromatic number problem, we create an instance of our 5G wireless game $I_w = (K, B, R)$ - For each node v in G, we create a user k_v in K. There are q RBs and $|B| = n * q$ base stations (n is the number of nodes in the graph), and each user k faces an interference of I_B from every base station she does not occupy. If an edge exists between two nodes u and v, the corresponding users k_u and k_v heavily interfere with each other if they share the same RB; where we define *heavy interference* to be the case when the interference signal is no less than the given value $I_B|B|(L+2)$. If two nodes are not neighbors, the corresponding users interfere with each other with value δ, where δ is a small positive value. The signal strength of each user is $g_{k,b}^r p_k^r = 1, \forall r, b$. Therefore, a user k facing no heavy interference obtains a utility bound

$$\gamma_{k,b}^r \geq \frac{1}{\sigma^2 + \delta|B| + I_B|B|},$$

and a user facing heavy interference from at least one other user obtains a utility bound

$$\gamma_{k,b}^r \leq \frac{1}{\sigma^2 + I_B|B|(L+2) + I_B(|B|-1) + (n-2)\delta}.$$

Given a social optimum, S_w in I_w, we show an equivalent solution for q-coloring I_c. If the social utility is $\max\min_k \gamma_{k,b}^r \geq \frac{1}{\eta+\delta|B|+I_B|B|}$, no two users that heavily interfere with each other can occupy the same resource block, since their utilities

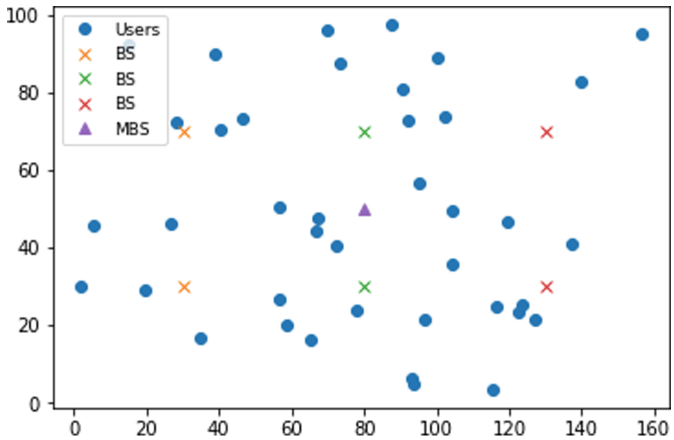


Fig. 2. Geometric distribution of Users and Base stations

would then be lesser. Then, for each RB r, no two occupying users will have their corresponding nodes in I_c as neighbors, i.e., the corresponding nodes of users in each RB r will be an independent set I_r. The converse can be proved similarly, implying that the problem is NP-Hard.

We now show that approximating the problem of finding the socially optimal solution is also NP-Hard. In I_w, consider the utility of a user k_v using RB r on BS b with no high-value interference from another user k_u (corresponding nodes in I_c are not neighbors). Assuming the noise σ^2 to be negligible, the bound on k_v's utility is then $\gamma_c \geq \frac{1}{\sigma^2+\delta|B|+I_B|B|}$ since the same resource block may be occupied on every possible BS. Similarly, if a user k_v faces heavy interference from no less than one other user (corresponding nodes in I_c are neighbors), her utility will be bounded as $\gamma_{nc} \leq \frac{1}{\sigma^2+I_B|B|(L+2)}$. Noting that L, n and $|B|$ are sufficiently large constants, the utilities u_c and u_{nc} corresponding to γ_c and γ_{nc} respectively are $u_c \geq \log\left(1+\gamma_c\right) = \log\left(1+\frac{1}{\delta|B|+I_B|B|}\right) \approx \frac{1}{\delta|B|+I_B|B|}$, and similarly $u_{nc} \leq \log\left(1+\gamma_{nc}\right) \approx \frac{1}{I_B|B|(L+2)}$. Let $\delta = \frac{I_B}{L+1}$, the ratio between the two utility bounds is then

$$\frac{u_c}{u_{nc}} \geq \frac{I_B|B|(L+2)}{(1+\frac{1}{L+1})I_B|B|} \geq \frac{L+2}{1+\frac{1}{L+1}} > L.$$

This gap can be used to show that finding an approximation better than factor $\frac{1}{L}$ is NP-Hard. □

4.1 Exchange-Method for Social Optimum

We now present Algorithm 1 which iteratively improves the social objective function. At each step, all users are considered in a round-robin order (line 6) and it is determined if the user can improve the social objective function by changing his own assignment (line 10) or by switching his assignment with another user's assignment (line 14) . The algorithm terminates when no improvements are possible.

Algorithm 1: Exchange-Method

1 $u_{old} = -1$, $u_{new} = 0$
2 Assign all users randomly to BSs and RBs.
3 Let list Q contain all unoccupied RB and BS pairs.
4 **while** $u_{new}! = u_{old}$ **do**
5     $u_{old} = u_{new}$
6   **for** $j \in 1 \cdots |K| - 1$ **do**
7         $\forall (r, b) \in Q$, let $A_{(r,b)}$ be the assignment when user k_j is assigned to (r, b); calculate minimum utility over all users: $u(A_{(r,b)}) = \min_k u^r_{k,b}$.
8         $(r^*, b^*) = \arg\max_{(r,b)\in Q} u(A(r, b))$.
9   Let $u^{'}_{min}$ be the minimum utility achieved after k_j is assigned (r^*, b^*).
10   **if** $u^{'}_{min} > u_{new}$ **then**
11   Let k_j picks the strategy (r^*, b^*) such that $u^{k_j}_{r,b} = u'_{min}$.
12             $u_{new} = u^{'}_{min}$
13   **else**
14   For all pairs (k,k_j) where $k \neq k_j$: check if switching k with k_j improves u_{min}
15             $u_{new} = u_{min}$

Approximation Ratio of the Exchange-Method: Note that the approximation ratio can be arbitrarily large - Consider an example with 3 users and 3 RBs, where δ is an arbitrarily small number, with signal strength matrix $\Gamma = \begin{bmatrix} 1 & \delta & \delta \\ \delta & 1 & \delta \\ \delta & \delta & 1 \end{bmatrix}$ where rows are users and columns are RBs. If we start with an assignment where k_2 is using r_1, k_3 is using r_2 and k_1 is using r_3, then each user has a signal strength of δ, i.e., user with minimum utility gets δ. The algorithm terminates with this solution. However, the optimal social solution would be the assignment with k_1 using r_1, k_2 using r_2 and k_3 using r_3, leading to the ratio of best-to-worst solution of the exchange method as $\frac{1}{\delta}$.

Complexity of the Exchange-Method: We note that the minimum utility has an upper bound, given by $\frac{\Gamma_{max}}{\sigma^2}$; and therefore, the exchange-method algorithm always terminates. Let T be the maximum number of strategy changes or user pairs switching. The inner *for* loop runs $|K|$ times and line 6 takes time proportional to $O(|R||B||K|)$. Line 13 takes time proportional to $O(K^2)$, implying a total algorithm running time of $O(T|K|^2|R||B|)$.

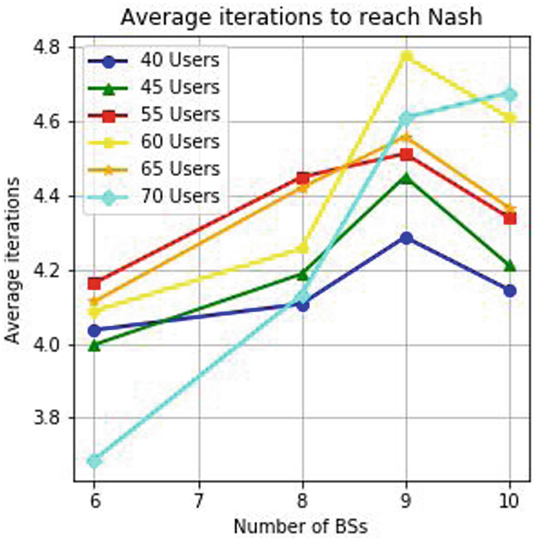


Fig. 3. Effect of number of Users and BSs on number of iterations on finding Nash equilibrium

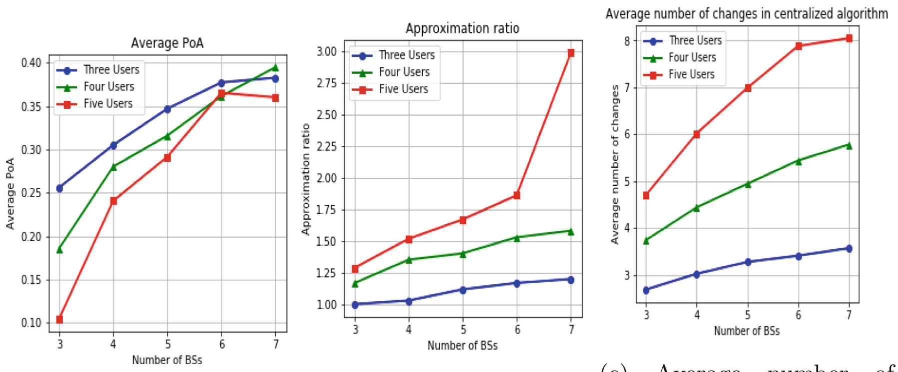
5 Simulations

In this section, we present simulations to study Nash equilibrium and price of anarchy in a realistic geographic setting. We provide experimental results on the best response method, examining the speed of convergence and the success rate of achieving Nash equilibrium. We also experimentally analyze the complexity of Algorithm 1 in terms of number of exchanges performed before termination.

The 5G wireless communication model consists of 40 to 65 users, 12 RBs and 6 to 10 BSs as is typical for these models, the reason we don't set number of RBs to a big number is to avoid the situation that most RBs are occupied by only one or two users with little interference. Users are located within a rectangular space of size $100 * 160$ m, the BSs are located on a grid contained within that space (illustrated in Fig. 2), and the MBS is located at the center of the map. The channel gain ranges from -15 dB to -31 dB and power ranges from 26 dBm to 50 dBm. These data ranges are based on a realistic 5G network. For our simulations, we pick random numbers within the specified ranges for each run.

5.1 Simulations to Analyze Nash Equilibrium and Price of Anarchy

The Nash equilibrium is determined via the best-response method as follows: Users are considered in a round-robin order to determine if they will switch out of the currently assigned resource block and the base station to an alternate resource block and base station pair, the pair being chosen in order to maximize the user's utility given the strategies of the other players.



(a) Average value of PoA (b) Approximation ratio via exchange-method (c) Average number of changes via exchange-method

Fig. 4. Effect of number of Users and BSs on inverse of the PoA and approximation ratio using exchange-method

The number of users in our simulations range from 40 to 65 in intervals of size 5, and the number of BSs range between 6 and 10. Figure 3 shows the number of iterations required to reach Nash equilibrium. When the algorithm converges to a Nash equilibrium, as it does in 99% of cases in our experiments, we observe that the number of iterations is at most 5, and generally increases with an increase in the number of users.

For PoA, we analyze for only a small number of users (ranging from 3 to 5) and base stations (ranging from 3 to 7) since computing the optimum solution using the max-min social objective is NP-hard. Figure 4a illustrates the average value of the PoA value achieved for with different number of users and BSs while the number of RBs are set to 2. We observe that the PoA (for the max-min social objective measure) increases from 0.105 to 0.4 as the number of BSs increases. This indicates that increase in number of BSs improves the value of the PoA, as expected. In this setting, we also consider the exchange method and compare the results with the actual social optimum, which we compute via a complete case analysis. Figure 4b shows the median approximation ratio defined as the ratio of the optimum social objective value to the social objective value obtained using Algorithm 1. We observe that this ratio is bounded (in fact at most 3 for the cases considered); clearly as the number of users and the number of BSs increase, the approximation ratio increases, i.e., the performance of the algorithm becomes worse.

Figure 4c illustrates the average number of the assignment changes being made during the implementation of the exchange-method. The number of changes is the sum of the number of times a user changes its strategy and the number of times two users swap assignments. We note that the increase in the number of changes is almost linear w.r.t the number of users and BSs; the number of average changes are 2.69, 3.74 and 4.71 as the number of users increase from 3 to 5 when there are 3 BSs.

6 Conclusions

We have studied 5G communication networks and developed a game model comprising mobile clients in a distributed setting. We showed that finding the social optimal using max-min metric is NP-Hard and provided an exchange-method heuristic. We also provided a best response method to determine Nash equilibrium. Simulation results show the effectiveness of our analyses and algorithms.

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