Average Age of Incorrect Information in Random Access Channels for IoT Systems

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Abstract. Age of incorrect information (AoII) has been proposed recently to overcome the shortcomings of age of information (AoI) in internet of things (IoT) systems. AoII takes into account the content of the information by penalizing the sink only when it has an incorrect perception of the monitored source. This is of paramount importance for scenarios where actuations are taken based on the current data sample. On the other hand, random access (RA) has been identified as a promising solution for supporting next-generation IoT systems. Therefore, a thorough understanding of the behaviors of RA policies from the perspective of AoII is key for the design of IoT systems. In this paper, we study two representative RA schemes, namely slotted ALOHA (SA) and irregular repetition slotted ALOHA (IRSA), with Markov sources. We track the AoII evolution for both schemes through a Markovian analysis, where state transition probabilities are derived and closed form expressions for the average AoII are obtained. Simulation results are provided to validate our analysis. The study reveals the influences of the Markov source on the system performance as well as the design trade-offs for IRSA. Furthermore, the performance of SA and IRSA are compared under various settings, showing the cases where IRSA can largely outperform SA in terms of average AoII.

Keywords: Age of incorrect information · Random access · Slotted ALOHA · Irregular repetition slotted ALOHA · IoT

1 Introduction

As a key component of the next-generation wireless communication network, internet of things (IoT) has served as a rich source of research problems at all protocol layers. In particular, massive machine type communications (mMTC) are characterized by a very large number of terminals sporadically communicating to a central gateway over a shared wireless channel in an unpredictable fashion. In this situation, at the link layer, traditional grant-based access policies

This research was funded by the National Natural Science Foundation of China under grant numbers 62071146 and 62171151 and the Fundamental Research Funds for the Central Universities (No. HIT.OCEF. 2021012).

J. Zhao (Ed.): WiSATS 2023, LNICST 509, pp. 109–129, 2023.
https://doi.org/10.1007/978-3-031-34851-8_9

become especially inefficient due to the high signaling overhead as well as the high complexity of coordination. Random access (RA) schemes, thanks to their grant-free nature and simplicity, have attracted more attention. Several IoT commercial solutions and standards (e.g., NB-IoT [19], SigFox [18], LoRa [8], etc.) are based on the simple ALOHA protocol and its variations.

In RA, packets transmitted by different terminals may collide with each other, which has limited the number of terminals that can be supported. Irregular repetition slotted ALOHA (IRSA) proposed in [7] represents one of the most appealing solutions to overcome packet collisions. The basic idea of IRSA is to transmit multiple packet replicas at the transmitter side within a frame consisting of slots and employ successive interference cancellation (SIC) at the receiver side to resolve packet collisions. The throughput of IRSA has been shown to be comparable to that of grant-based schemes. Due to its potential, the ETSI DVB-RCS2 standard has included IRSA as the link layer protocol for return-link satellite communications [4] and IRSA variations [14,17] have been developed, which further improve the performance of IRSA.

On the other hand, many IoT applications (e.g., environmental monitoring, industrial automation, etc.) see the need to have a best real-time estimation of the remote process monitored by each terminal. In these cases, a fresh view of each terminal at the gateway is critical for correct and efficient decision making. Traditional metrics (e.g., throughput, delay, packet loss rate (PLR)) commonly used for designing RA schemes cannot well capture the notion of information freshness and age of information (AoI) [5] has emerged as a pioneering metric to overcome the shortcomings of traditional metrics. AoI is defined as the time elapsed since the generation time stamp of the last received packet at the receiver. Triggered by this novel concept, existing RA schemes designed based on traditional metrics have been revisited and redesigned.

The formula for the average AoI (AAoI) of slotted ALOHA (SA) was first derived in [20], which was then extended to pure ALOHA in [21]. It was revealed that the AAoI of SA and the system throughput are inversely proportional. To break this limitation, a threshold-based SA policy relying on the feedback from the receiver was proposed in [2], where terminals keep silent before their ages reach a fixed age threshold. The age performance of this policy was later analyzed in [22], showing a significant improvement over SA. In parallel, the first analysis of AoI for IRSA was presented in [11] and [10], providing a powerful tool for designing age-optimal IRSA schemes. It was demonstrated that IRSA exhibits a remarkable potential in terms of AoI compared to the SA strategy. Another independent analysis and optimization of AoI for IRSA was reported in [15] with different settings on the packet generation model and the SIC process. Leaning on the results in [10], the optimization of IRSA with heterogeneous terminals requiring different levels of AoI was performed in [13].

Although AoI has been widely deemed as a fundamental metric for communication systems, it has some intrinsic limitations. That the value of AoI increases linearly with time may become unreliable for quantifying the effect of stale information. In view of this, a more general age penalty function following a power law of the time elapsed has been investigated for SA in Gilbert-Elliot channels in [12]. Another important limitation is that AoI does not account for the information content of the update packets nor the current knowledge at the receiver. For instance, the age penalty keeps growing even when the receiver has perfect knowledge of the monitored process. To remedy this, a new metric called age of incorrect information (AoII) has been proposed in [9]. AoII is defined as the time elapsed since the receiver has an incorrect perception of the monitored process. Therefore, AoII measures the staleness of the information content rather than of the time stamps. Motivated by this, a recent work [6] studied the problem of scheduling multiple terminals, each monitoring a Markov process, with the aim to minimize the mean AoII.

To the best of our knowledge, AoII has not been explored for RA schemes. Since RA is of great significance for supporting mMTC, it is necessary to look at RA schemes from an AoII viewpoint. In this paper, we provide the first study of the AoII metric for SA and IRSA with Markov sources. Specifically, closed form expressions of average AoII are derived for SA and IRSA. Based on this, the influences of Markov sources on the average AoII performance are analyzed. Our results reveal in what cases IRSA can largely outperform SA and how the optimal operating frame length of IRSA heavily depends on the nature of Markov sources being tracked. The analytical results can be readily used as an efficient tool for determining optimal system operating parameters, which is important for deploying RA schemes for mMTC.

The rest of the paper is organized as follows. Section 2 introduces the system model and some preliminaries. Then, derivations of average AoII for IRSA and SA are provided in Sect. 3. Simulation results are presented and discussed in Sect. 4. Finally, Sect. 5 concludes the paper.

2 System Model and Preliminaries

2.1 Network Model

We focus on a communication system where N_u terminals, each equipped with sensors, monitor processes of interest and send update packets over a shared wireless channel to a central gateway using RA strategies. The gateway updates estimations of these remote processes based on the last received packets. The aim of the communication system is to have the best real-time estimation of the process at each terminal.

Time is divided into slots of same duration, each fitting one packet transmission and all terminals are synchronized to this pattern. In the rest of this paper, we consider time to be discrete and normalized to the slot duration.

More concretely, terminal i observes a discrete-time random process $(X_i(t))_{t\in\mathbb{N}}$ and the gateway maintains an estimation of the process, denoted by $(\hat{X}_i(t))_{t\in\mathbb{N}}$. Packets are generated by sampling the process and delivered to the gateway over a RA channel with delay D. Denote $I_i(t)$ as the indicator for packet reception such that $I_i(t) = 1$ if an update packet from terminal i is successfully decoded at time

t and $I_i(t) = 0$ otherwise. Assume that the sampling time is negligible compared to the transmission delay. Then, the estimation process is updated by the gateway following

$$\hat{X}_i(t) = \begin{cases} X_i(t-D) & I_i(t) = 1 \\ \hat{X}_i(t-1) & I_i(t) = 0. \end{cases} \tag{1}$$

It is important to observe that the information brought by the update packet is not real-time due to the time delay. And the estimation process keeps its value when no packet is successfully decoded.

The random process $(X_i(t))_{t\in\mathbb{N}}$ monitored by terminal i is considered to be a discrete-time Markov chain with N_s states $\{S_1, \ldots, S_{N_s}\}$. The one-step transition probabilities are given by

$$\begin{aligned} p_s(j,k) &= P(X_i(t+1) = S_k \mid X_i(t) = S_j) \\ &= \begin{cases} p_R & j = k \\ p_t & j \neq k, \end{cases} \end{aligned} \tag{2}$$

where $0 < p_R < 1$ denotes the probability of remaining at the same state and $0 < p_t < 1$ the probability of transitioning to another state. It follows immediately that

$$p_R + (N_s - 1)p_t = 1. \tag{3}$$

We assume that processes at different terminals are independent of each other.

Next, we define two quantities regarding this Markov chain, which will be useful in Set. III and IV. Without risk of confusion, we drop the terminal index i. Let T_t denote the first passage time from state S_j to S_k $(j \neq k)$, i.e., $T_t = \min\{n \geq \ 2 \text{suchthat} X(n) = S_k \mid X(1) = S_j\} - 1$. Using the method in [3, Section 7.4], the expectation of T_t can be obtained as

$$\overline{T_t} = \frac{1}{p_t}. \tag{4}$$

Note that $\overline{T_t}$ can also be interpreted as the mean time it takes for the Markov chain to transit back to the same state after it leaves the state.

Let T_R be the time duration of staying at the same state. Then, its expectation can be readily calculated as

$$\overline{T_R} = \sum_{k=0}^{\infty} k p_R^k (1 - p_R) + 1 = \frac{1}{1 - p_R}. \tag{5}$$

2.2 Random Access Schemes

As for RA strategies, SA and IRSA are considered for transmitting update packets. The simple collision model is adopted for packet reception, i.e., packets in collisions are considered to be lost and packets without collisions are always successfully decoded. For both schemes, no feedback nor retransmissions are considered.

Slotted ALOHA (SA). In SA, each terminal becomes active independently at each slot with probability μ. Each active terminal samples its process of interest at the start of the slot and transmits an update packet to the gateway. Each packet transmission consumes one slot, thus delay $D = 1$.

The channel load, defined as the average number of active terminals per slot, is

$$G_{sa} = N_u \mu.$$

Denote PLR_{sa} as the probability for a packet to be lost in SA. Based on the collision model, a packet can be successfully decoded when no other terminals are active at the same slot. Therefore, $PLR_{\text{sa}} = 1 - (1 - \mu)^{(N_u - 1)}$. Then, the throughput of SA, defined as the average number of successfully decoded packets per slot, can be obtained as

$$S_{sa} = G_{sa}(1 - PLR_{\text{sa}}) = N_u \mu (1 - \mu)^{(N_u - 1)}. \tag{6}$$

Irregular Repetition Slotted ALOHA (IRSA) [7]. In IRSA, slots are organized into frames, each consisting of m slots. Each terminal becomes active at each frame independently with probability ρ. Each active terminal samples its process of interest at the start of the frame and generates an update packet. Then, r replicas of this packet are created with probability $\Lambda(r)$ and these replicas are transmitted at r slots uniformly selected within the frame. The degree distribution of IRSA is defined as

$$\Lambda(x) = \sum_{r=1}^{r_m} \Lambda(r) x^r,$$

where r_m denotes the maximum number of replicas. Similarly to SA, the channel load for IRSA is defined as

$$G_{irsa} = N_u \rho / m.$$

At the gateway, after a whole frame is received, the SIC process is performed to decode packets. We assume that the time of executing the SIC process is negligible in comparison with the frame length. Then, due to the frame structure, each packet in IRSA is decoded with delay $D = m$. Denoting the probability for a packet in IRSA not to be successfully decoded as PLR_{irsa}, the throughput of IRSA can be expressed as

$$S_{irsa} = G_{irsa}(1 - \text{PLR}_{irsa}) = \frac{N_u \rho}{m}(1 - \text{PLR}_{irsa}). \tag{7}$$

The calculation of PLR_{irsa} is known to be difficult, for which no exact closed form expressions exist in the literature. In this paper, we adopt the method introduced in [10], which combines the results from [1] and [16] and achieves good approximations for calculating PLR_{irsa}. Specifically, PLR_{irsa} is calculated as

$$\text{PLR}_{irsa} = P_{wf}(G_{irsa}, \Lambda(x)) + P_{ef}(G_{irsa}, \Lambda(x)), \tag{8}$$

where P_{wf} and P_{ef} represent the PLR of IRSA in the waterfall and error-floor region respectively. The expressions for these two functions can be found in [1] and [16] respectively.

2.3 Age of Incorrect Informaiton (AoII) [9]

The gateway relies on estimation $\hat{X}_i(t)$ to make decisions or drive feedback loops. Therefore, the gateway should be increasingly penalized when staying in an erroneous state ($\hat{X}_i(t) \neq X_i(t)$) and not penalized when having perfect knowledge of the monitored process. The metric AoII well captures this notion. Mathematically, AoII for terminal i at time t is defined as [9]

$$\Delta_i(t) = (t - V_i(t))\mathbb{1}\{\hat{X}_i(t) \neq X_i(t)\}, \tag{9}$$

where $\mathbb{1}\{.\}$ is the indicator function and

$$V_i(t) = \max\{t_0 < t \text{ such that } \hat{X}_i(t_0) = X_i(t_0)\}$$

denotes the last time instant where the estimation is correct. An example of AoII evolution with $N_s = 3$ according to (9) is provided in Fig. 1. At slot 1, 2 and 5, the gateway has perfect estimation of the process, leading to zero penalty. In contrast, AoII increases at slot 3 and 4, resulting from the erroneous estimation of the process.

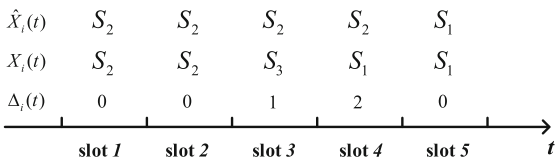


Fig. 1. An example of AoII evolution with $N_s = 3$.

In this paper, we are interested in the average AoII

$$\overline{\Delta} = \frac{1}{N_u}\sum_{i=1}^{N_u}\overline{\Delta}_i, \tag{10}$$

where $\overline{\Delta}_i$ is the average AoII of terminal i, which is defined as

$$\overline{\Delta}_i = \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\Delta_i(t). \tag{11}$$

3 Derivations of Average AoII for SA and IRSA

In this section, we derive the expressions of average AoII for SA and IRSA by means of a Markovian analysis similar to that in [10]. Note that all terminals in the system operate independently of each other, thus

$$\overline{\Delta} = \frac{1}{N_u}\sum_{i=1}^{N_u}\overline{\Delta}_i = \overline{\Delta}_i. \tag{12}$$

Based on this, in the following, we concentrate on the evolution of $\Delta_i(t)$ for a generic terminal i.

From (9) and Fig. 1, we can observe that the value of AoII at next slot will either increase by 1 if the gateway stays in an erroneous state or be reset to 0 if the gateway has a correct estimation, which can be expressed as

$$\Delta_i(t+1) = \begin{cases} \Delta_i(t)+1 & \hat{X}_i(t+1) \neq X_i(t+1) \\ 0 & \hat{X}_i(t+1) = X_i(t+1). \end{cases} \quad (13)$$

The random process $X_i(t)$ is characterized by the Markov chain introduced in Sect. 2, while the estimation process $\hat{X}_i(t)$ is related to the RA scheme. We first analyze the case of SA.

3.1 Slotted ALOHA Average AoII

Recall that in SA, each packet transmission consumes one slot, i.e., $D = 1$. According to (1), $\hat{X}_i(t+1)$ is obtained as

$$\hat{X}_i(t+1) = \begin{cases} X_i(t) & I_i(t+1) = 1 \\ \hat{X}_i(t) & I_i(t+1) = 0. \end{cases} \quad (14)$$

Leaning on this and noticing that all terminals operate independently over successive slots, the stochastic process $(\Delta_i(t))_{t\in\mathbb{N}}$ in SA is therefore Markovian and can be fully characterized by a discrete-time Markov chain with state space $\mathbb{N}_0$. Next, we derive one-step transition probabilities

$$p_{sa}(j, k) = P(\Delta_i(t+1) = k \,|\, \Delta_i(t) = j), \quad j, k \in \mathbb{N}_0 \quad (15)$$

describing this Markov chain by considering two cases:

$\boldsymbol{\Delta_i(t) = 0}$. In this case, the gateway has perfect knowledge of the remote process at current slot, i.e., $\hat{X}_i(t) = X_i(t)$. According to (14), the estimation at next slot will be updated as $\hat{X}_i(t+1) = X_i(t)$. By (13), $\Delta_i(t+1)$ will equal 0 if the remote process remains at its current state at next slot, which happens with probability p_R; $\Delta_i(t+1)$ will equal 1 if the remote process leaves its current state, which occurs with probability $1 - p_R$.

$\boldsymbol{\Delta_i(t) \neq 0}$. In this case, the gateway has an incorrect perception of the remote process at current slot, i.e., $\hat{X}_i(t) \neq X_i(t)$. According to (14), the estimation at next slot depends on the packet reception result. Specifically, $\hat{X}_i(t+1) = X_i(t)$ if terminal i generates a packet and the packet is received with no collisions. This event happens with probability $\mu(1 - PLR_{sa}) = S_{sa}/N_u$. In this situation, $\Delta_i(t+1)$ will equal 0 if the remote process remains at its current state, which happens with probability p_R and $\Delta_i(t)+1$ if the remote process leaves its current state, which occurs with probability $1 - p_R$.

On the other hand, if no packet is successfully decoded, then $\hat{X}_i(t+1) = \hat{X}_i(t) \neq X_i(t)$. In this situation, $\Delta_i(t+1)$ will equal 0 if the remote process transits to state $\hat{X}_i(t)$, which happens with probability p_t and $\Delta_i(t)+1$ if the process remains at its current state or transits to other $N_s - 2$ states, which occurs with probability $1 - p_t$.

We summarize above results by expressing (15) as follows:

$$p_{sa}(j,k) = \begin{cases} p_R & j=0, k=0 \\ S_{sa}/N_u p_R + (1 - S_{sa}/N_u)p_t & j \neq 0, k=0 \\ 1 - p_{sa}(j,0) & k = j+1 \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

Leaning on (16), we are able to track AoII for SA at any time given the initial AoII. Nevertheless, we are interested in the long-term average AoII, for which a closed-from expression is provided as follows:

Proposition 1. *The average AoII of a SA system monitoring remote Markov processes of Sect. 2, measured in slots, is given by*

$$\overline{\Delta}_{sa} = \frac{1 - p_R}{\alpha(\alpha + 1 - p_R)}, \tag{17}$$

where $\alpha = S_{sa}/N_u p_R + (1 - S_{sa}/N_u)p_t$.

Proof. We first show that the Markov chain characterizing the stochastic process $(\Delta_i(t))_{t\in\mathbb{N}}$ of SA is irreducible, which requires that there exist $n > 0$ such that the n-step transition probability between any state-pair (j,k) is strictly positive. Based on (16), we can observe that for $k > j$, the transition from j to k can take place in $k - j$ steps with probability $(1-\alpha)^{k-j} > 0$ if $j \neq 0$ and $(1-p_R)(1-\alpha)^{k-j-1} > 0$ if $j = 0$. Otherwise, the transition can occur by first moving to state 0 in one step and then to state k in k steps. This event happens with probability $\alpha(1-p_R)(1-\alpha)^{k-1} > 0$ if $j \neq 0$ and $p_R > 0$ if $j = 0$. Therefore, the statement of irreducibility holds. From (16), we can also observe that state 0 has period 1, indicating that the chain is aperiodic. Since the chain is irreducible and aperiodic, steady-state probabilities $\{\pi_k\}$, $k \in \mathbb{N}_0$ exist and can be obtained by solving the balance equations:

$$\begin{cases} \pi_k = \pi_0 p_R + \sum_{k=1}^{\infty} \pi_k \alpha \overset{(a)}{=} \pi_0 p_R + (1-\pi_0)\alpha & k=0 \\ \pi_k = \pi_0 (1-p_R)(1-\alpha)^{k-1} & k > 0 \end{cases} \tag{18}$$

where (a) is due to the normalization equation $\sum_{k=0}^{\infty} \pi_k = 1$. For $k = 0$, we can readily compute π_k from the first equation in (18) as

$$\pi_0 = \frac{\alpha}{1 + \alpha - p_R}. \tag{19}$$

Otherwise π_k can be obtained directly by substituting (19) into the second equation in (18).

It can be concluded that the process $(\Delta_i(t))_{t\in\mathbb{N}}$ is ergodic because it has positive steady-state probabilities. Based on this ergodicity, we have

$$\begin{aligned}\overline{\Delta}_{sa} &= \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\Delta_i(t) = \sum_{k=0}^{\infty}\pi_k k \\ &= \sum_{k=1}^{\infty}\frac{\alpha}{1+\alpha-p_R}(1-p_R)(1-\alpha)^{k-1}k \\ &= \frac{1-p_R}{\alpha(\alpha+1-p_R)}\end{aligned} \tag{20}$$

concluding the proof. □

3.2 IRSA Average AoII

Recall that in IRSA, due to the frame structure, each update packet is decoded with delay $D = m$. By (1), the estimation process in IRSA should be updated as

$$\hat{X}_i(t) = \begin{cases} X_i(t-m) & I_i(t) = 1 \\ \hat{X}_i(t-1) & I_i(t) = 0. \end{cases} \tag{21}$$

Without loss of generality, we start tracking the AoII process of IRSA at the beginning of a generic frame by setting $t = 1$ at the first slot of the frame. According to the IRSA protocol, the SIC decoding process starts after a whole frame is received. In other words, packets transmitted at current frame are decoded at the start of the next frame. Therefore, in IRSA, the estimation process can only be updated at time

$$t \in \Phi = \{t_l = (l-1)m + 1, l \in \mathbb{N}\},$$

i.e., at the first slot of each frame. No packets are decoded at other time, i.e., $I_i(t) = 0$ at time $t \notin \Phi$. Based on this observation, we divide time into two sets: $t \in \Phi$ and $t \notin \Phi$. According to (21), the estimation process at $t \notin \Phi$ keeps its value

$$\hat{X}_i(t_l + k) = \hat{X}_i(t_l), \quad k \in \{1, \ldots, m-1\} \tag{22}$$

and the estimation process at $t \in \Phi$ is updated as

$$\hat{X}_i(t_{l+1}) = \begin{cases} X_i(t_l) & I_i(t_{l+1}) = 1 \\ \hat{X}_i(t_{l+1}-1) & I_i(t_{l+1}) = 0 \end{cases} \tag{23}$$

Now let us focus on the evolution of $\Delta_i(t)$. Denote $T_l = \{t_l, \ldots, t_{l+1}-1\}$ as the set of time indexes within the l-th frame. By (13) and (22), we can see that the random process $(\Delta_i(t))_{t\in T_l}$ within frame l is Markovian. It can be characterized by a discrete-time Markov chain with initial state $\Delta_i(t_l)$ and state space $\mathbb{N}_0$. The one-step state transition probabilities

$$q_{irsa}(j,k) = P(\Delta_i(t+1) = k \mid \Delta_i(t) = j), \quad j, k \in \mathbb{N}_0 \tag{24}$$

for this chain can be derived by following similar steps in SA except that no packet reception needs to be considered. Therefore, we have

$$q_{irsa}(j,k) = \begin{cases} p_R & j = 0, k = 0 \\ p_t & j \neq 0, k = 0 \\ 1 - q_{irsa}(j,0) & k = j+1 \\ 0 & \text{otherwise.} \end{cases} \tag{25}$$

Next, we concentrate on AoII at the first slot of each frame, i.e., the random process $(\varDelta_i(t))_{t\in\varPhi}$, where the packet can be decoded and then the estimation process is updated. Since each terminal operates independently over successive frame in IRSA, $(\varDelta_i(t))_{t\in\varPhi}$ is also Markovian, which can be described by a discrete-time Markov chain with state space $\mathbb{N}_0$. The one-step transition probabilities

$$p_{irsa}(j,k) = P(\varDelta_i(t_{l+1}) = k \mid \varDelta_i(t_l) = j), \quad j,k \in \mathbb{N}_0 \tag{26}$$

can be obtained by considering two cases similar to that in SA, however, with important differences:

$\boldsymbol{\varDelta_i(t_l) = 0}$. In this case, $\hat{X}_i(t_l) = X_i(t_l)$. According to (23), $\hat{X}_i(t_{l+1})$ will be updated as $X_i(t_l)$ regardless of the packet reception result at time t_{l+1}. Therefore, $\varDelta_i(t_{l+1})$ can be viewed as the state at the m-th step for the Markov chain described by (25) given initial state $\varDelta_i(t_l) = 0$. Thus we have

$$P(\varDelta_i(t_{l+1}) = k \mid \varDelta_i(t_l) = 0) = q_{irsa}^{(m)}(0,k), k \in \{0,\ldots,m\}, \tag{27}$$

where $q_{irsa}^{(m)}$ denotes the m-step transition probability.

$\boldsymbol{\varDelta_i(t_l) \neq 0}$. In this case, $\hat{X}_i(t_l) \neq X_i(t_l)$ and $\hat{X}_i(t_{l+1})$ depends on the packet reception result at time t_{t+1}. If no packet is successfully decoded, then $\hat{X}_i(t_{l+1}) = \hat{X}_i(t_l)$, which happens with probability $(1-\rho) + \rho PLR_{irsa} = 1 - S_{irsa}m/N_u$. In this case, the distribution of $\varDelta_i(t_{l+1})$ is determined by the m-step transition probability of the Markov chain in (25)

$$\begin{aligned} &P(\varDelta_i(t_{l+1}) = k \mid \varDelta_i(t_l) \neq 0) = q_{irsa}^{(m)}(\varDelta_i(t_l), k), \\ &k \in \{0,\ldots,m-1, \varDelta_i(t_l)+m\},\ I_i(t_{l+1}) = 0. \end{aligned} \tag{28}$$

On the other hand, if an update packet is successfully decoded, then $\hat{X}_i(t_{l+1}) = X_i(t_l)$. Since the Markov chain describing the remote process is symmetric, without loss of generality, we can assume that $X_i(t_l) = S_1$. Then $\varDelta_i(t_{l+1})$ will equal 0, if the remote process remains at state S_1 at time t_{l+1}, i.e., $X_i(t_{l+1}) = S_1$, which happens with probability $p_s^{(m)}(1,1)$, denoting the m-th step transition probability of the Markov chain describing the remote process in (2).

$\varDelta_i(t_{l+1})$ will equal $\varDelta_i(t_{l+1}-1)+1$ if $X_i(t_{l+1}) \neq S_1$. Recall that the random process $\varDelta_i(t)$ within a frame is characterized by the Markov chain described by (25). The probability mass function (PMF) for random variable $\varDelta_i(t_{l+1}-1)$ given initial state $\varDelta_i(t_l) \neq 0$ can be obtained by

$$\begin{aligned} &P(\varDelta_i(t_{l+1}-1) = k \mid \varDelta_i(t_l) \neq 0) = q_{irsa}^{(m-1)}(\varDelta_i(t_l), k), \\ &k \in \{0,\ldots,m-2, \varDelta_i(t_l)+m-1\}. \end{aligned} \tag{29}$$

Therefore, the distribution of $\Delta_i(t_{l+1}) = \Delta_i(t_{l+1}-1)+1$ is given by

$$\begin{aligned}
&P(\Delta_i(t_{l+1}) = k+1 \,|\, \Delta_i(t_l) \neq 0) \\
&= q_{irsa}^{(m-1)}(\Delta_i(t_l), k) P(X_i(t_{l+1}) \neq S_1 \,|\, \Delta_i(t_{l+1}-1) = k) \\
&= q_{irsa}^{(m-1)}(\Delta_i(t_l), k)(1-p_t), \\
&k \in \{0, \dots, m-2, \Delta_i(t_l)+m-1\},\ I_i(t_{l+1}) = 1,
\end{aligned} \tag{30}$$

where $P(X_i(t_{l+1}) \neq S_1 \,|\, \Delta_i(t_{l+1}-1) = k)$ denotes the probability that the source process will not transition to state S_1 at next slot given current AoII $\Delta_i(t_{l+1}-1) = k$. This probability depends on the value of k. Specifically, if $k = 0$, which implies $X_i(t_{l+1}-1) = \hat{X}_i(t_{l+1}-1) = \hat{X}_i(t_l) \neq S_1$, then the probability can be readily obtained as $1-p_t$. If $k \neq 0$, the situation becomes more complex. Here we approximate this probability as $1-p_t$ for all k.

To summarize above results, we can write

$$\begin{aligned}
\alpha_0 =& P(\Delta_i(t_{l+1}) = 0 \,|\, \Delta_i(t_l) \neq 0) \\
=& (1-\frac{S_{irsa}m}{N_u}) q_{irsa}^{(m)}(\Delta_i(t_l), 0) + \frac{S_{irsa}m}{N_u} p_s^{(m)}(1,1), \\
\alpha_k =& P(\Delta_i(t_{l+1}) = k \,|\, \Delta_i(t_l) \neq 0) \\
=& (1-\frac{S_{irsa}m}{N_u}) q_{irsa}^{(m)}(\Delta_i(t_l), k) + \frac{S_{irsa}m}{N_u}(1-p_t) \\
& q_{irsa}^{(m-1)}(\Delta_i(t_l), k-1),\ k \in \{1, \dots, m-1\}, \\
\alpha_m =& P(\Delta_i(t_{l+1}) = \Delta_i(t_l)+m \,|\, \Delta_i(t_l) \neq 0) \\
=& 1-\alpha_0-\sum_{k=1}^{m-1}\alpha_k,
\end{aligned} \tag{31}$$

and (26) can be expressed as

$$p_{irsa}(j,k) = \begin{cases} q_{irsa}^{(m)}(0,k) & j = 0, k = \{0, \dots, m\} \\ \alpha_k & j \neq 0, k = \{0, \dots, m-1\} \\ \alpha_m & j \neq 0, k = j+m \\ 0 & \text{otherwise.} \end{cases} \tag{32}$$

In the following, we derive expressions for $p_s^{(m)}(1,1)$ and $q_{irsa}^{(m)}(j,k)$ in (27) and (31) so that we can get a full description of (32).

In general, for a Markov chain, m-step probabilities can be calculated from the m-th power of its one-step transition probability matrix. However, it is difficult to get closed form expressions using this method. In this paper, we consider the case where the average first passage time $\overline{T_t} = 1/p_t \gg m$, which will be explained in the next section. In this situation, some approximations can be made and closed form expressions are obtained. Specifically, there exist many routes that the chain can follow such that it starts from a certain initial state and reaches the destination state after m steps. Some of these routes have probabilities much larger than others, which we call the dominate routes in the rest of the paper. By calculating probabilities of those dominate routes, we can achieve a good approximation.

$p_s^{(m)}(1,1)$ represents the probability that starting from state S_1 the remote process will remain at the same state after m slots. Recall that $\overline{T_t}$ represents the average time needed for the source process to transit back to S_1 after it leaves S_1. Since $\overline{T_t} \gg m$, the remote process will remain at S_1 with very small probability if it leaves state S_1 during the m slots. Therefore, we can approximate $p_s^{(m)}(1,1)$ by considering the dominate route, where the remote process never leaves the initial state:

$$p_s^{(m)}(1,1) = p_R^m. \tag{33}$$

Recall that $q_{irsa}^{(m)}(j,k)$ denotes the probability that starting from $\Delta_i(t) = j$, the AoII will transit to $\Delta_i(t+m) = k$ after m slots. Note that during the m slots, no packets can be decoded and the estimation process keeps the same value. Without loss of generality, we assume that the initial state of the remote process $X_i(t) = S_1$. We need to consider two cases: $\Delta_i(t) = 0$ and $\Delta_i(t) \neq 0$.

In the case of $\Delta_i(t) = 0$, $\hat{X}_i(t) = X_i(t) = S_1$. $\Delta_i(t)$ will be reset to 0 at next slot if the remote process remains at S_1 and increase by 1 otherwise. Since $\overline{T_t} \gg m$, once the remote process leaves S_1, it will be unlikely for the process to return to S_1 at rest slots, which indicates that $\Delta_i(t)$ will increase till the end of the m slots. Therefore, we can approximate $q_{irsa}^{(m)}(0,k)$ by considering the dominate route, where the source process leaves the initial state S_1 after $m-k$ slots for $k \neq 0$ and never leaves the initial state for $k = 0$:

$$q_{irsa}^{(m)}(0,k) = \begin{cases} p_R^m & k = 0 \\ p_R^{m-k}(1-p_R) & k \in \{1,\ldots,m\}. \end{cases} \tag{34}$$

In the case of $\Delta_i(t) \neq 0$, $\hat{X}_i(t) \neq X_i(t)$. $\Delta_i(t+m)$ will equal $\Delta_i(t)+m$ if the remote process never visits $\hat{X}_i(t)$ during the m slots. This event has probability $(1-p_t)^m$. Otherwise, $\Delta_i(t+m)$ will take a value in $\{0,\ldots,m-1\}$. First, we consider $\Delta_i(t+m) = 0$. The dominate routes leading to $\Delta_i(t+m) = 0$ are those where the source process first transits to $\hat{X}_i(t)$ at time $t+t_0$, $t_0 \in \{1,\ldots,m\}$ and then remains at $\hat{X}_i(t)$ for the rest of the slots. By adding the probabilities of these routes and observing that $\overline{T_t} = 1/p_t \gg m > 1$ we have

$$\begin{aligned} q_{irsa}^{(m)}(j,0) &= \sum_{t_0=1}^{m} (1-p_t)^{t_0-1} p_t p_R^{m-t_0} \\ &= p_t \frac{1-p_R^m}{1-p_R} \overset{(3)}{=} \frac{1-p_R^m}{N_s-1},\ j \neq 0. \end{aligned} \tag{35}$$

For $\Delta_i(t+m) = k \in \{1, \ldots, m-1\}$, the dominate routes are those where the remote process first reaches state $\hat{X}_i(t)$ at time $t+m-k$ and leaves the state at the next slot, which happens with probability

$$\begin{aligned} q_{irsa}^{(m)}(j,k) &= q_{irsa}^{(m-k)}(j,0)(1-p_R) \\ &\overset{(35)}{=} \frac{1-p_R^{m-k}}{N_s-1}(1-p_R),\ j \neq 0,\ k \in \{1, \ldots, m-1\}. \end{aligned} \tag{36}$$

Substituting (33), (35) and (36) into (31) we can get

$$\begin{aligned} \alpha_0 =& (1-\frac{S_{irsa}m}{N_u})\frac{1-p_R^m}{N_s-1}+\frac{S_{irsa}m}{N_u}p_R^m, \\ \alpha_k =& (1-\frac{S_{irsa}m}{N_u})\frac{1-p_R^{m-k}}{N_s-1}(1-p_R)+\frac{S_{irsa}m}{N_u}(1-p_t) \\ & \frac{1-p_R^{m-k}}{N_s-1}(1-p_R) \overset{p_t \ll 1}{\approx} \frac{1-p_R^{m-k}}{N_s-1}(1-p_R), \\ & k \in \{1, \ldots, m-1\}, \\ \alpha_m =& 1-\alpha_0-\sum_{k=1}^{m-1}\alpha_k \\ =& 1-\alpha_0-p_t\left[m-1-(1-p_R^{m-1})\frac{p_R}{1-p_R}\right]. \end{aligned} \tag{37}$$

Plugging (34) and (37) into (32), we can obtain one-step transition probabilities for the Markov chain describing the evolution of AoII at the first slot of each frame in IRSA, i.e., $p_{irsa}(j,k)$ defined in (26). Leaning on these results, the expression for the average AoII of IRSA is provided as follows.

Proposition 2. *The average AoII of a IRSA system monitoring remote Markov processes of Sect. 2, measured in slots, is given by*

$$\overline{\Delta}_{irsa} = \overline{\Delta}_{irsa,0} + \overline{\Delta}_{irsa,+}, \tag{38}$$

where

$$\begin{aligned} \overline{\Delta}_{irsa,0} =& \theta_0\left\{\frac{m-1}{2}+\gamma_R/m\left[\gamma_R(1-p_R^{m-1})+1-m\right]\right\}, \\ \gamma_R =& \frac{p_R}{1-p_R}, \\ \theta_0 =& \frac{\alpha_0}{1+\alpha_0-p_R^m}, \\ \overline{\Delta}_{irsa,+} =& (1-\theta_0)\frac{m-1}{2}+(1-p_R)\frac{\theta_0 m\alpha_m}{1-\alpha_m}+\frac{m\alpha_m}{(1-\alpha_m)^2} \\ & \left[p_R(1-p_R^{m-1})\frac{N_s\theta_0-1}{N_s-1}+(1-\theta_0)(m-1)p_t\right] \\ & +\frac{p_t}{1-\alpha_m}\left[\gamma_R(m-1-\gamma_R+\gamma_R p_R^{m-1})(N_s\theta_0-1)\right. \\ & \left.+(1-\theta_0)m(m-1)/2\right]. \end{aligned}$$

Proof. Following similar steps in proofing (17) for SA, it can be shown that the Markov chain characterizing the random process $(\varDelta_i(t))_{t\in\varPhi}$ is irreducible and aperiodic. Therefore, steady-state probabilities $\{\theta_k\}$, $k \in \mathbb{N}_0$ exist and can be obtained by solving the balance equations:

$$\begin{cases} \theta_k = \theta_0 p_R^m + \sum_{j=1}^{\infty} \theta_j \alpha_0 & k = 0 \\ \theta_k = \theta_0 p_{irsa}(0,k) + \sum_{j=1}^{\infty} \theta_j \alpha_k & k \in \{1,\ldots,m-1\} \\ \theta_k = \theta_{\beta_k}(p_{irsa}(\beta_k, \beta_k + m))^{\psi_k} & k \geq m, \end{cases} \tag{39}$$

where $k = \psi_k m + \beta_k$. With the probability normalization equation $\sum_{j=0}^{\infty} \theta_j = 1$, we can write $\theta_0 = \theta_0 p_R^m + (1-\theta_0)\alpha_0$. Therefore,

$$\theta_0 = \frac{\alpha_0}{1 + \alpha_0 - p_R^m}. \tag{40}$$

Similarly, for $k \in \{1,\ldots,m-1\}$, we can write

$$\begin{aligned} \theta_k =& \theta_0 p_{irsa}(0,k) + (1-\theta_0)\alpha_k \\ =& \theta_0 p_R^{m-k}(1-p_R) + (1-\theta_0)\alpha_k \end{aligned} \tag{41}$$

Finally, for $k \geq m$, by referring to (32) and (34) we can obtain

$$\theta_k = \begin{cases} \theta_0 (p_R^m)^{\psi_k} & \beta_k = 0 \\ \theta_{\beta_k}(\alpha_m)^{\psi_k} & \beta_k \in \{1,\ldots,m-1\}. \end{cases} \tag{42}$$

It can be concluded that the random process $(\varDelta_i(t))_{t\in\varPhi}$ is ergodic because it has positive steady-state probabilities.

Segmenting time into frames, the average AoII can be expressed as

$$\begin{aligned} \overline{\varDelta}_{irsa} &= \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \varDelta_i(t) \\ &= \lim_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} \frac{1}{m} \sum_{j=0}^{m-1} \varDelta_i(t_l + j) \\ &= \lim_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} \overline{\varDelta}_i^{(l)}, \end{aligned} \tag{43}$$

where $\overline{\varDelta}_i^{(l)} = \frac{1}{m}\sum_{j=0}^{m-1} \varDelta_i(t_l + j)$ represents the average AoII during the l-th frame. Recall that $T_l = \{t_l, \ldots, t_l + m - 1\}$ represents the set of time indexes within the l-th frame and the random process $(\varDelta_i(t))_{t\in T_l}$ is Markovian with initial state $\varDelta_i(t_l)$ and one-step transition probabilities given in (25).

Next, we derive expressions for the expectation of $\overline{\varDelta}_i^{(l)}$ given initial state $\varDelta_i(t_l) = k$, which is denoted as $\mathbb{E}\left[\overline{\varDelta}_i^{(l)} | \varDelta_i(t_l) = k\right]$, by considering two cases: $\varDelta_i(t_l) = 0$ and $\varDelta_i(t_l) \neq 0$.

For $\varDelta_i(t_l) = 0$, with $\overline{T_t} \gg m$, dominate routes for the Markov chain starting from 0 can be identified as that $\varDelta_i(t)$ will stay at state 0 for the first j slots and increase in the following slots to $m-1-j$, whose probability can be approximated as $p_R^j(1-p_R)$. Thus, we have

$$\begin{aligned}
&\mathbb{E}\left[\overline{\varDelta}_i^{(l)} | \varDelta_i(t_l) = 0\right] \\
&= \sum_{j=1}^{m-2} p_R^j (1-p_R) \frac{1}{m} \sum_{k=1}^{m-1-j} k \\
&= \sum_{j=1}^{m-2} p_R^j (1-p_R) \frac{(j-m)^2 + (j-m)}{2} \\
&= \frac{m-1}{2} + \frac{1}{m} \gamma_R \left[\gamma_R (1 - p_R^{m-1}) + 1 - m\right].
\end{aligned} \tag{44}$$

For $\varDelta_i(t_l) \neq 0$, with $\overline{T_t} \gg m$, we consider only one dominate route where $\varDelta_i(t)$ keeps increasing till the end of the frame. Therefore,

$$\begin{aligned}
&\mathbb{E}\left[\overline{\varDelta}_i^{(l)} | \varDelta_i(t_l) = k \neq 0\right] = \frac{1}{m} \sum_{l=0}^{m-1} (k+l) \\
&= k + \frac{m-1}{2}.
\end{aligned} \tag{45}$$

Now we can write

$$\begin{aligned}
\overline{\varDelta}_{irsa} &= \lim_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} \overline{\varDelta}_i^{(l)} \\
&\overset{(b)}{=} \sum_{k=0}^{\infty} \theta_k \mathbb{E}\left[\overline{\varDelta}_i^{(l)} | \varDelta_i(t_l) = k\right] \\
&= \theta_0 \mathbb{E}\left[\overline{\varDelta}_i^{(l)} | \varDelta_i(t_l) = 0\right] + \sum_{k=1}^{\infty} \theta_k \mathbb{E}\left[\overline{\varDelta}_i^{(l)} | \varDelta_i(t_l) = k\right],
\end{aligned} \tag{46}$$

where (b) stems from the ergodicity of the random process $(\varDelta_i(t))_{t\in\varPhi}$. Substituting (40) and (44) into (46) we get

$$\overline{\varDelta}_{irsa,0} = \theta_0 \mathbb{E}\left[\overline{\varDelta}_i^{(l)} | \varDelta_i(t_l) = 0\right].$$

Plugging (41), (42) and (45) into (46), after some manipulations, we get

$$\overline{\varDelta}_{irsa,+} = \sum_{k=1}^{\infty} \theta_k \mathbb{E}\left[\overline{\varDelta}_i^{(l)} | \varDelta_i(t_l) = k\right],$$

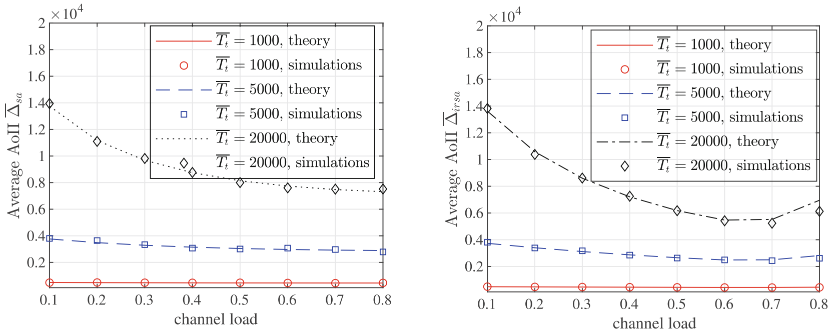
concluding the proof. □

4 Numerical Results and Discussion

In this section, we validate the derived analytical expressions via simulations and study the behavior of SA and IRSA in terms of average AoII. In all simulations, we set number of terminals $N_u = 5000$, and degree distribution $\Lambda(x) = x^3$ is employed for IRSA.

The remote Markov process introduced in Sect. 2 is mainly characterized by two parameters: p_t and p_R. Here, we investigate the impact of p_t and p_R from the perspective of $\overline{T_t} = 1/p_t$ and $\overline{T_R} = 1/(1 - p_R)$, which is more conceptually intuitive. Recall that $\overline{T_R}$ represents the mean time duration that the remote process will stay at current state, and $\overline{T_t}$ denotes the mean time needed for the remote process to reach a specific state starting from a different state.

The value of AoII will increase by 1 at each slot if the gateway has an incorrect estimation of the remote process. And it will be reset to 0 whenever the gateway has an correct estimation. This can happen in two circumstances: either the remote process transits to the estimation state at the gateway or an update packet has been successfully decoded at the gateway and the content of the packet brings the correct information of the remote process; one is related to the characteristics of the remote process itself and the other is determined by the RA scheme.



(a) SA (b) IRSA

Fig. 2. Average AoII vs. channel load for SA and IRSA; frame size $m = 100$; number of terminals $N_u = 5000$; $\Lambda(x) = x^3$ for IRSA; $\overline{T_R} = 1000$; $\overline{T_t} = 500, 1000, \text{and } 20000$.

4.1 Impact of $\overline{T_t}$

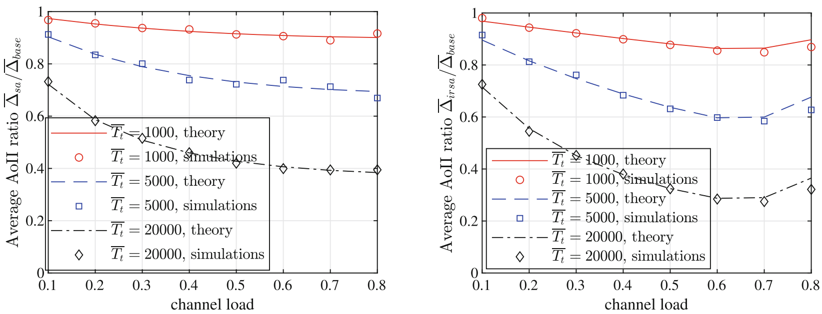
We first study the impact of $\overline{T_t}$ on the average AoII performance for SA and IRSA. The strategy where all terminals keep silent is considered as the baseline scheme for comparison. The average AoII for this scheme can be obtained by setting $S_{sa} = 0$ in (17):

$$\overline{\Delta}_{base} = \frac{1 - p_R}{p_t(p_t + 1 - p_R)}.$$

Figure 2 reports the trends of $\overline{\Delta}_{sa}$ and $\overline{\Delta}_{irsa}$ as a function of the channel load measured in active terminals per slot. It can be observed that both strategies exhibit higher average AoII values with larger $\overline{T_t}$. Indeed, with larger $\overline{T_t}$, it takes longer time for the remote process to transit to the estimation state at the gateway, i.e., to reset AoII to 0, thus leading to higher average AoII.

More interestingly, Fig. 2 indicates that the effect of terminal activity on average AoII highly depends on the value of $\overline{T_t}$. In particular, with $\overline{T_t} = 1000$, both schemes experience approximately the same average AoII at all channel loads. As discussed above, the frequency of resetting AoII to 0 is determined by the Markov source and the packet reception at the gateway. Specifically, the source spends on average $\overline{T_t}$ slots transiting to the estimation state at the gateway while it consumes on average $1/(S_{sa}/N_u)$ and $1/(S_{irsa}/N_u)$ slots to successfully decode an update packet for SA and IRSA respectively. Note that under the collision channel model, since $S_{irsa} < 1$ and $S_{sa} < 1$, we have $1/(S_{sa}/N_u) > N_u$ and $1/(S_{irsa}/N_u) > N_u$. Therefore, when $\overline{T_t} < N_u$, the evolution of AoII will be dominated by the Markov source itself, and the terminal activity will have little effect on the average AoII.

Figure 3 further explores this phenomenon by comparing $\overline{\Delta}_{sa}$ and $\overline{\Delta}_{irsa}$ with $\overline{\Delta}_{base}$ through their ratios, namely $\overline{\Delta}_{sa}/\overline{\Delta}_{base}$ and $\overline{\Delta}_{irsa}/\overline{\Delta}_{base}$. It can be seen that the improvement over $\overline{\Delta}_{base}$ is rather limited for $\overline{T_t} < N_u$, which is of little practical interest. In the following, we mainly focus on the cases where $\overline{T_t} > N_u$. Since we are interested in the mMTC scenario, the number of terminals is much smaller than the number of available resources, i.e., $m \ll N_u$. In this case, we have $m \ll \overline{T_t}$, which has been assumed when deriving the average AoII for IRSA in Sect. 3.



(a) SA (b) IRSA

Fig. 3. Average AoII ratios $\overline{\Delta}_{sa}/\overline{\Delta}_{base}$ and $\overline{\Delta}_{irsa}/\overline{\Delta}_{base}$ vs. channel load for SA and IRSA; frame size $m = 100$; number of terminals $N_u = 5000$; $\Lambda(x) = x^3$ for IRSA; $\overline{T_R} = 1000$; $\overline{T_t} = 500, 1000, \text{and } 20000$.

4.2 Impact of $\overline{T_R}$ and Frame Size m

A successfully decoded update packet with correct information content will reset current AoII to 0, leading to average AoII reduction. The throughput metric denotes the average number of update packets successfully decoded per slot. Accordingly, a higher throughput with more frequent updating is beneficial for improving the average AoII performance.

On the other hand, the content of the update packet may become incorrect due to the transmission delay, which is 1 for SA and m for IRSA. Although IRSA performs better in terms of throughput compared with SA, it has a longer delay resulted from the frame structure, which may lead to obsolete update packet and overcome the gain brought by throughput. Therefore, for IRSA, there exists an important trade-off between throughput and delay: increasing the frame size m achieves a better throughput performance, but at the same time results in a longer delay. This trade-off obviously depends on $\overline{T_R}$ as $\overline{T_R}$ determines the probability that the update packet will become obsolete with a certain delay.

Figure 4 investigates the role of frame size m in IRSA with two different values of $\overline{T_R}$: 500 and 5000. It can be seen that in the case of a lower $\overline{T_R} = 500$, a larger frame size $m = 300$ penalizes the average AoII at all channel loads in comparison with $m = 50$, indicating that the long delay induced by the larger frame size has overcome the benefits of a higher throughput. While in the case of $\overline{T_R} = 5000$, as the source tends to stay at the current state for a longer time, we observe a preference for a larger frame size at high channel loads (> 0.6).

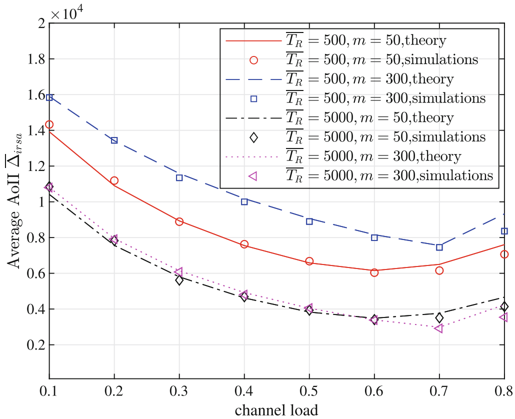


Fig. 4. Average AoII vs. channel load for IRSA; frame size $m = 50, 300$; number of terminals $N_u = 5000$; $\Lambda(x) = x^3$ for IRSA; $\overline{T_R} = 500, 5000$; $\overline{T_t} = 20000$.

This naturally raises the problem of identifying the optimal working point, i.e., the optimal frame size m^* for IRSA given the source parameter T_R and the channel load, which is solved in Fig. 5 based on expression (38) which characterizes the average AoII of IRSA. In general, the optimal frame size increases with the system channel load. At low channel loads with sparse terminal activity, the PLR of IRSA is relatively low. In this situation, increasing the frame size brings limited gains in throughput with a high risk of receiving obsolete information. In contrast, at high loads, the channel becomes more congested, the improvement in throughput with a larger frame size tends to outweigh the risk of receiving an obsolete packet. However, if T_R is small, the risk of an update packet becoming obsolete is relatively high. Then even at high loads, a small frame size is preferred, which is the case for $T_R = 500$ in Fig. 5. In all cases, our derived closed-form expression can be used as an efficient tool to find the optimal frame size for IRSA.

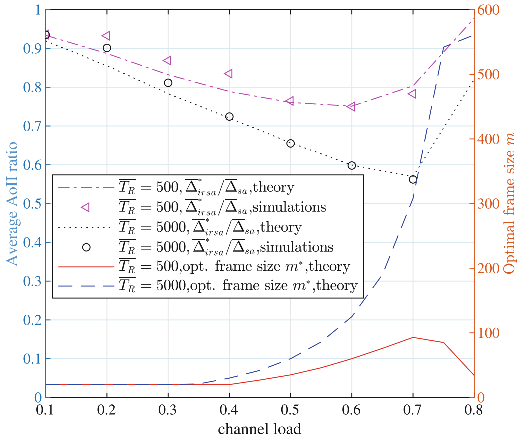


Fig. 5. Optimal frame size m^* for IRSA and Average AoII ratios $\overline{\Delta}^*_{irsa}/\overline{\Delta}_{sa}$ vs. channel load; number of terminals $N_u = 5000$; $\Lambda(x) = x^3$ for IRSA; $\overline{T_R} = 500, 5000$; $\overline{T_t} = 20000$.

We also compare the performance of IRSA adopting optimal frame sizes with SA in Fig. 5, where the average AoII of IRSA is denoted as $\overline{\Delta}^*_{irsa}$ and the comparison is presented in terms of average AoII ratio: $\overline{\Delta}^*_{irsa}/\overline{\Delta}_{sa}$. It can be observed that IRSA outperforms SA at most channel loads. For remote process with large $\overline{T_R}$ IRSA is able to benefit from its advantage of high throughput without risking receiving obsolete packets. However, when $\overline{T_R}$ decreases, IRSA will suffer from the obsolete information and its advantage over SA diminishes. Therefore, for remote process with small $\overline{T_R}$, SA might be an appropriate choice for its simplicity. In this sense, our derived analytical results can be utilized as a convenient tool for choosing the best RA scheme for IoT systems with the knowledge of the characteristics of the Markov remote process to be monitored.

5 Conclusion

In this paper, we investigated RA policies SA and IRSA from the perspective of AoII for IoT systems with Markov sources. Leaning on a Markovian analysis, we tracked the AoII evolution and derived state transition probabilities for SA and IRSA respectively. Closed form expressions of the average AoII were provided for both schemes, which are validated through simulation results. The trade-off between throughput and the risk of receiving packets with incorrect content was explored for IRSA and the key roles played by the frame size m and the Markov source character $\overline{T_R}$ were analyzed and highlighted. The analytical results can be used as an efficient tool for finding the optimal working point for IRSA. Finally, we showed that IRSA can outperform SA significantly under large $\overline{T_R}$. For sources with small $\overline{T_R}$, SA might be a more appropriate choice due to its simplicity.

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