



Accurate SER Quantization for Uplink Multi-user NOMA System

Yu Xu, Zhenyong Wang^(✉), Chen Cui, and Qing Guo

School of Electronics and Information Engineering, Harbin Institute of Technology,
Harbin 150001, China
{xu_yu, ZYWang, cuichen, QGuo}@hit.edu.cn

Abstract. Non-orthogonal multiple access (NOMA) is presented as a promising solution to support a massive number of communication devices in wireless networks. Due to the error propagation problem of successive interference cancellation (SIC) decoding, it is very difficult to quantify error rate performance accurately for the NOMA system. In this paper, accurate quantization expression of symbol error rate (SER) for the uplink NOMA system over additive white Gaussian noise (AWGN) channel is given. The quantified expression is effective regardless of the number of access users. Finally, we validate the provided expression and confirm the effectiveness through Monte Carlo experiments.

Keywords: Non-orthogonal multiple access (NOMA) · Multi-user · Symbol error rate (SER) · Error propagation

1 Introduction

NOMA, where multiple users are multiplexed in the power domain, is identified as a fundamental technique that can enable trillions of Machine-Type Communication (MTC) devices to communicate with the base station (BS) in cellular IoT use cases [1]. Due to its potential, NOMA is studied thoroughly in recent years [2–4]. Among these contributions, many problems are addressed from the perspective of system performance, such as system capacity and outage probability based on the Shannon formula. Channel codes currently used are designed to approach the channel capacity for long data packets. However, the transmission of small data packets associated with IoT poses a challenge. That is, due to the significant performance degradation, we cannot use the channel codes to approach the channel capacity of the NOMA system. In this regard, evaluating the error rate performance helps to develop effective algorithms, which can be utilized to improve the performance of the entire NOMA system [5, 6]. Unfortunately, this has not been studied in depth so far.

Due to the error propagation problem of SIC [7], it is very intractable to analyze error rate performance for the NOMA system. Only a few works addressed this issue. Some papers quantified the error rate performance accurately [8–10],

but the authors limited their research to two-user cases. It is worth mentioning that NOMA aims to serve as many users as possible with good performance, i.e., not suggested to serve only two users. Meanwhile, in [11–13], the error rate performance is analyzed and focused on multi-user cases. However, this research can only provide approximate results when considering the error propagation problem of SIC. Given the above situation, an accurate SER quantization expression for the uplink NOMA system over the AWGN channel is given in this paper. This expression is effective under multi-user scenarios. As current cellular IoT technologies like NB-IoT use low-rate modulations, i.e., BPSK and QPSK. We obtain the results in the BPSK case, then we extend our results to the QPSK case.

The rest of the paper is organized as follows. In Sect. 2, an uplink NOMA system with L users is presented. The SIC decoder of the NOMA system is introduced and the error propagation problem is analyzed. In Sect. 3, the SER performance of the multi-user NOMA system is quantified. The expression is verified through Monte Carlo experiments in Sect. 4. The paper concludes in Sect. 5.

2 Preliminary

2.1 System Model

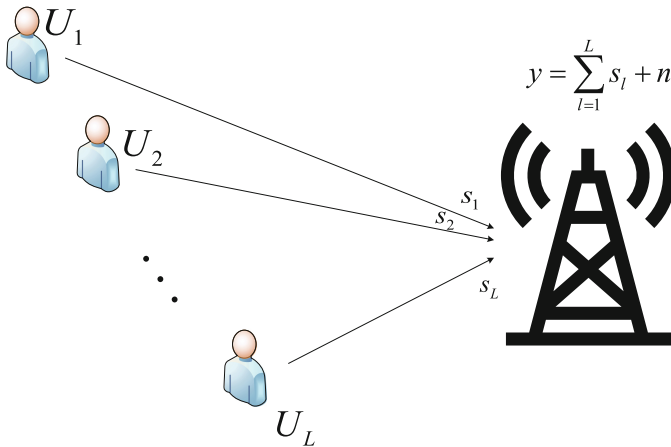


Fig. 1. Uplink NOMA systems with L users.

An uplink NOMA system with L users, denoted by U_1, U_2, \dots, U_L , and one BS is illustrated in Fig. 1. For simplicity, we assume that both the BS and MTC devices are equipped with a single antenna. All users adopt the same modulation, that is BPSK or QPSK. Besides, the carrier is recovered perfectly and the symbol

is synchronized at the BS. We suppose that different users are multiplexed in the power domain by transmitting equal maximum power. Because of their different channel conditions, their received power at the BS is different. Therefore, the superimposed received signal at the BS is given by

$$y = \sum_{l=1}^L s_l + n, \tag{1}$$

where s_l is the corresponding signal transmitted by U_l with received power ε_l at the BS; n is the zero mean AWGN with variance $N_0/2$. Without loss of generality, we assume that users are sorted in ascending order according to their received power, i.e. $\varepsilon_1 < \varepsilon_2 < \dots < \varepsilon_L$; ε_L and ε_1 correspond to the maximum and minimum received power, respectively.

2.2 Successive Interference Cancellation Decoding

At the BS, the receiver adopts SIC to decode all users' data. The user's signal, which has the strongest power, is detected first while treating others' signals as interference. The first user's signal is detected using maximum likelihood (ML) detection according to the following rule

$$\hat{s}_L = \arg \min_{\hat{s}_L \in D_L} |y - s_L|, \tag{2}$$

where D is the set of signal constellation. If U_l is BPSK modulated, D_l can be denoted by $D_l = \{\sqrt{\varepsilon_l}, -\sqrt{\varepsilon_l}\}$. Once the receiver decodes s_L correctly, it reconstructs s_L and subtracts s_L from the received signal y . Then s_{L-1} is decoded. Repeating this operation in descending order according to their received power until all users' information has been decoded.

Since the received signal of the U_l is interfered by the rest $l - 1$ users' signals, the BS should select users whose channel conditions are significantly different and multiplex them in the power domain. To fulfill this condition, here we assume that

$$\sqrt{\varepsilon_l} > \sum_{j=1}^{l-1} \sqrt{\varepsilon_j}, 1 < l \leq L. \tag{3}$$

2.3 Error Propagation Problem of SIC

We use $k_j, l < j < L$, to indicate whether the former U_j 's information is correctly decoded. $k_j = 1$ denotes U_j 's information is decoded correctly and $k_j = -1$ denotes decoded incorrectly. For $U_l, 1 \leq l < L$, the process of demodulating is given as

$$\hat{s}_l = \arg \min_{\hat{s}_l \in D_l} \left| y - \sum_{j=l+1}^L k_j s_j - s_l \right|. \tag{4}$$

If the messages of previous users are not correctly decoded at the receiver, the wrong reconstructed signal is subtracted from the composite multiuser signal, thus resulting in interference to the remaining users. It leads to the accumulation of decoding errors. The error propagation problem of SIC makes the SER quantization for the NOMA system intractable.

3 SER Quantization for Multi-user NOMA System

In this section, we derive the expression of the SER performance for the NOMA system. First, we quantify SER performance focus on two-user cases. Then we extend to the multi-user cases. For simplicity, we assume that all users adopt BPSK modulated signals. The results for the QPSK modulation case are given at the end of this section.

Considering the system model of $L = 2$, the BS receives the superimposed signal transmitted by U_1 and U_2 with power ε_1 and ε_2 , respectively. We can model the received signal as a Gaussian mixture model. A mixture distribution is made up of several component distributions and the corresponding probability distribution function is expressed as

$$p_2 = \sum_{s_1 \in D_1, s_2 \in D_2} \alpha \phi(x|\theta), \tag{5}$$

where p_i denotes the Gaussian mixture probability distribution consist of i users. The components $\phi(x|\theta)$ are Gaussian distribution. Each component has a separate parametrized mean $\theta = s_1 + s_2$ and uniformly covariance $N_0/2$. The parameters of a Gaussian mixture specify the prior probability α given to each component. In our case, the signal s_l transmitted by U_l , $l \in \{1, 2\}$, is chosen from the set $D_l = \{\sqrt{\varepsilon_l}, -\sqrt{\varepsilon_l}\}$ with equal probability of $1/2$ at the receiver. There are four components therefore α equals $1/4$. The following is our main result that SER performance of two-user uplink NOMA system:

Theorem 1. *We use P_{i-j} to indicate the SER of U_j while i users are multiplexed in the power domain. Therefore, the SER of U_2 can be expressed as*

$$P_{2-2} = \frac{1}{2} [Q(\sqrt{\frac{2\varepsilon_2}{N_0}} + \sqrt{\frac{2\varepsilon_1}{N_0}}) + Q(\sqrt{\frac{2\varepsilon_2}{N_0}} - \sqrt{\frac{2\varepsilon_1}{N_0}})], \tag{6}$$

and the SER of U_1 , is expressed as

$$P_{2-1} = \frac{1}{2} [2Q(\sqrt{\frac{2\varepsilon_1}{N_0}}) - Q(\sqrt{\frac{2\varepsilon_1}{N_0}} + \sqrt{\frac{2\varepsilon_2}{N_0}}) - Q(\sqrt{\frac{2\varepsilon_1}{N_0}} - \sqrt{\frac{2\varepsilon_2}{N_0}}) + Q(\sqrt{\frac{2\varepsilon_1}{N_0}} + 2\sqrt{\frac{2\varepsilon_2}{N_0}}) + Q(\sqrt{\frac{2\varepsilon_1}{N_0}} - 2\sqrt{\frac{2\varepsilon_2}{N_0}})], \tag{7}$$

where Q function is defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$.

Proof. The receiver decodes s_2 while treating s_1 as interference.

$$\begin{aligned}
 P_{2-2} &= 2 \int_{-\infty}^0 p_2(s_2 = \sqrt{\varepsilon_2}) dx \\
 &= \frac{1}{2} \int_{-\infty}^0 \mathcal{N}(\sqrt{\varepsilon_2} + \sqrt{\varepsilon_1}, \frac{N_0}{2}) + \mathcal{N}(\sqrt{\varepsilon_2} - \sqrt{\varepsilon_1}, \frac{N_0}{2}) dx \\
 &= \frac{1}{2} [Q(\sqrt{\frac{2\varepsilon_2}{N_0}} + \sqrt{\frac{2\varepsilon_1}{N_0}}) + Q(\sqrt{\frac{2\varepsilon_2}{N_0}} - \sqrt{\frac{2\varepsilon_1}{N_0}})],
 \end{aligned} \tag{8}$$

where $\mathcal{N}(m, \sigma)$ denotes the probability density functions of Gaussian random variables with mean m and covariance σ . The (6) is obtained.

When U_2 's signal is correctly decoded, the SER of U_1 can be expressed as

$$\begin{aligned}
 P_{2-1}' &= 2[\int_{-\infty}^0 p_2(s_1 = \sqrt{\varepsilon_1}, s_2 = \sqrt{\varepsilon_2}) dx + \int_{-\infty}^0 p_2(s_1 = \sqrt{\varepsilon_1}, s_2 = -\sqrt{\varepsilon_2}) dx] \\
 &= \frac{1}{2} [\int_{-\sqrt{\varepsilon_2}}^0 \mathcal{N}(\sqrt{\varepsilon_1}, \frac{N_0}{2}) dx + \int_{-\infty}^0 \mathcal{N}(\sqrt{\varepsilon_1}, \frac{N_0}{2}) dx] \\
 &= \frac{1}{2} [2Q(\sqrt{\frac{2\varepsilon_1}{N_0}}) - Q(\sqrt{\frac{2\varepsilon_1}{N_0}} + \sqrt{\frac{2\varepsilon_2}{N_0}})].
 \end{aligned} \tag{9}$$

The receiver adopts SIC to decode s_2 , reconstructs, and subtracts s_2 from the received signal as shown in (9). It's worth noting that when $s_1 = \sqrt{\varepsilon_1}$, $s_2 = \sqrt{\varepsilon_2}$, the subtraction process of SIC affects the lower bound of the integration interval.

Similarly, when U_2 's signal is incorrectly decoded, the SER of U_1 can be expressed as

$$\begin{aligned}
 P_{2-1}'' &= \frac{1}{2} [\int_{-\infty}^0 \mathcal{N}(\sqrt{\varepsilon_1} + 2\sqrt{\varepsilon_2}, \frac{N_0}{2}) dx + \int_{-\sqrt{\varepsilon_2}}^0 \mathcal{N}(\sqrt{\varepsilon_1} - 2\sqrt{\varepsilon_2}, \frac{N_0}{2}) dx] \\
 &= \frac{1}{2} [Q(\sqrt{\frac{2\varepsilon_1}{N_0}} + 2\sqrt{\frac{2\varepsilon_2}{N_0}}) + Q(\sqrt{\frac{2\varepsilon_1}{N_0}} - 2\sqrt{\frac{2\varepsilon_2}{N_0}}) - Q(\sqrt{\frac{2\varepsilon_1}{N_0}} - \sqrt{\frac{2\varepsilon_2}{N_0}})].
 \end{aligned} \tag{10}$$

The total SER of U_1 can be expressed as

$$P_{2-1} = P_{2-1}' + P_{2-1}'' \tag{11}$$

Thus, the (7) is obtained. ■

In multi-user case, we rewrite the received signal as $s_l = i_l \sqrt{\varepsilon_l}$, $1 \leq l \leq L$, where i_l is chosen from the set $\{1, -1\}$ with equal probability.

Theorem 2. *Considering the system with L users, the SER for U_L can be expressed as*

$$P_{L-L} = \frac{1}{2^{L-1}} \sum_{i_1, i_2, \dots, i_{L-1} \in \{-1, 1\}} Q(\sqrt{\frac{2\varepsilon_L}{N_0}} + \sum_{j=1}^{L-1} i_j \sqrt{\frac{2\varepsilon_j}{N_0}}). \tag{12}$$

For $1 < l < L$, the SER of U_l , can be calculated as

$$\begin{aligned}
 P_{L-l} &= \frac{1}{2^{L-1}} \sum_{i_1, \dots, i_{l-1} \in \{-1, 1\}} \left\{ \sum_{i_l, \dots, i_L \in \{-1, 1\}} \sum_{k_{l+1}, \dots, k_L \in \{-1, 1\}} \right. \\
 & 2^{\frac{1}{2} \sum_{n=l+1}^L (k_n+1)} Q\left[\sqrt{\frac{2\varepsilon_l}{N_0}} + \sum_{j=1}^{l-1} i_j \sqrt{\frac{2\varepsilon_j}{N_0}} \right. \\
 & \left. + \sum_{n=l+1}^L (1 - k_n) i_n \sqrt{\frac{2\varepsilon_n}{N_0}} \right] - \sum_{m=l+1}^L \sum_{i_{l+1}, \dots, i_L \in \{-1, 1\}} \sum_{k_{m+1}, \dots, k_L \in \{-1, 1\}} \\
 & 2^{\frac{1}{2} \sum_{n=m+1}^L (k_n+1)} Q\left[\sqrt{\frac{2\varepsilon_l}{N_0}} + \sum_{j=1}^{l-1} i_j \sqrt{\frac{2\varepsilon_j}{N_0}} + \sum_{n=l+1}^m i_n \sqrt{\frac{2\varepsilon_n}{N_0}} \right. \\
 & \left. \left. + \sum_{m \neq L, n=m+1}^L (1 - k_n) i_n \sqrt{\frac{2\varepsilon_n}{N_0}} \right] \right\}. \tag{13}
 \end{aligned}$$

Specially, the SER for U_1 can be expressed as

$$\begin{aligned}
 P_{L-1} &= \frac{1}{2^{L-1}} \left\{ \sum_{i_2, \dots, i_L \in \{-1, 1\}} \sum_{k_2, \dots, k_L \in \{-1, 1\}} 2^{\frac{1}{2} \sum_{n=2}^L (k_n+1)} \right. \\
 & Q\left[\sqrt{\frac{2\varepsilon_1}{N_0}} + \sum_{n=2}^L (1 - k_n) i_n \sqrt{\frac{2\varepsilon_n}{N_0}} \right] \\
 & - \sum_{m=2}^L \sum_{i_2, \dots, i_L \in \{-1, 1\}} \sum_{k_{m+1}, \dots, k_L \in \{-1, 1\}} 2^{\frac{1}{2} \sum_{n=m+1}^L (k_n+1)} \\
 & \left. Q\left[\sqrt{\frac{2\varepsilon_1}{N_0}} + \sum_{n=2}^m i_n \sqrt{\frac{2\varepsilon_n}{N_0}} + \sum_{m \neq L, n=m+1}^L (1 - k_n) i_n \sqrt{\frac{2\varepsilon_n}{N_0}} \right] \right\}. \tag{14}
 \end{aligned}$$

Proof. The derivation process of (12) is similar to (8). For $1 < l < L$, the SER can be expressed as

$$\begin{aligned}
 P_{L-l} &= 2 \int_{-\infty}^0 p_L(s_l = \sqrt{\varepsilon_l}) dx \\
 &= \frac{1}{2^{L-1}} \left[\sum_{i_1, \dots, i_{l-1}, i_{l+1}, \dots, i_L \in \{-1, 1\}} \sum_{k_{l+1}, \dots, k_L \in \{-1, 1\}} \right. \\
 & \left. \int_{LB}^0 \phi(x|s_1, \dots, s_l = \sqrt{\varepsilon_l}, \dots, s_L) dx \right], \tag{15}
 \end{aligned}$$

where

$$s_j = \begin{cases} i_j \sqrt{\varepsilon_j}, & \text{if } 1 \leq j < l, \\ (1 - k_j) i_j \sqrt{\varepsilon_j}, & \text{if } l < j \leq L. \end{cases} \tag{16}$$

LB is the lower bound of the integration interval and can be calculated as follows

$$LB = \begin{cases} -\sqrt{\varepsilon_m}, & \text{if } m \notin \emptyset \text{ and } m = l + 1, \\ -\sqrt{\varepsilon_m} + \sum_{n=l+1}^{m-1} \sqrt{\varepsilon_n}, & \text{if } m \notin \emptyset \text{ and } m \neq l + 1, \\ -\infty, & \text{if } m \in \emptyset, \end{cases} \quad (17)$$

where m is intermediate variable and can be calculated as

$$m = \min_{i_j k_j > 0, l < j \leq L} j. \quad (18)$$

We substitute (16)–(18) into (15) and (13)–(14) are obtained. \blacksquare

When all users use QPSK modulated signals, it is in fact two binary phase-modulation signals in phase quadrature. The SER can be calculated as

$$P'_{i-j} = 1 - (1 - P_{i-j})^2, \quad (19)$$

where P_{i-j} and P'_{i-j} correspond to SER when all users adopt BPSK and QPSK modulation, respectively.

4 Numerical Results

In this section, several sets of experiments using Monte Carlo simulation are designed to verify the expression obtained beforehand. We first illustrate the limitations of the approximate expression used in [12], which we call the approximate results, through the case of a two-user NOMA system. Then we show that our quantization expression is effective under the multi-user case.

In Fig. 2 and Fig. 3, we illustrate the SER performance comparisons between our method and the approximate quantization method through a two-user NOMA system. In our experiment, we change the received power ε_1 of U_1 while keeping the received power ε_2 of U_2 unchanged. The x-axis represents the ε_1/N_0 of U_1 . Both users adopt BPSK modulation and we set $\varepsilon_2/N_0 = 12$ dB and $\varepsilon_2/N_0 = 10$ dB respectively. Monte Carlo simulation results, our quantified expression and the approximate quantified expression shows consistency in Fig. 2. However, in Fig. 3, the approximate quantization method cannot fit the SER performance curve of user 1. It is only suitable for occasions where the error propagation phenomenon is not obvious, and cannot accurately quantify the performance.

Figure 4 illustrates the SER performance curve of four-user NOMA system. All users adopt BPSK modulation. It shows perfect consistency between our expression and Monte Carlo simulation. In our experiment, we change the received power ε_1 of U_1 while keeping the received power of other users unchanged, to explore the relationship between the SER performance of each user and the corresponding ε_1/N_0 . We set the received power of other users as follows. $\varepsilon_4/N_0 = 23.2$ dB, $\varepsilon_3/N_0 = 17.2$ dB, $\varepsilon_2/N_0 = 11.1$ dB. It can be observed

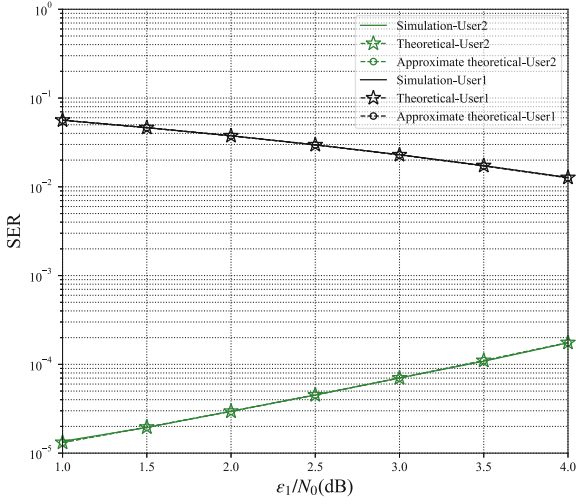


Fig. 2. SER performance comparisons of two-user NOMA system in AWGN channel ($\epsilon_2/N_0 = 12$ dB).

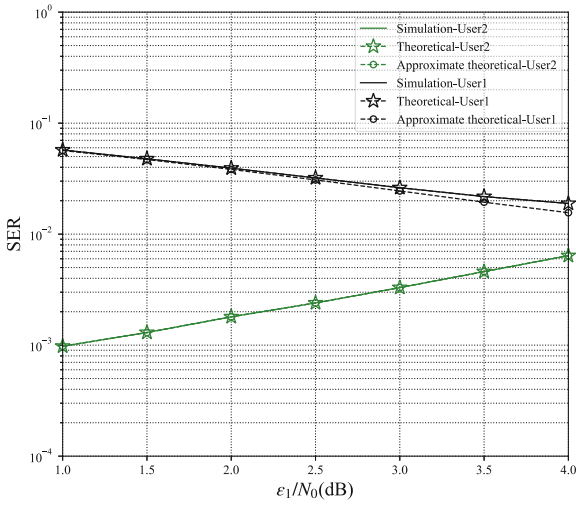


Fig. 3. SER performance comparisons of two-user NOMA system in AWGN channel ($\epsilon_2/N_0 = 10$ dB).

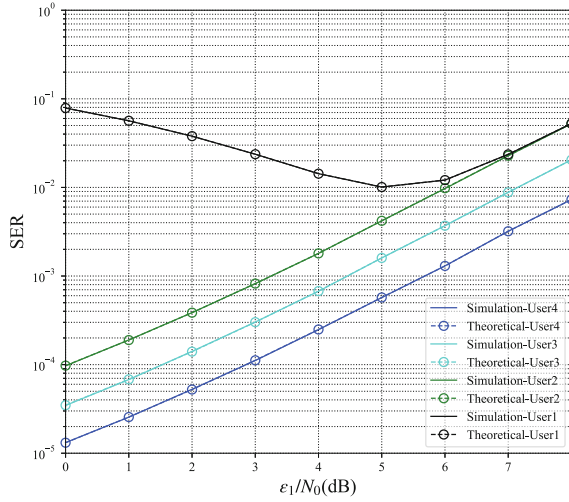


Fig. 4. SER performance of 4-user NOMA system in AWGN channel.

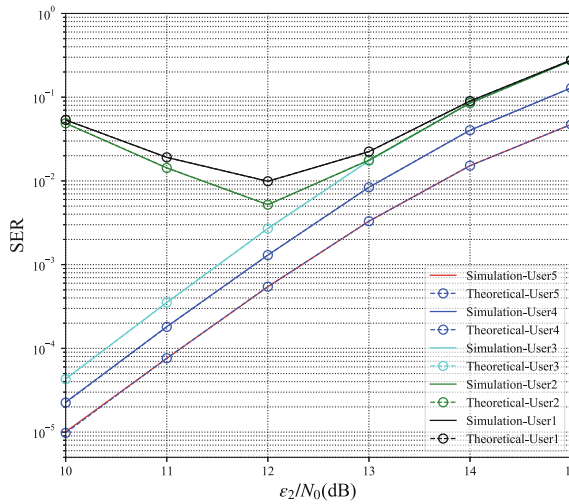


Fig. 5. SER performance of 5-user NOMA system in AWGN channel.

from the figure that the SER performance of other users decreases as ϵ_1/N_0 increases. This is because when other users are being decoded, U_1 's signal is regarded as interference. The stronger the power of U_1 , the stronger the interference, which causes performance degradation. Meanwhile, the SER of U_1 increases and then decreases as ϵ_1/N_0 increases. The main reason for this phenomenon is that when ϵ_1/N_0 is lower than 5 dB (5 dB is the approximate minimum point. The exact point can be calculated by the obtained expression), the SER

performance improves as the signal power increases. When ε_1/N_0 exceeds 5 dB, the extra power causes previous users to suffer more interference during decoding, which leads to error propagation. In the decoding process of U_1 , the performance degradation caused by error propagation exceeds the performance improvement brought about by the increase of its power, and the overall performance is degraded. We use the same experimental method to explore the performance of the QPSK case. In Fig. 5, the SER performance of the five-user NOMA system is tested. In this experiment, we change the received power ε_2 of U_2 while keeping the received power of other users constant. The received power settings of other users are as follows. $\varepsilon_5/N_0 = 30.0$ dB, $\varepsilon_4/N_0 = 24.0$ dB, $\varepsilon_3/N_0 = 18.0$ dB, $\varepsilon_1/N_0 = 6.0$ dB. From the figure we can draw the same conclusion as in Fig. 4. It is worth noting that when ε_2/N_0 is higher (greater than 14 dB), the SER curves of U_1 , U_2 and U_3 coincide. This is because in this case, the error propagation phenomenon is severe, and the decoding failure of the previous user almost certainly causes the decoding failure of the subsequent users. As can be seen from Fig. 4 and Fig. 5, our quantized expression can fit the performance curve accurately even when the error propagation is severe. That confirms the effectiveness.

5 Conclusion

NOMA is identified as a promising technology for massive IoT applications. This paper quantified the SER performance accurately for the uplink multi-user NOMA system over the AWGN channel. The corresponding simulation results verified the correctness of the derived expression. Based on the expression proposed in this paper, we can solve many problems effectively which cannot be solved by the traditional Shannon formula, thus improve the performance of the entire system.

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