



The Conjunction of Deterministic and Probabilistic Events in Realistic Scenarios of Outdoor Infections

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Abstract. The aim of this paper is the derivation of an robust formalism that calculates the so-called social distancing as already determined in the ongoing Corona Virus Disease 2019 (Covid-19 in short) being established in various places in the world between 1.5 m and 2.5 m. This would constitutes a critic space of separation among people in the which aerosols might not be effective to infect healthy people. In addition to wearing masks and face protection, the social distancing appears to be critic to keep people far of infections and consequences produced from it. In this way, the paper has opted by the incorporation of a full deterministic model inside the equation of Weiss, by the which it fits well to the action of outdoor infection when wind manages the direction and displacement of aerosols in space. Thus, while a deterministic approach targets to propose a risk's probability, a probabilistic scenario established by Weiss in conjunction to the deterministic events would yield an approximated model of outdoor infection when there is a continuous source of infected aerosols that are moving through air in according to a wind velocity. The simulations have shown that the present approach is valid to some extent in the sense that only the 1D case is considered. The model can be extended with the implementation of physical variables that can attenuate the presence of disturbs and random noise that minimizes the effectiveness of present proposal.

Keywords: Covid-19 · Weiss probabilistic equation · Outdoor infection

1 Introduction

The ongoing pandemic Covid-19 has been firstly identified in Wuhan in China the last of 2019 [1, 2]. The virus has been randomly propagated overall continents and phases of global infection were observed, some points that are noteworthy constitutes the number of new infections as well as the continue increasing of fatalities [3]. As well-known up to two well-defined waves have been observed: at middle of 2020, end 2020 in conjunction to the beginning of 2021. Furthermore,

a third wave might be happening nowadays as result of the mutation of strain. In this manner, countries are adopting social policies to face and contain the incoming worst scenarios [4]. In fact, curfew, quarantines and social restrictions were firmly adopted. Before the vaccination seasons the so-called social distancing was imposed as a response to reduce the positive exponential increasing of new cases that in most cases is consisting of vulnerable population [5] as well as for a solid return of a harmonic social-economy scenario [6]. Even the vaccination has been completed, one might to expect the arrival of new variants that would trigger again a new wave of pandemics. To counteract this, one might to appeal to distance-based policies that are demonstrated to be effective to some extent [7,8]. Some of these restrictions as:

- Vaccination of up to two doses,
- Usage of mask and face protection,
- Social distancing,
- Permanent usage of alcohol and disinfectant,
- Lock-down if needed.

The rest of this paper is as follows: In second section the deterministic scenario of outdoor infection is presented. Here is incorporated the wind velocity that emerges in an inherent manner, important equations are derived. In third section the equation of Weiss is presented and its importance as mathematical formulation that engages the action of outdoor infection is presented. In fourth section, the conjunction of deterministic and probabilistic formulations is presented in order to postulate the equation of outdoor infection. Here computational simulations are presented. The social distancing is estimated. Finally in last section the conclusion of paper is done.

2 Theory of Outdoor Infection: Main Equations

In order to construct a theory of outdoor infection, a few requirements are needed in order to derive deterministic equations. These are as follows:

- A finite number of healthy people and at least one infected,
- The infected one (or ones) are carrying out a loud speech,
- The infected one exhales a finite number of aerosols, being them that are transporting the virus,
- There is wind velocity that might to be pointing the place where are people,
- Depending at climate conditions and aerodynamics aerosols can be negligible a few meters after that were exhaled.

2.1 Derivation of Model

Consider the distribution of net number of aerosols per unit of time and distance $\mathbf{n}(r, t)$ that can be defined as:

$$\mathbf{n}(r, t) = \frac{\Delta N}{\Delta r \Delta t} \tag{1}$$

with ΔN the number of aerosols that were exhaled. Therefore, the instantaneous number $\mathbf{N}(T, R)$ that are scattered along the space can be written as:

$$\mathbf{N}(T, R) = \int_0^T \int_0^R dt dr \mathbf{n}(r, t). \tag{2}$$

Because aerosols is a fluid that is propagating along air, then the diffusion equation can also be applied. Therefore for the unidimensional case (that of course is a special case, a 3D modeling is beyond the scope of this paper), can be written as:

$$\frac{d\mathbf{n}(r, t)}{dt} = D \frac{d^2\mathbf{n}(r, t)}{dr^2} \tag{3}$$

and from this a crude solution can be done as:

$$\mathbf{n}(r, t) = D \int dt \frac{d^2\mathbf{n}(r, t)}{dr^2} \tag{4}$$

so that Eq. 2 might be more accurate in the form as given below:

$$\mathbf{N}(R, T) = D \int_0^T dt \int_0^R dr \int_0^t d\tau \frac{d^2\mathbf{n}(r, \tau)}{dr^2}. \tag{5}$$

The integration over dt turns out to be trivial, so that one has:

$$\mathbf{N}(R, T) = DT \int_0^R dr \int_0^t d\tau \frac{d^2\mathbf{n}(r, \tau)}{dr^2}. \tag{6}$$

Nevertheless, the left-side can still be simplified in the sense that

$$\mathbf{N}(R, T) = DT \int_0^R dr \int_0^t d\tau \frac{d}{dr} \frac{d\mathbf{n}(r, \tau)}{dr} = DT \int_0^R dr \int_0^t d\tau \frac{d}{dr} \frac{d\mathbf{n}(r, \tau)}{dr} \tag{7}$$

Consider the human speech in the scenario of loud speak so that one can expect that a substantial amount aerosols are being scattered. Without physical considerations of dehydration and a certain direction of wind then one can attribute a periodical distribution for example.

2.2 Incorporation of Wind Velocity

It is trivial that a velocity appears after a straightforward insertion of the chain rule in Eq. 2:

$$\mathbf{N} = \int_0^T dt \int_0^R dr \frac{d\mathbf{n}}{dr} \frac{dr}{dt} = \int_0^T dt \int_0^R dr v(t) \frac{d\mathbf{n}}{dr} = \int_0^T dt \frac{d\mathbf{n}}{dr} v(t). \tag{8}$$

In this manner from Eq. 6 one can generalize to a finite number of sources, so that one gets:

$$\mathbf{N}(r) = \sum_q \int_0^T dt \frac{d\mathbf{n}_q(r, t)}{dr} v_q(t), \tag{9}$$

where q runs over the possible sources of aerosols [9] and that in praxis denote all those people that were infected and contain the strain and actions of loud speech would be an imminent manner to infect healthy people. One can wonder about the change in time of $\mathbf{N}(r)$ that can be written as

$$\frac{d\mathbf{N}(r)}{dt} = \sum_q \frac{d\mathbf{n}_q(r, t)}{dr} v_q(t), \tag{10}$$

with $v(t)$ the 1-dimension direction of wind. This leads to define a new relation that involve a fraction of net number of aerosols that might to be under the influence of wind:

$$\frac{d\mathbf{n}_q}{dr} = \frac{\eta\mathbf{N}}{\tilde{\mathbf{n}}_q} \tag{11}$$

so that one can see that under the presence of wind, \mathbf{n} is now a fraction of all those sources exhaling aerosols with $\tilde{\mathbf{n}}_q$, and η a random number. Thus Eq. 11 in Eq. 12 one arrives to:

$$\frac{d\mathbf{N}(r, t)}{dt} = \sum_q \frac{\eta\mathbf{N}(r, t)}{\mathbf{n}_q} v_q(t) = \eta\mathbf{N}(r, t) \sum_q \frac{v_q(t)}{\mathbf{n}_q}. \tag{12}$$

This indicates that a change at $\mathbf{N}(r, t)$ is proportional to it times the wind velocity $v_q(t)$. Therefore one can go through a trivial integration in both sides:

$$\int \frac{d\mathbf{N}(r, t)}{\mathbf{N}(r, t)} = \int_0^t \eta \sum_q \frac{v_q(t)}{\mathbf{n}_q} dt, \tag{13}$$

yielding the familiar exponential distribution:

$$\mathbf{N}(r, t) = N_0 \text{Exp} \left[\frac{\pm\eta}{Q} \sum_q \int_0^{\mathbf{T}_{\text{DH}}} \frac{v_q(t)}{\mathbf{n}_q} dt \right] \tag{14}$$

where the integer Q at the denominator indicates the average over all sources. Here \mathbf{T}_{DH} denotes the time of dehydration for all aerosols.

In Fig. 1 the logarithm of Eq. 14 has been plotted under the assumption that $Q = 2, 3, 4$ and 5 . The velocity $v_q = 0.05(t + 2 * q)^2 \text{Sin}[q * t]$ and $\mathbf{n}_q = 1 + 20^q$ the number of aerosols. Also, $\pm\eta/Q = -1/2$, $N_0 = 1/1.2$. One can see that the distribution exhibits a kind of minimum at 3.5 s.

One can attribute these minimum as the time by the which the dehydration takes place. The why one can see the up of distribution from $t = 4$ s is because the wind velocity v_q that has been modeled through a sinusoidal function.

And it is actually encompassing the fact that in outdoor the wind direction is a composition of various ones due to the complexity of terrain and obstacles.

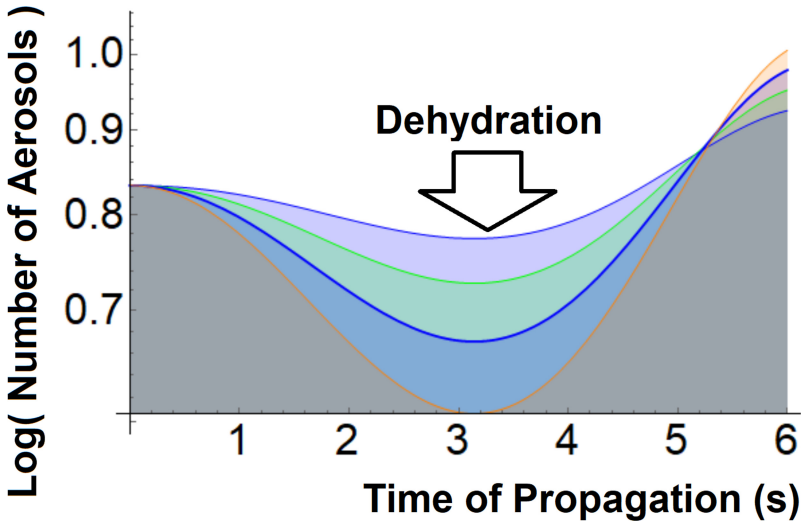


Fig. 1. Illustration of Eq. 14 showing a minimal at the normalized logarithm distribution of number of aerosols for $t = 3.5$ s supposed to be caused by the dehydration of aerosols in air. The up seen at the distributions can be explained in terms of wind velocity. Plot and the next ones were done with Wolfram [11]

3 The Stochastic View of Weiss Equation

As it is well-known, outdoor infection can be done with a high effectiveness in such scenarios as displayed in Fig. 2. In fact, while there is fix wind velocity, aerosols can be displaced some meters before their dehydration producing massive infection.

3.1 The Weiss Equation

In circumstances as shown in Fig. 2, to talk about that outdoor infection is done through deterministic laws might have a weak sustain so that, one can complement to it a formulation created under probabilistic grounds. In fact, the equation of Weiss considers the following action: Assume a finite number of N objects are thrown to a finite number of Q cells. All cells exhibit same morphology so that no any preference exists there. When this action is repeated M times, then one arrives to the probability \mathbf{P} that a cell is unoccupied¹. This can be written below as:

$$\mathbf{P} = \frac{Q \text{Log} Q}{NM} \tag{15}$$

¹ Obviously the probability of occupancy is done by $1 - \mathbf{P}$.



Fig. 2. Example of a potential scenario of random infection when at least one participant of chorus is spreading aerosols transporting virus. Taken from [10].

3.2 Intuitive Derivation

Consider the simplest case: the action that N objects are thrown to Q cells. One can wonder how many tries can be done if one requires that all cells are occupied. Clearly for this it is demanded a large number of M tries. Then the probability that exists unoccupied ones is given by:

$$\mathbf{P} = \frac{Q}{NM}. \tag{16}$$

For example, consider $Q = 10$ cells and $N = 1000$ objects. Thus for the first event one has that $\mathbf{P} = 10^{-2} = 0.01$ the probability of having 10 unoccupied cells. When Eq. 16 is written as:

$$\int d\mathbf{P} = \int \frac{dQ}{NM}. \tag{17}$$

Thus, both M and N are totally independent of Q , so that one can write below:

$$\frac{1}{Q} \frac{d\mathbf{P}}{dQ} = \frac{1}{NM} \frac{1}{Q} \tag{18}$$

by the which both members were divided by Q . With this one has that:

$$\frac{d\mathbf{P}}{Q} = \frac{1}{NM} \frac{dQ}{Q} \tag{19}$$

whose integration in both members yields Eq. 15. On the other hand the probability there is Q occupied cells can be written as $\mathbf{p} + \mathbf{P} = 1$, so that from this the one gets:

$$\mathbf{p} = 1 - \frac{Q \text{Log} Q}{NM}. \tag{20}$$

Interestingly, from the following assumption:

$$\left(\frac{Q \text{Log} Q}{NM}\right)^\ell \approx 0 \tag{21}$$

$\forall \ell \geq 2$ that is true only if $NM \gg Q \text{Log} Q$. With this then one can arrive to a new definition for the occupancy probability:

$$\mathbf{p} = \text{Exp}\left(-\frac{Q \text{Log} Q}{NM}\right). \tag{22}$$

In Fig. 3, the occupied, unoccupied and total probabilities are plotted. All of them are function of number of cells. From it one can see that all have a coincidence for $Q = 118$ cells with a common probability of 0.56. For this exercise $NM = 1000$, equivalent to $N = 200$ objects and $M = 5$ times that were thrown the objects to the available cells. It is easy to see that occupancy decreases with the number of cells. At the other side, the unoccupancy increases with Q resulting to be rather logic since for example, the probability of having 45 unoccupied of 100 total cells is 0.45 when have been thrown 200 objects 5 times. In the inverse case one gets 65 occupies cells of 100 available ones².

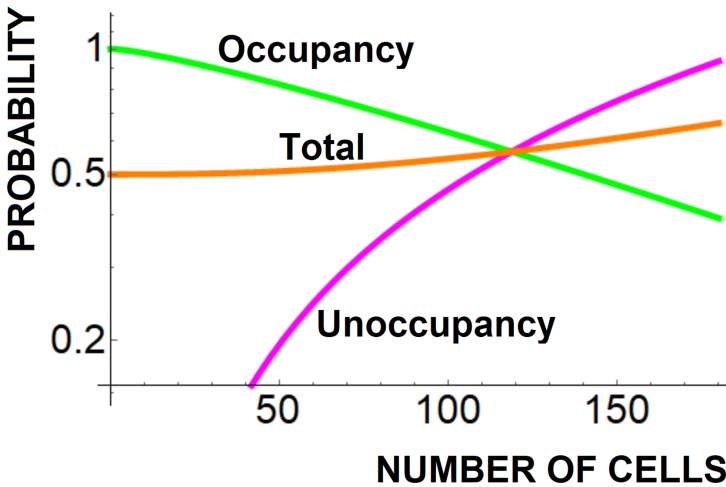


Fig. 3. The Weiss probabilities (i) Occupancy (green), (ii) Unoccupancy (magenta) and (iii) Total (orange), for $N = 200$ thrown objects in up to 5 times (Color figure online)

² One can see that sum yields 110 cells, fact that can be understood in terms of errors of model.

4 The Stochastic and Deterministic Synergy

The Weiss equation Eq. 15 has a notable application³ in the field of Infection Dynamics, a discipline that is nowadays in constant progress with the apparition of Covid-19 pandemic. With the emergence of global pandemic, a series of policies were imposed. Among them one can see that well-known social distancing that has been established of being between 1.5 m to 2 m. Once known that virus is mainly transmitted by aerial paths because the exhalation and the subsequent emission of aerosols, then one clearly has two scenarios: (i) indoor: that has been deeply studied, and (ii) outdoor: whose scenario is not well established since in most studied the wind velocity and dehydration time were not taken into account. In this manner one can associate the Weiss equation to the following scenario: one or various infected people that are carrying out loud speech. This action is generating the production of aerosols that do not only stay there but also can be traveling along the wind direction if this is enough stable to produce the displacement of virus enclosed in the aerosol. From Eq. 15 one can establish that N objects are denoting the N aerosols, Q cells denote Q healthy people, M denotes the times that aerosols is arriving to people, and finally \mathbf{P} the probability that people is not reached by infected aerosol.

4.1 The Frequency of Aerosols Arrival

It should be noted that in Eq. 15, the integer number M denoting the number of times by the which the aerosols are falling into the healthy people can also be seen as a frequency. In fact, because mathematically one does not find not any contradiction neither inconsistency, then it is feasible to rewrite again Eq. 15 as with $Q \rightarrow q$:

$$\mathbf{P}(\omega) = \frac{\omega q \text{Log}(q)}{N} \tag{23}$$

that can be read as the probability $\mathbf{P}(\omega)$ of q healthy people becomes infected after of N aerosols have fallen on them. Here one can see that the probability is now depending on the frequency in the which the aerosols are arriving to people. Imminently while ω increases the probability does it. Taking into account all this, one can adjudicate to ω the name of **risk frequency**. Because N the number of aerosols has been proposed in Eq. 14, then it enters inside the denominator of Eq. 23:

$$\mathbf{P}(\omega) = \frac{\omega q \text{Log}(q)}{N_0 \text{Exp} \left[\frac{\pm \eta}{Q} \sum_q^Q \int_0^{\mathbf{T}_{\text{DH}}} \frac{v_q(t)}{\mathbf{n}_q} dt \right]}. \tag{24}$$

The sign \pm is reverted when Eq. 24 acquires the following form:

$$\mathbf{P}(\omega) = \frac{\omega q \text{Log}(q) \text{Exp} \left[\frac{\pm \eta}{Q} \sum_q^Q \int_0^{\mathbf{T}_{\text{DH}}} \frac{v_q(t)}{\mathbf{n}_q} dt \right]}{N_0}. \tag{25}$$

³ Among others not less relevant, such as nano particles and drug delivery carriers.

The probability for all allowed frequencies after the integration over them can be written as:

$$\begin{aligned} \mathbf{P} &= \int \mathbf{P}(\omega) d\omega = \int \frac{\omega q \text{Log}(q) \text{Exp} \left[\frac{\pm \eta}{Q} \sum_q^Q \int_0^{\mathbf{T}_{\text{DH}}} \frac{v_q(t)}{\mathbf{n}_q} dt \right]}{N_0} d\omega \\ &= \int \mathbf{P}(\omega) d\omega = \frac{\omega^2 q \text{Log}(q) \text{Exp} \left[\frac{\mp \eta}{K} \sum_k^K \int_0^{\mathbf{T}_{\text{DH}}} \frac{v_k(t)}{\mathbf{n}_k} dt \right]}{N_0}. \end{aligned} \quad (26)$$

In this manner the probability can be written as:

$$\mathbf{P} = \frac{\omega^2 q \text{Log}(q)}{N_0} \text{Exp} \left[\frac{(\mp \eta)}{K} \sum_k^K \int_0^{\mathbf{T}_{\text{DH}}} \frac{v_k(t)}{\mathbf{n}_k} dt \right]. \quad (27)$$

In order to test the model in a quantitative manner, only one source is considered, thus one gets:

$$\mathbf{P} = \frac{\omega^2 q \text{Log}(q)}{N_0} \text{Exp} \left[(\mp \eta) \int_0^{\mathbf{T}_{\text{DH}}} \frac{v(t)}{\mathbf{n}} dt \right]. \quad (28)$$

Here it is important to recall that the left-side admits a first time derivative so that one gets:

$$\frac{d}{dt} \mathbf{P} = \frac{d}{dt} \left(\frac{\omega^2 q \text{Log}(q)}{N_0} \text{Exp} \left[(\mp \eta) \int_0^{\mathbf{T}_{\text{DH}}} \frac{v(t)}{\mathbf{n}} dt \right] \right) \quad (29)$$

because \mathbf{P} depends explicitly on r the one gets an extra velocity at the left-side due to chain rule:

$$\frac{d\mathbf{P}}{dr} \frac{dr}{dt} = \frac{\omega^2 q \text{Log}(q)}{N_0} \frac{d}{dt} \left(\text{Exp} \left[(\mp \eta) \int_0^{\mathbf{T}_{\text{DH}}} \frac{v(t)}{\mathbf{n}} dt \right] \right). \quad (30)$$

The velocity at the left-side can be recognized as the wind velocity:

$$\frac{d\mathbf{P}}{dr} \mathbf{V} = \frac{\omega^2 q \text{Log}(q)}{N_0} \left(\mp \eta \frac{v(t)}{\mathbf{n}} \right) \left(\text{Exp} \left[(\mp \eta) \int_0^{\mathbf{T}_{\text{DH}}} \frac{v(t)}{\mathbf{n}} dt \right] \right). \quad (31)$$

With this, an universal hybrid formulation for the outdoor infection has been obtained. One can write again Eq. 31 as:

$$\frac{d\mathbf{P}}{dr} = \mp \eta \frac{\omega^2 q \text{Log}(q)}{N_0 \mathbf{n}} \left(\frac{v(t)}{\mathbf{V}} \right) \left(\text{Exp} \left[(\mp \eta) \int_0^{\mathbf{T}_{\text{DH}}} \frac{v(t)}{\mathbf{n}} dt \right] \right). \quad (32)$$

One can note that now the derivative with respect to distance r is directly proportional to the rate v/\mathbf{V} with \mathbf{V} the arriving velocity of aerosols. It is actually the final velocity of how are arriving aerosols to people, therefore it is

also denoting the velocity of all those aerosols once were exhaled.

Under some circumstances one can assume that:

$$(\mp\eta) \int_0^{\mathbf{T}_{\mathbf{DH}}} \frac{v(t)}{\mathbf{n}} dt = (\mp\eta) \int_0^{\mathbf{T}_{\mathbf{DH}}} \frac{dr}{\mathbf{n}dt} dt = (\mp\eta) \int_0^{\mathbf{r}_{\mathbf{DH}}} \frac{dr}{\mathbf{n}} = \frac{\mp\eta \mathbf{r}_{\mathbf{DH}}}{\mathbf{n}}. \quad (33)$$

Inserting Eq. 33 into Eq. 32, one gets:

$$\frac{d\mathbf{P}}{dr} = \mp\eta \frac{\omega^2 q \text{Log}(q)}{N_0 \mathbf{n}} \left(\frac{v(t)}{\mathbf{V}} \right) \left(\text{Exp} \left[\frac{\mp\eta \mathbf{r}_{\mathbf{DH}}}{\mathbf{n}} \right] \right). \quad (34)$$

As done in Eq. 1 \mathbf{n} has units of number of aerosols per units of time and distance. In this manner, one can wonder from Eq. 34 if $\frac{d\mathbf{P}}{dr}$ exhibits a particular morphology when it is dependent on \mathbf{n} . With Eq. 34 the full probability of risk (with the potential of being infected in outdoor spaces) can be expressed as:

$$\mathbf{P} = \int \left[\mp\eta \frac{\omega^2 q \text{Log}(q)}{N_0 \mathbf{n}} \left(\frac{v(t)}{\mathbf{V}} \right) \left(\text{Exp} \left[\frac{\mp\eta \mathbf{r}_{\mathbf{DH}}}{\mathbf{n}} \right] \right) \right] dr, \quad (35)$$

in the which one arrives to the infection probability in the outdoor case:

$$\mathbf{P}_{\text{INF}} = 1 - \int \left[\mp\eta \frac{\omega^2 q \text{Log}(q)}{N_0 \mathbf{n}} \left(\frac{v(t)}{\mathbf{V}} \right) \left(\text{Exp} \left[\frac{\mp\eta \mathbf{r}_{\mathbf{DH}}}{\mathbf{n}} \right] \right) \right] dr. \quad (36)$$

5 Simulations and Discussion

Equation 34 denotes the change of probability of q people are not being infected due to \mathbf{n} aerosols [12]. Thus, while is kept both as independent variables, in Fig. 4 the contour plot is showing the distribution of the probabilities. Here, not any particular direction of wind was assumed [13]. Therefore, one can see the lowest probabilities along the blue color. Nevertheless the red color at the values of $\mathbf{n} = 1$ and $q = 50$ is telling us that $d\mathbf{P}/dr$ has a maximum so that a low number of aerosols can be reaching all 50 people. It is the case when $v(t)/\mathbf{V}$ exhibits same magnitude [14].

In Fig. 5 a density plot of Eq. 34 is displayed. Here have been considered as independent variables both $\mathbf{r}_{\mathbf{DH}}$ and $v(t)$ the dehydration distance and wind velocity respectively. What one can learn from it is the existence of up to three different phases categorized in three colors: blue, orange and red. Clearly a probability of 0.5 for the range between 1.5 m and 2.0 m is seen as a potential scenario. In Fig. 6 same as previous but done as a contour plot. Here the apparition of various colors helps us to differentiate the sectors of high risk as for example let us to case of a wind velocity of 1 m/s. For this then would correspond a dehydration length of 1.5 m with a low probability of not being infected. It should be noted that even for small wind velocities and short dehydration length there is high probability of not being infected. These numerical results ar fully in according with policies of the so-called **social distancing** the which establishes distances of 1.5 m⁴.

⁴ However neither temperatures nor specifying indoor and outdoor were done.

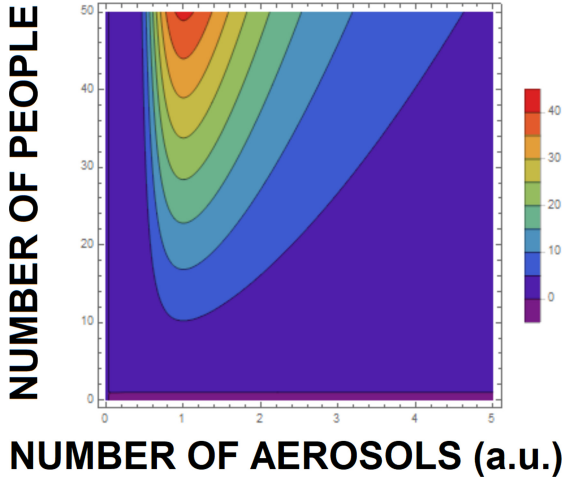


Fig. 4. The Weiss probabilities for $N = 200$ thrown objects in up to 5 times as function of number of people under risk of infection and number of aerosol (in arbitrary units). As seen the red color with a low number of aerosols, still the risk is high. (Color figure online)

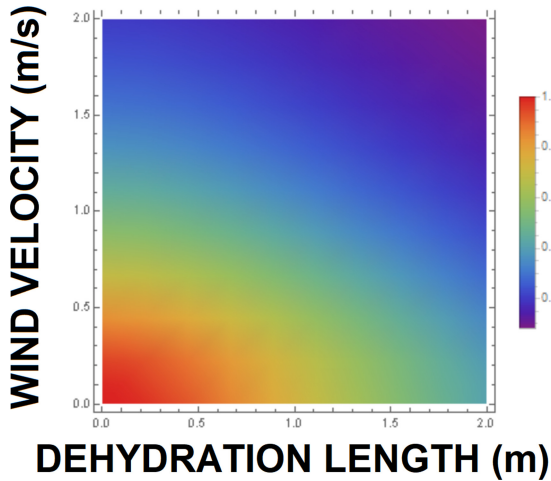


Fig. 5. The Weiss probabilities as function of wind velocity and dehydration length. One can see that in the lowest wind velocities and short dehydration lengths the risk is negligible (in full concordance to common sense and realistic cases) for wind velocities greater than 1.4 m/s fact that would have influence on the dehydration time.

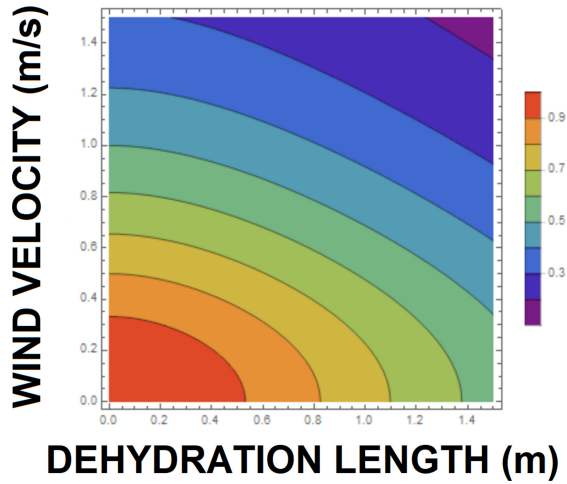


Fig. 6. The Weiss probabilities for $N = 200$ thrown objects in up to 5 times, same as Fig. 5 but done in contour plot showing the bands that belong to a concrete risk of being infected. The blue area is perceived as (Color figure online)

6 Conclusion

In this paper, a study about the estimation of the so-called social distancing was presented. In essence it consider the equation of Weiss as principal structure of this investigation. In fact, it was found that there is a firm correspondence between the Weiss equation and the phenomenon of outdoor infection that although suggested as a policy of protection against the human-human transmission of Covid-19, it is not clear if the established distances are considering the wind velocity that in some scenarios that might be realistic in praxis, can boost massive infections in open spaces. Thus, from the results of this paper is suggested to implement well-designed policies in outdoor spaces in order to guarantee that in crowded places the probability of infection is negligible in comparison to the indoor scenario [4], even with the random presence of infected people, and a strong wind velocity as well. In a future work the problem of outdoor infection but in a 3D scenario shall be treated.

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