A Game Theory Approach for Water Exchange in Eco-Industrial Parks: Part 2 - A Case Study with Regeneration Units

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Abstract. Part 2 of the paper presents the optimal design of the water exchange network in the eco-industrial parks (EIP) using the single-leader multi-follower (SLMF) game methodology. The SLMF game methodology is suitable on the case study of water management in EIP with regeneration units. The main goal of regeneration units is to reduce the contaminant concentration with different quality specifications. From there, the enterprises can not only use the wastewater from other enterprises but also reuse recycled water to reap economic benefit. At the same time, the environmental performance is also enhanced, since water scarcity is counteracted by replacing freshwater usage with wastewater and/or recycled water. The benefits of regeneration units in the EIP design will be discussed and the comparison with the case study without regeneration units presented in Part 1 [4] is also considered.

Keywords: Game theory · Nash equilibrium · Single-leader multi-follower game · Mixed-integer programming · Eco-industrial park

1 Introduction

The growing scarcity of freshwater coupled with an increasing demand for water suggests that the need to reuse and recycle water is essential. For this reason, the concept of EIPs was born [1]. In an EIP, the attempt of businesses cooperate with the others is to reduce waste and pollution, efficiently share resources (e.g., materials, energy, water, by-products, and so on). It also helps achieve sustainable development, with the aim to increase economic gains and to reduce the impact of contaminants on the environment. To implement EIP in a sustainable way, companies need methods and optimization tools to design appropriate inter-enterprises exchanges. EIP problems for managing industrial water can be solved by mathematical programming procedures. Furthermore, two scenarios are considered in the literature for designing an EIP: EIP without regeneration units (see, e.g., [3,6]), and EIP with regeneration units (see, e.g., [2,3,5,6]).

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As presented in Part 1 [4], we design and optimize the water exchange networks in EIPs without regeneration units, based on a game theory approach. And this paper is devoted to the optimal design of water exchange networks in EIPs with regeneration units also based on the game theory approach.

The remainder of this paper is organized as follows: Sect. 2 is dedicated the methodology and model formulation which briefly describes the problem addressed in this article and present a water management model in EIPs with regeneration units, based on a single-leader multi-follower model. The reformatting of the EIP modeling problem as a mixed integer linear programming problem is addressed in Sect. 3. The benefits of regeneration units in the EIP design will be discussed and the comparison with the case study without regeneration units presented in Part 1 [4] is considered in Sect. 4. Finally, conclusions and perspectives are presented in Sect. 5.

2 Methodology and Model Formulation

2.1 Problem Statement

Let n, m denote the given number of enterprises and regeneration units; and $I_P := \{1, \ldots, n\}$, $I_R = \{n+1, \ldots, n+m\}$ denote the index set of enterprises and regeneration units, respectively; and 0 denote a sink node. The sink node is a place to store contaminated wastewater. Thus, we define $I = I_P \cup I_R$ and $I_0 = \{0\} \cup I$. Each enterprise has its own pre-defined water input requirement and quality characteristics, as well as the quantity and quality of available output wastewater. For each enterprise, the resource consumption can be freshwater, wastewater from other enterprises, and/or from regeneration units. Indeed, the polluted water from an enterprise can be sent to the sink node, to other enterprises, and/or to regeneration units.

The objective of the model is to determine a network of connections of water streams among them so that both the total freshwater consumption and the annualized operating cost of each enterprise in the park are minimized, while satisfying all process and environmental constraints.

2.2 Minimizing Operating Costs

Each enterprise $i \in I_P$ may receive the wastewater from other enterprises, and/or from regeneration units within the EIP. Nevertheless, for technical constraints on the enterprise i, the contaminant concentration of total flux, delivered by the other enterprises and/or by the regeneration units, cannot exceed a certain maximum value denoted here by $C_{i,\text{in}}$ [ppm]. On the other hand each enterprise $i \in I_P$ generates a mass of contaminant M_i [g/h] due to its own working, that needs to be diluted before exiting the enterprise. To do so, enterprise i should buy an amount of freshwater z_i [T/h] such that, after dilution, the output pollutant concentration is lower than a limit concentration $C_{i,\text{out}}$ [ppm]. Actually considering that enterprise i will optimize his process, it is assumed that each

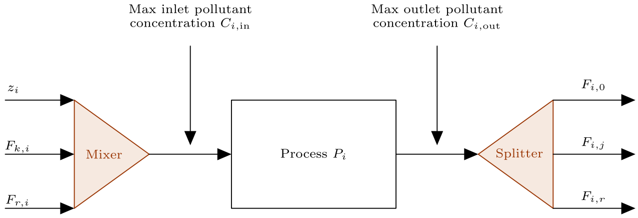


Fig. 1. Process around an enterprise i.

enterprise $i \in I$ will consume the exact amount of the freshwater it needs to attain concentration constraints $C_{i,\text{out}}$, and therefore, its output pollutant concentration will have a concentration equivalent to this constant $C_{i,\text{out}}$. Obviously we have that $C_{i,\text{in}} \leq C_{i,\text{out}}$. This structure is illustrated in Fig. 1.

In addition, each regeneration unit $r \in I_R$ has a given output contaminant concentration, that is $C_{r,\text{out}}$ [ppm]. Moreover, the enterprises can send the wastewater to the regeneration units in order to reduce the contaminant concentration of wastewater. Then, then enterprises will buy and use the recycle water. The process around a regeneration unit is illustrated in Fig. 2.

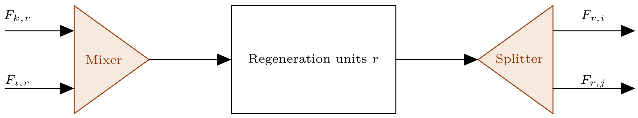


Fig. 2. Process around a regeneration unit r.

There is an exchange of materials between the enterprises in EIPs. On the other hand, let E be the configuration for the water exchange network in EIPs such that $(i,j) \in E$ then the enterprise i can pump his wastewater to enterprise j. Especially if the enterprise i uses the connection $(i,0)$, it means that it is discharging polluted water outside the park.

Defining the *stand-alone* and *complete* configuration, respectively, as follows

$$E_{\text{st}} := \{(i,0) \ : \ i \in I_P\}$$
$$E_{\max} := \{(i,j) \ : \ i \in I, \, j \in I_0\} \cup \{(r,k) \ : \ r \in I_R, \, k \in I_P\},$$

thus a valid configuration E must satisfy that $E_{\text{st}} \subset E \subset E_{\max}$. We denote $\mathcal{E}$ as the set of valid configurations for the EIP. In addition, for any $E \in \mathcal{E}$, we denote by $E^c = E_{\max} \backslash E$ the family of connections that are not present in E.

In term of variables, each enterprise $i \in I_P$ sends polluted water to $j \in I_0$, taken into account by variable $F_{i,j}$ [T/h]. In addition, we set $F = (F_{i,j} \ : \ (i,j) \in E_{\max})$ to be the full vector of fluxes through the exchange network.

Furthermore, for each enterprise $i \in I$, we denote $F = (F_i, F^P_{-i}, F^R)$ where $F_i = (F_{i,j} \ : \ j \in I)$, $F^P_{-i} = (F_{k,j} \ : \ k \in I_P \backslash \{i\})$, and $F^R = (F_r \ : \ r \in I_R)$, to emphasize the vector of fluxes between enterprise $i \in I_P$. Then, for a fixed network E, the EIP model with regeneration units must satisfy the following constraints:

1. Water mass balance constraint around an enterprise $i \in I_P$:

$$z_i + \sum_{(k,i)\in E} F_{k,i} = \sum_{(i,j)\in E} F_{i,j}. \tag{1}$$

2. Contaminant mass balance constraint around an enterprise $i \in I_P$:

$$M_i + \sum_{(k,i)\in E} C_{k,\text{out}} F_{k,i} = C_{i,\text{out}} \sum_{(i,j)\in E} F_{i,j}, \tag{2}$$

3. Inlet/outlet concentration constraints around an enterprise $i \in I_P$:

$$\sum_{(k,i)\in E} C_{k,\text{out}} F_{k,i} \le C_{i,\text{in}} \left(z_i + \sum_{(k,i)\in E} F_{k,i} \right). \tag{3}$$

4. Contaminant concentration constraints around a regeneration unit $r \in I_R$:

$$\sum_{(k,r)\in E} C_{k,\text{out}} F_{k,r} \ge C_{r,\text{out}} \sum_{(r,j)\in E} F_{r,j}. \tag{4}$$

5. Mass balance around a regeneration unit $r \in I_r$:

$$\sum_{(k,r)\in E} F_{k,r} = \sum_{(r,j)\in E} F_{r,j}. \tag{5}$$

6. Positivity of fluxes and null fluxes outside the connections: we need all the fluxes to be positive:

$$\begin{cases} F_{i,j} \ge 0, & \forall (i,j) \in E \\ z_i \ge 0, & \forall i \in I_P. \end{cases} \tag{6}$$

   Of course, we also put

$$\forall (i,j) \in E^c,\ F_{i,j} = 0, \tag{7}$$

   that is, the effective fluxes can only pass through the existing connections in E.

By combining Eqs. (1) and (2) we obtain:

$$M_i + \sum_{(k,i)\in E} C_{k,\text{out}} F_{k,i} = C_{i,\text{out}} \left(z_i + \sum_{(k,i)\in E} F_{k,i} \right), \quad \forall i \in I_p. \tag{8}$$

From Eq. (8) the freshwater consumption of the enterprise $i \in I$ is defined by

$$z_i(F_{-i}) = \frac{1}{C_{i,\text{out}}} \left(M_i + \sum_{(k,i)\in E} \left(C_{k,\text{out}} - C_{i,\text{out}} \right) F_{k,i} \right). \tag{9}$$

Thus, each enterprise $i \in I_P$ wants to minimize his operating cost, given by

$$\mathrm{Cost}_i(F_i, F_{-i}^P, F^R, E) = A\Bigg[c \cdot z_i(F_{-i}) + \gamma_{i,0}F_{i,0} + \sum_{r\in I_R} \gamma_{i,r}F_{i,r} + \sum_{r\in I_R} \gamma_{r,i}F_{r,i} \sum_{k\in I_P} \gamma_{k,i}F_{k,i} + \sum_{j\in I_P} \gamma_{i,j}F_{i,j} + \sum_{r\in I_R} \mu_r F_{r,i}^{\psi}\Bigg],$$

where A [h] stands for the annual EIP operating hours, c [\$/T] for the price to buy freshwater, $\gamma_{i,0}[\$/T]$ for the price of wastewater discharge, $\gamma_{p,q}[\$/T]$ for the cost of sending wastewater from enterprise p to q, and $\mu_r[\$/T]$ for the cost of regenerating water. The power $\psi < 1$ in the regenerated water cost term accounts for economy scale, namely the larger the volume of regenerated water, the lower the operating cost of the regeneration unit.

With all these considerations, each enterprise's $i \in I_P$ optimization problem is given by $P_i\left(F_{-i}^P, F^R, E\right)$:

$$\begin{aligned} &\min_{F_i} \ \mathrm{Cost}_i\left(F_i, F_{-i}^P, F^R, E\right) \\ &s.t. \Big\{\text{Equations (1)-(3)-(6)-(7).} \end{aligned} \tag{10}$$

For an operation of the regeneration units F^R and a configuration of the exchange network $E \in \mathcal{E}$, the family of equilibria for F^R and E at the lower-level problem is given by $\mathrm{Eq}(F^R, E)$. Furthermore,

$$\mathrm{Eq}(F^R, E) \Longleftrightarrow \forall i \in I_P,\ F_i \,\text{solves the problem}\, P_i\left(F_{-i}^P, F^R, E\right).$$

Remark 1. *If no enterprises and no regeneration units send water to enterprise $i \in I_P$, the amount of freshwater consumed by enterprise i should be*

$$z_i = \frac{M_i}{C_{i,\mathrm{out}}}.$$

Then, the cost of stand-alone configuration is given by

$$\mathrm{STC}_i = A \cdot (c + \gamma_{i,0})\frac{M_i}{C_{i,\mathrm{out}}}.$$

2.3 Minimizing Consumption of Natural Resources

In the model, the EIP authority tries to optimize the total freshwater consumption, and so he wants to minimize the objective function

$$Z(F) = \sum_{i\in I_P} z_i(F_{-i}). \tag{11}$$

The EIP authority must guarantee that each enterprise $i \in I_P$ participating in the EIP reduces its operating cost compared to the stand-alone case, that is,

$$\mathrm{Cost}_i(F_i, F_{-i}^P, F^R, E) \leq \alpha \cdot \mathrm{STC}_i. \tag{12}$$

Now, the optimization problem of the EIP authority is given by

$$\min_{F \in \mathbb{R}^{|E_{\max}|}, E \in \mathcal{E}} Z(F)$$
$$s.t. \begin{cases} \text{Equations (4)-(5)-(6)}, \\ F^P \in \text{Eq}(F^R, E), \\ \text{Cost}_i(F_i, F^P_{-i}, F^R, E) \le \alpha_i \cdot \text{STC}_i, \qquad \forall i \in I_P. \end{cases} \tag{13}$$

3 Mixed-Integer Programming Reduction

As we can observe the EIP authority problem (13) is mathematical programming with equilibrium constraints, and thus it is difficult to solve. In the following, however, this type of problem will be reformulated as a single mixed-integer programming problem.

3.1 Characterization of Equilibria

In order to characterize the set of the equilibria $\text{Eq}(F^R, E)$ as a system of equalities and inequalities, for each enterprise $i \in I_P$, we define the set

$$E_{i,\text{act}} := \left\{ (i,j) \in E \ : \ \gamma_{i,j} = \gamma_i^* := \min_{(i,k) \in E} \gamma_{i,k} \right\}. \tag{14}$$

Theorem 2. *For any valid exchange network $E \in \mathcal{E}$ and $F^R \ge 0$, and denoting $S(E)$ by the set*

$$S(F^R, E) = \left\{ F^P \ : \forall i \in I_P, \begin{array}{l} z_i(F_{-i}) + \sum_{(k,i) \in E} F_{k,i} = \sum_{(i,j) \in E} F_{i,j} \\ \sum_{(k,i) \in E} C_{k,\text{out}} F_{k,i} \le C_{i,\text{in}} \left(z_i + \sum_{(k,i) \in E} F_{k,i} \right) \\ F_i \big|_{E^c_{i,\text{act}}} = 0 \\ F_i \ge 0 \\ z_i(F_{-i}) \ge 0 \end{array} \right\} \tag{15}$$

then, one has $S(F^R, E) = \text{Eq}(F^R, E)$. Furthermore, any optimal solution (F, E) of the mathematical programming problem

$$\min_{F \in \mathbb{R}^{|E_{\max}|}, E \in \mathcal{E}} Z(F)$$
$$s.t. \begin{cases} \text{Equations}(4) - (5) - (6), \\ F^P \in S(F^R, E), \\ \text{Cost}_i(F_i, F^P_{-i}, F^R, E) \le \alpha_i \cdot \text{STC}_i, \qquad \forall i \in I_P. \end{cases} \tag{16}$$

is an optimal solution of the SLMF problem (13).

For the proof of Theorem 2, we prefer the reader to Appendix A.

3.2 Mixed-Integer Formulation

Constraint $F_i\big|_{E^c_{i,\text{act}}} = 0, \forall i \in I$ depends on the configuration of the water exchange network and is therefore difficult to implement numerical experiments. Thus, we will reduce the single optimization problem (13) to the mixed-integer programming problem.

Now, let $(i,j) \in E_{\max}$, we define

$$C(i,j) := \begin{cases} \{(i,k) \in E_{\max} \;:\; \gamma_{i,k} = \gamma_{i,j}\} & \text{if } i \in I_P \\ \{(i,k) \in E_{\max}\} & \text{if } i \in I_R \end{cases} \tag{17}$$

the *arc class* of (i,j). Moreover, we denote by $\mathcal{C}_i = \{C(i,j) \;:\; (i,j) \in E_{\max}\}$ the set of all arc classes of enterprise i.

If there exists a class $C(i,j) \in \mathcal{C}_i$ satisfying

$$E_{i,\text{act}} \subseteq C(i,j), \tag{18}$$

then it is called the *active class* of E of the enterprise i, and we will denote it as $C_i(E)$.

Now, let $D = \bigcup_{i \in I} \mathcal{C}_i$, the family of all arc classes of enterprises. For each enterprise $i \in I$ and each arc class $C \in \mathcal{C}_i$, we introduce the boolean variable $y = (y_C)_{C \in D} \in \{0,1\}^{|D|}$ in the following way:

$$y_C = \begin{cases} 1 & \text{if } C \text{ is the active class of } i, \\ 0 & \text{otherwise.} \end{cases}$$

From this new boolean variable $y \in \{0,1\}^{|D|}$, we will build a configuration associated to y as

$$E(y) = \left(\bigcup \{C \;:\; y_C = 1\} \right) \cup \{(i,0) \;:\; i \in I_P\} \cup \{(r,j) \in E_{\max} \;:\; r \in I_R\}.$$

Now, let's consider the following Mixed-Integer optimization problem:

$$\min_{F \in \mathbb{R}^{|E_{\max}|}, y \in \{0,1\}^{|D|}} Z(F)$$
$$s.t. \begin{cases} \text{Equations (1)-(3)-(4)-(5)-(6)}, & \\ \sum_{C \in \mathcal{C}_i} y_C = 1, & \forall i \in I_P, \\ \sum_{(i,j) \in C} F_{i,j} \le K \cdot y_C, & \forall C \in D, \\ \text{Cost}_i(F_i, F^P_{-i}, F^R, E) \le \alpha_i \cdot \text{STC}_i, & \forall i \in I_P, \end{cases} \tag{19}$$

Here, K is a constant large enough.

Theorem 3. *If (F, E) is an optimal solution of the problem (16), then (F, y^E) is an optimal solution of the problem (19), where $y^E \in \{0,1\}^{|\mathcal{D}|}$ is given by*

$$y_C^E = \begin{cases} 1 & \text{if } C = C_i(E) \text{ for some } i \in I, \\ 0 & \text{otherwise.} \end{cases}$$

If (F, y) is an optimal solution of the problem (19), then $(F, E(y))$ is an optimal solution of the problem (16).

For the proof of Theorem 3, we prefer the reader to Appendix B.

3.3 Null Class as Exit Option

We can observe that the stand-alone configuration $E_{\rm st}$ is always a feasible configuration for the EIP model. However, with the constraint (12), the problem may become infeasible. Therefore, we need to take into account the possibility of excluding some enterprises from the network when the EIP authority does not ensure to satisfy constraint (12) for such enterprises.

Now, for each enterprise $i \in I$, we introduce a boolean variable $y_{i,\rm null} \in \{0,1\}$ such that

$$y_{i,\rm null} = \begin{cases} 1 & \text{if } i \text{ breaks the constraint (12),} \\ 0 & \text{otherwise.} \end{cases}$$

With this boolean variable, we will add the following constraints to problem (19):

1. For each enterprise $i \in I_P$,

$$y_{i,\rm null} + \sum_{C \in \mathcal{C}_i} y_C = 1, \tag{20}$$

   namely, only one class is active.
2. For each enterprise $i \in I_P$,

$$\sum_{(i,j) \in C(i,0)} F_{i,j} \le K \cdot (y_{C(i,0)} + y_{i,\rm null}), \tag{21}$$

$$\sum_{(i,j) \in E_{\max}, j \neq 0} F_{i,j} \le K \cdot (1 - y_{i,\rm null}). \tag{22}$$

   Constraints (21) and (22) are to ensure that, if the enterprise breaks the constraint, then the enterprise e do not share polluted water with other enterprises and will use the connection $(i, 0)$.
3. For each enterprise $i \in I_P$,

$$\sum_{(k,i) \in E_{\max}} F_{k,i} \le K \cdot (1 - y_{i,\rm null}). \tag{23}$$

   This constraint is to ensure that, if the enterprise breaks the constraint (12), then no enterprises can send him any polluted water.

4. For each enterprise $i \in I_P$,

$$\text{Cost}_i(F_i, F_{-i}, E(y)) \leq \alpha_i \cdot \text{STC}_i \cdot (1 - y_{i,\text{null}}) + \text{STC}_i \cdot y_{i,\text{null}}. \tag{24}$$

We denote by $\overline{D} = D \cup \{\text{Null}_i \ : \ i \in I_P\}$, where Null_i is the null class, associated to $y_{i,\text{null}}$, and $D_0 = D \backslash \{C(i,0) \ : \ i \in I_P\}$. Denoting

$$\text{STC}_i(y_{i,\text{null}}) := \alpha_i \cdot \text{STC}_i \cdot (1 - y_{i,\text{null}}) + \text{STC}_i \cdot y_{i,\text{null}},$$

With all the foregoing, problem (19) becomes

$$\min_{F \in \mathbb{R}^N, y \in \{0,1\}^{|\overline{D}|}} Z(F)$$
$$s.t. \begin{cases} \text{Equations (1)-(3)-(4)-(5)-(6)-(20)-(21)-(22)-(23)}, & \\ \sum_{(i,j) \in C} F_{i,j} \leq K \cdot y_C, & \forall C \in D_0, \\ \text{Cost}_i(F_i, F_{-i}, E(y)) \leq \text{STC}_i(y_{i,\text{null}}), & \forall i \in I_P. \end{cases} \tag{25}$$

4 Numerical Experiments

4.1 Case Study

Now, we simulate numerical examples of the model described in Sect. 2. We assume that

$$\gamma_{i,j} = \begin{cases} \delta & \text{if } j \in I_P, \\ 2\delta & \text{if } j \in I_R, \\ \beta & \text{if } j = 0. \end{cases} \tag{26}$$

From (26), we can observe that each enterprise i has to pay for both water receipt and water deposit. In the case of regeneration units, the enterprises must pay for the operation of the regeneration units. Hence, for each enterprise $i \in I_P$, the set $\mathcal{C}_i = \{C_{i,p}, C_{i,r}, C_{i,0}\}$ where

$$C_{i,p} = \{(i,j) \in E_{\max} \ : \ j \in I_P\}$$
$$C_{i,r} = \{(i,r) \in E_{\max} \ : \ r \in I_R\}$$
$$C_{i,0} = \{(i,0)\}.$$

Now, for each enterprise $i \in I_P$, we introduce four integer variables, $y_{i,p}, y_{i,r} y_{i,0}$, $y_{i,\text{null}} \in \{0,1\}$ as follows:

– If $y_{i,p} = 1$, it means the connections in $C_{i,p}$ are included in the network.
– If $y_{i,r} = 1$, it means the connections in $C_{i,r}$ are included in the network.
– If $y_{i,0} = 1$, it means the connection $(i,0)$ is the only exit connection for i, and i participates in the EIP.
– If $y_{i,\text{null}} = 1$, it means the connection $(i,0)$ is the only exit connection for i, and i does not participate in the EIP.

Note that only one of these integer variables takes the value 1, i.e., $y_{i,\text{null}} + y_{i,p} + y_{i,r} + y_{i,0} = 1, \forall i \in I$, and in doing so it determines the network E to be implemented and the operation that each enterprise can do within this network. We denote $y \in \{0,1\}^{4n}$ as the vector of all integer variables of all enterprises.

Since the optimization problem (25) can have several solutions and to get a solution with more enterprises involved, we replace $Z(F)$ by

$$Z(F) + \text{Coef} \cdot \sum_{i \in I_P} y_{i,\text{null}}, \tag{27}$$

where Coef ≥ 0 is a coefficient to penalize the objective function. Then, the optimization problem (25) becomes:

$$\min_{F,y} \quad Z(F) + \text{Coef} \cdot \sum_{i \in I} y_{i,\text{null}}$$
$$s.t. \begin{cases} \text{Equations (1)-(3)-(4)-(5)-(6)}, & \\ y_{i,\text{null}} + y_{i,p} + y_{i,r} + y_{i,0} = 1, & \forall i \in I_P, \\ \sum_{(i,j) \in C_{i,p}} F_{i,j} \leq K \cdot y_{i,p}, & \forall i \in I_P, \\ \sum_{(i,j) \in C_{i,r}} F_{i,j} \leq K \cdot y_{i,r}, & \forall i \in I_P, \\ F_{i,0} \leq K \cdot (y_{i,0} + y_{i,\text{null}}), & \forall i \in I_P, \\ \sum_{(i,j) \in E_{\max}, j \neq 0} F_{i,j} \leq K \cdot (1 - y_{i,\text{null}}), & \forall i \in I_P, \\ \sum_{(k,i) \in E_{\max}} F_{k,i} \leq K \cdot (1 - y_{i,\text{null}}), & \forall i \in I_P, \\ \text{Cost}_i(F_i, F_{-i}, E(y)) \leq \cdot \text{STC}_i(y_{i,\text{null}}), & \forall i \in I_P. \end{cases} \tag{28}$$

We use input data on the scale of an EIP of 15 enterprises. The data of enterprises are given in Table 1 and Table 2 of paper [4]. It is supposed that the EIP operates A = 1 h. Additionally, we assume that there are 3 different regeneration units that are distinguished by their ability to regenerate water. The operating parameters of regeneration units are illustrated in Table 6 of the paper [6].

All simulations below performed using Julia `Julia v1.0.5` programming language, using `CPLEX V12.10.0` as a solver.

4.2 Results and Discussion

The resulting optimized EIP network is shown in Fig. 3, and it corresponds to $\alpha = 0.90$ and Coef $= 1$. This optimal network provides operating cost of each enterprise and total freshwater consumption that are lower than a stand-alone network as shown in Table 1.

Table 1. Summary of results of the EIP.

Enterprise	Freshwater stand-alone (T/h)	Freshwater in EIP (T/h)	Cost_i stand-alone (MMUSD/hour)	Cost_i in EIP (MMUSD/hour)	% Reduction in Cost_i
1	20.00	20.00	7.00	3.00	57.14
2	25.00	25.00	8.75	3.50	60.00
3	30.00	12.50	10.50	7.33	30.15
4	50.00	18.52	17.50	10.31	41.08
5	66.67	0.00	23.33	14.01	39.94
6	8.33	0.00	2.92	1.60	45.07
7	25.00	0.00	8.75	5.90	32.55
8	37.50	0.00	13.12	8.51	35.18
9	100.00	0.00	35.00	28.13	19.62
10	33.33	0.00	11.67	7.46	36.01
11	33.33	0.00	11.67	9.92	14.94
12	25.00	0.00	8.75	7.67	12.38
13	27.27	0.00	9.54	8.19	14.24
14	4.28	0.00	1.50	0.13	91.18
15	10.00	0.00	3.50	1.45	58.55
Total	495.72	76.02	173.50	117.13	32.49

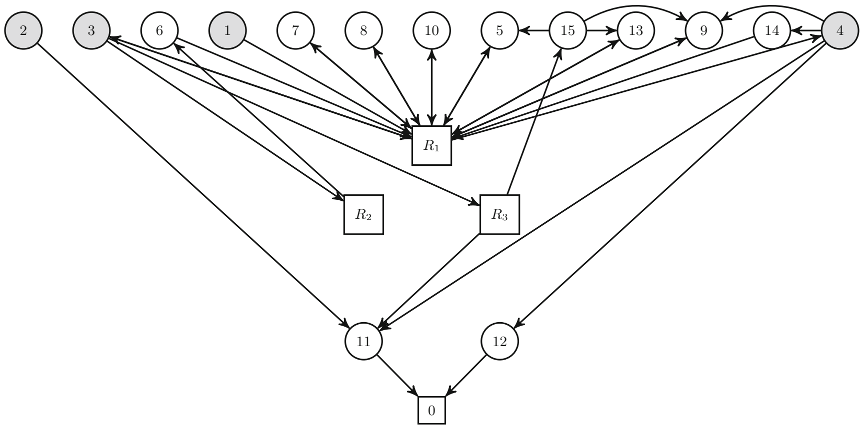


Fig. 3. The optimal configuration in the case $\alpha_i = 0.90$ and Coef = 1. Gray nodes are active consuming fresh water.

When enterprises operate in an EIP with an optimal configuration, the benefits of using water regeneration units are obvious. First, total operating cost is reduced compared to the stand-alone case, as expected, from 495.72 (T/h) to 76.02 (T/h), which equates to a reduction of 84.66%. Furthermore, the water demand of enterprises 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 are entirely supplied by other enterprises and/or by regeneration units. Secondly, the EIP authority ensures that each enterprise gets at least a 10% reduction in costs. Enterprise 14 achieves the highest percentage reduction of operating cost corresponding to 91.18% while enterprise 12 has the lowest reduction corresponding to 12.38% with respect to the stand-alone configuration. Total operating cost is reduced compared to the stand alone case, as expected, from 173.50 ($/h) to 117.13 ($/h), which means a decrease of 32.49%.

4.3 Comparison Between the EIP Model Without Regeneration Units and the EIP Model with Regeneration Units

The design and optimization of industrial water networks in eco-industrial parks are studied by formulating and solving SLMF game problems. The SLMF game methodology is suitable on a case study of water management in EIPs without and with regeneration units. It is therefore important to compare the optimal results of both approaches. The criteria we used for comparing the two models are as follows: first, the ability to reduce freshwater consumption; second, the number of enterprises involved into the optimal EIP and last, the ability to reduce the operating costs of each enterprise.

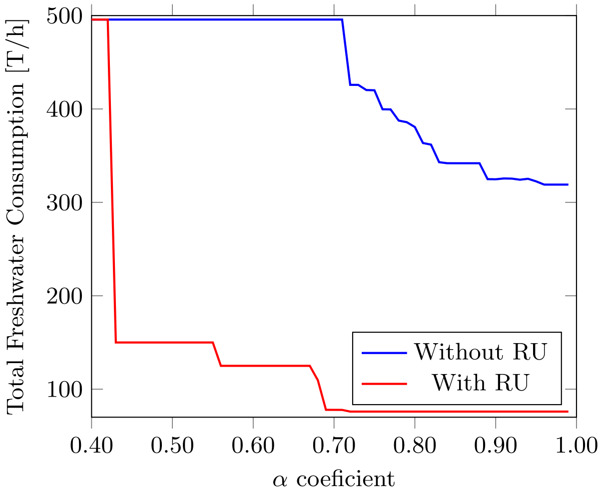
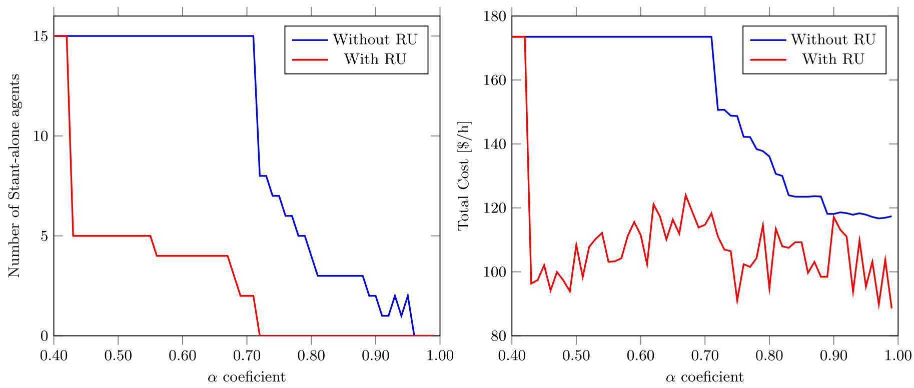


Fig. 4. Sensitivity analysis for $\alpha \in [0.40, 0.99]$ and Coef $= 1$. Total freshwater consumption for both study cases: without and with regeneration units.

First of all and for both models, the total of freshwater consumption and wastewater discharge decreases when α increases as shown in Fig. 4. But the case study of water integration in EIP with regeneration units allows an strong reduction of freshwater consumption compare to the case study of water integration in EIP without regeneration units. In the optimized networks, the EIP with regeneration units achieves a minimum total of freshwater consumption is 76.02

(T/h) corresponding to 84.66% compared to the stand-alone case, while the EIP without regeneration units achieves a minimum total freshwater consumption is 319.04 (T/h) corresponding to 35.64% with respect to the stand-alone configuration.



(a) Number of stand-alone enterprises in the park.

(b) Total operating cost in the park.

Fig. 5. Sensitivity analysis for $\alpha \in [0.40, 0.99]$ and Coef = 1.

The number of enterprises which operate stand-alone for both models is shown in Fig. 5a. Roughly speaking, for $\alpha \in [0.40, 0.99]$, the number of enterprises operating stand-alone in the EIP with regeneration units is always less than that of enterprises in the EIP without regeneration units. Moreover, with $\alpha \in [0.43, 0.71]$, the EIP with regeneration units shows its clear superiority on the EIP without regeneration units. Indeed in the EIP with regeneration units the designer can build a park for which not only a reduction of freshwater consumption is achieved compare to EIP without regeneration units results but also the designer can attract the exigent enterprises by guaranteeing a relative gain of more than 29% while with the EIP model without regeneration units only proposes full stand alone situation. Another interesting feature is that if $\alpha \geq 0.72$ then the EIP with regeneration units can ensure that all enterprises will participate in the park while the EIP without regeneration units reach this full involvement only for $\alpha \geq 0.96$. Finally, for $\alpha \leq 0.42$, the optimal solution is the stand-alone configuration for both models, thus 0.42 playing the role of a threshold value for the relative gain.

As observed from Fig. 5b, the total operating cost in the EIP with regeneration units is always less than the one without regeneration units. In the optimized networks, the EIP with regeneration units achieves a minimum total operating cost is 88.58 ($/h) corresponding to 48.94% with respect to the stand-alone configuration, while the EIP without regeneration units achieves a minimum total operating cost is 116.70 (\$/h) corresponding to 32.74% compared to the stand-alone configuration.

5 Conclusion and Perspectives

In this work, we have presented a game theory methodology to optimize the water exchange networks in the EIP. The SLMF game methodology is suitable on a case study of water management in EIP with regeneration units. This study shows that regeneration units yield very significant gains in the EIP design. More precisely, if the enterprises work in the EIP with regeneration units, then the total freshwater consumption and total operating cost in the optimal configuration have been reduced by 84.66% and 32.49%, respectively, while if the enterprises work in the EIP without regeneration units, then the total freshwater consumption and total operating cost in the optimal configuration have been reduced by 34.5% and 31.94%, respectively.

A methodology taking into account in this present work only the single contaminant case is implemented. Thus, we would like to address the EIP design with multi contaminant case in the future. Moreover, the eco-industrial park concept can be extended by sharing not only water but also more resources such as energy or other materials.

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Appendix A

Proof of Theorem 2

We only need to prove the equality $S(F^R, E) = \mathrm{Eq}(F^R, E)$ since the second part of the statement is a direct consequence by replacing the constraint "$F \in \mathrm{Eq}(F^R, E)$" with "$F \in S(F^R, E)$".

First, we show that $S(F^R, E) \subseteq \mathrm{Eq}(F^R, E)$. Indeed, let $F^P \in S(F^R, E)$. Since $E_{i,\mathrm{act}} \subset E$, hence F_i is a feasible point of $P_i(F^P_{-i}, F^R, E)$, for any $i \in I_P$.

Now, let F'_i be another feasible point of $P_i(F^P_{-i}, F^R, E)$, for any $i \in I_P$. Then, $F'_i \geq 0$ and the water mass balance constraint (1) is satisfied. Therefore, one has

$$\begin{aligned}\mathrm{Cost}_i(F'_i, F^P_{-i}, F^R, E) - \mathrm{Cost}_i(F_i, F^P_{-i}, F^R, E) &= \sum_{(i,j)\in E} \gamma_{i,j} F'_{i,j} - \gamma^*_i \left(\sum_{(i,j)\in E_{i,\mathrm{act}}} F_{i,j} \right) \\ &\geq \gamma^*_i \left(\sum_{(i,j)\in E} F'_{i,j} - \sum_{(i,j)\in E_{i,\mathrm{act}}} F_{i,j} \right).\end{aligned}$$

Furthermore, the mass balance constraint (1) is satisfied for F'_i and F_i, thus

$$\sum_{(i,j)\in E} F'_{i,j} = z_i(F_{-i}) + \sum_{(k,i)\in E} F_{k,i} = \sum_{(i,j)\in E} F_{i,j} = \sum_{(i,j)\in E_{i,\mathrm{act}}} F_{i,j}.$$

Hence, $\text{Cost}_i(F_i', F_{-i}^P, F^R, E) \geq \text{Cost}_i(F_i, F_{-i}^P, F^R, E)$. Thus, F_i solves $P_i(F_{-i}, E)$, and since this holds for every $i \in I$, we conclude that $F \in \text{Eq}(F^R, E)$.

Now, we show that $\text{Eq}(F^R, E) \subseteq S(F^R, E)$. Indeed, let $F \in \text{Eq}(F^R, E)$, and assume that $F \notin S(F^R, E)$. Since F_i is a feasible point of $P_i(F_{-i}^P, F^R, E)$ for each $i \in I_P$, thus $F \notin S(F^R, E)$ if there exists $i_0 \in I_P$ such that $F_{i_0}\big|_{E^c_{i_0,\text{act}}} \neq 0$. Hence, there exists $(i_0, j_0) \in E \backslash E_{i_0,\text{act}}$ such that $F_{i_0,j_0} > 0$. Let $(i_0, j_1) \in E_{i,\text{act}}$ and consider the vector

$$F'_{i_0,k} = \begin{cases} F_{i_0,k} & \text{if } k \in I\backslash\{j_0, j_1\}, \\ 0 & \text{if } k = j_0, \\ F_{i_0,j_1} + F_{i_0,j_0} & \text{if } k = j_1. \end{cases}$$

We have that $F'_{i_0} \geq 0$ and also

$$z_i(F_{-i_0}) + \sum_{(k,i_0)\in E} F_{k,i_0} = \sum_{(i_0,j)\in E} F_{i_0,j} = \sum_{(i_0,j)\in E} F'_{i_0,j}.$$

Hence F'_{i_0} is a feasible point of $P_i(F_{-i_0}^P, F^R, E)$. Furthermore, we have that

$$\begin{aligned} \text{Cost}_{i_0}(F'_i0, F_{-i_0}^P, F^R, E) - \text{Cost}_{i_0}(F_{i_0}, F_{-i_0}^P, F^R, E) &= \sum_{(i_0,j)\in E} \gamma_{i_0,j} F'_{i_0,j} - \sum_{(i_0,j)\in E} \gamma_{i_0,j} F_{i_0,j} \\ &= (\gamma_{i_0,j_1} - \gamma_{i_0,j_0})\, F_{i_0,j_0} \\ &= (\gamma^* - \gamma_{i_0,j_0})\, F_{i_0,j_0} < 0, \end{aligned}$$

since, by construction, $\gamma_{i_0,j_0} > \gamma^*$. This show that F_{i_0} doesn't solve $P_i(F_{-i_0}^P, F^R, E)$, which is a contradiction. Therefore, $F \in S(F^R, E)$.

Appendix B

Proof of Theorem 3

Let (F, E) be an optimal solution of problem (16). Since, for each enterprise $i \in I$, $E_{i,\text{act}} \subseteq C(i,j)$ thus there exists a unique active class $C_i(E)$. Let us define $y^E \in \{0,1\}^{|D|}$ with

$$y_C^E = \begin{cases} 1 & \text{if } C = C_i(E) \text{ for some } i \in I, \\ 0 & \text{otherwise.} \end{cases}$$

Then, for every $i \in I_P$, $\sum_{C\in\mathcal{C}_i} y_C^E = 1$. Now, fix a class $C \in D$, and let $i \in I_P$ such that $C \in \mathcal{C}_i$. We have that

$$\sum_{(i,j)\in C} F_{i,j} \leq \begin{cases} K = K \cdot y_C^E & \text{if } C = C_i(E), \\ 0 = K \cdot y_C^E & \text{if } C \neq C_i(E), \end{cases}$$

For an enterprise $i \in I_P$, the fact that $E_{i,\text{act}} \subseteq E(y^E)$ lead us to the fact that

$$\text{Cost}_i(F_i, F^P_{-i}, F^R, E(y^E)) = \text{Cost}_i(F_i, F^P_{-i}, F^R, E),$$

and so, the constraint (12) is satisfied. We deduce that (F, y^E) is a feasible point of (19), since all other constraints are directly satisfied given that (F, E) is feasible for problem (16). Therefore, (F, y^E) is an optimal solution of the problem (19).

Now, let (F, y) be an optimal solution of problem (19). For each $i \in I_P$ and let C_i be the unique class in $\mathcal{C}_i$ satisfying $y_{C_i} = 1$. Then, we have

$$E(y)_{i,\text{act}} = C_i \text{ and } \sum_{(i,j)\in E_{\max}\setminus C_i} F_{i,j} \leq K \cdot \sum_{C\in\mathcal{C}_i\setminus\{C_i\}} y_C = 0.$$

We then infer that

$$F_i\big|_{E(y)^c_{i,\text{act}}} = 0.$$

Since this constraint is valid for every active enterprise $i \in I_P$, we can now rewrite the water mass balance constraint in problem (19) as

$$z(F_{-i}) + \sum_{(k,i)\in E(y)} F_{k,i} = \sum_{(i,j)\in E(y)} F_{i,j}, \forall i \in I.$$

We conclude that $(F, E(y))$ is a feasible point of problem (16), thus $(F, E(y))$ is an optimal solution of problem (16).

References

1. Boix, M., Montastruc, L., Azzaro-Pantel, C., Domenech, S.: Optimization methods applied to the design of eco-industrial parks: a literature review. J. Clean. Prod. **87**, 303–317 (2015)
2. Boix, M., Montastruc, L., Pibouleau, L., Azzaro-Pantel, C., Domenech, S.: A multiobjective optimization framework for multicontaminant industrial water network design. J. Environ. Manag. **92**(7), 1802–1808 (2011)
3. Boix, M., Montastruc, L., Pibouleau, L., Azzaro-Pantel, C., Domenech, S.: Industrial water management by multiobjective optimization: from individual to collective solution through eco-industrial parks. J. Clean. Prod. **22**(1), 85–97 (2012)
4. Cao Van, K.: A game theory approach for water exchange in eco-industrial parks: part 1 - a case study without regeneration units. Accepted to 7th EAI International Conference on Nature of Computation and Communication, p. 15pp (2021)
5. Montastruc, L., Boix, M., Pibouleau, L., Azzaro-Pantel, C., Domenech, S.: On the flexibility of an eco-industrial park (EIP) for managing industrial water. J. Clean. Prod. **43**, 1–11 (2013)
6. Ramos, M., Boix, M., Aussel, D., Montastruc, L., Domenech, S.: Water integration in eco-industrial parks using a multi-leader-follower approach. Comput. Chem. Eng. **87**, 190–207 (2016)