Optimization-Based Robust PID Controller to Enhance the Performance of Conical Tank System

Mulugeta Debebe(✉) and Beza Nekatibeb

College of Electrical and Mechanical Engineering, Addis Ababa Science and Technology University, Addis Ababa, Ethiopia
bezan.nekatibeb@aastu.edu.et

Abstract. This paper uses a particle swarm optimization (PSO) approach in order to identify nominal model parameters and to tune a Proportional- Integral - Derivative (PID) controller. In order to analyze the nominal model uncertainty, lumped multiplicative uncertainty structure is used to account unmodeled dynamics and nonlinearties. Moreover, first order weighting function through trial and error approximation method that limits the upper bound of the multiplicative uncertainty has been found. The Nominal model is used to design PSO optimized PID controller; and then the simulated results are compared with ACO based PI controller. The results show that the PSO optimized PID controller shows smooth and enhanced performance in terms of speed of response. And PSO based approach can efficiently applied in identifying the parameters of the system and tuning the PID controller for this particular system.

Keywords: PID controllers · Nominal model · Uncertain system · PSO

1 Introduction

Many food and chemical industries use conical tanks for their applications. In most of these industries, it is common the use a PID Controller for control of the conical tank system [1]. However, controlling the conical system is challenging as a result of the nonlinear nature of the system. There are many literatures on PID controller tuning for industrial process application based on the first order process with time delay (FOPTD) model. Different approaches have been introduced to tuning the PID controller for the past decades [2]. The approached which is proposed by Ziegler and Nichols (Z-N) has been proven to be successful to some level of control. The main weakness of the Z-N based approach is the necessity of previous knowledge about plant model. Moreover, there is still considerable disagreement regarding the possibility of the optimum coefficient of PID controller. Moreover, determination of PID controller tuning parameters is a vexing problem in many applications [3].

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Recently, modern optimization technique for complex engineering applications is becoming a research interest. Some of to mention are Genetic Algorithms (GA) [4], PSO [5, 6], ant colony optimization (ACO) [7] etc. These heuristic methods are the intelligence optimization algorithms that are based on natural behavior and characteristics. Mohideen K. Asan et al. [4] have used Labview to carry out a system identification technique to find the model of hybrid tank system and then used this model to tune off-line PID parameters by using Real-coded Genetic Algorithm (RGA). Wei-Der Chang et al. [5] proposed the PSO algorithm in order to optimally estimate the parameters for the Genesio-Tesi chaotic nonlinear systems. Mercy.D etal in [6] proposed improved PSO algorithm and made a comparison among the improved PSO, the conversional PSO and the classical PID. In [7], S.M.GiriRajkumar et al. ant ACO algorithm introduced to tune the optimal control parameters of a Proportional Integral (PI) controller for a nonlinear conical tank system. This work also presents system identification technique to estimate the parameters of the model based on open loop response of the system for four different heights of conical tank system. A key limitation proposed in [7] is that it does not suitable for analysis and practical implementation due to too many models and controller for a single system. In 1995 Kennedy and Eberhart were the first to propose PSO Algorithm. They inspired by nature based on the behavior of bird flocking. Group of birds find their food in a special way. More details of PSO algorithm can be found in Changhe Li PhD thesis [8].

The organization of this paper is as follows. The Sect. 2 gives a brief overview of the model formulation framework. The Sect. 3 examines the frequency analysis of uncertain system. Section 4 addresses PSO-PID tuning results. Conclusions are drawn in the Sect. 5.

2 PSO Algorithm- Based Model Formulations.

In the literatures various system identification methods have been proposed to obtain the model of the system both in linear and nonlinear structure. However, in this paper, an attempt has been made to address nominal model for the overall conical tank system as (FOPTD) shown in (1) by taking the models estimated from [7].

$$G(s) = \frac{Ke^{-Ts}}{\tau s + 1} \tag{1}$$

Where τ denotes the time constant, T is the time delay and K is the steady state gain.

In [7], various experiments were conducted for different operating ranges (flow) ranges and valve openings to get a typical response curve. The models of the conical tank were obtained by dividing the height into four levels to account the non-linearity in the shape of the conical tank: model-1 form 0 to 15 cm, model-2 from 15 to 27 cm, model-3 from 27 to 36 cm and 36 to 43 cm as model-4 as shown in (2) to (5) respectively.

$$\mathrm{G(s)_{model1}} = \frac{2.74\mathrm{e}^{-10.64\mathrm{s}}}{4.24\mathrm{s} + 1} \tag{2}$$

$$\mathrm{G(s)_{model2}} = \frac{2.19\mathrm{e}^{-13.98\mathrm{s}}}{8.9\mathrm{s} + 1} \tag{3}$$

$$\mathrm{G(s)_{model3}} = \frac{1.6\mathrm{e}^{-15.51\mathrm{s}}}{12.19\mathrm{s}+1} \tag{4}$$

$$\mathrm{G(s)_{model4}} = \frac{1.36\mathrm{e}^{-19\mathrm{s}}}{15.56\mathrm{s}+1} \tag{5}$$

Therefore, by taking these models, one nominal model that adequately enough to describe the overall conical tank system can be obtained using the PSO algorithm. Before PSO is executed, the objective function must be defined properly. Here well-known 2-norm error signal in (6) is used as the performance index to select the adjustable parameters of the model so that the step response error between the given models shown in (1) to (4) and the desired optimum model is minimized. Accordingly T^*, K^*, *and*, τ^* are estimated parameters of the nominal model, Y_{nom} is step response of the nominal model and Y_i is step response of the given models. The search lower and upper bounds for parameters τ, T and K are constrained in the interval (13, 20).

$$\|\boldsymbol{Y_{nom}} - \boldsymbol{Y_i}\|_2 = \underset{T,K,\tau}{\arg min} \left(\sqrt{\sum_{i=1}^{4}(\boldsymbol{Y_{nom}} - \boldsymbol{Y_i})^2}\right) \tag{6}$$

If any resulting parameter outside the interval during the search, set it the corresponding bounds, i.e.

$$parameter\ value = \begin{cases} 1.3\ if\ \ search\ value\ \le\ 1.3 \\ 20\ if\ \ search\ value \ge 20 \end{cases} \tag{7}$$

PSO is described by a well-known evolution equation and a particle $\boldsymbol{j}$ at iteration $\boldsymbol{k}$ is characterized by the following set of Eq. (8)

$$\begin{aligned} \hat{X}_k^j &= \hat{X}_{k-1}^j + \hat{V}_k^j \\ \hat{V}_k^j &= W * \hat{V}_{k-1}^j + C_1 R_1 * \left(P_b^j - \hat{X}_{k-1}^j\right) + C_2 R_2 * \left(G_b - \hat{X}_{k-1}^j\right) \end{aligned} \tag{8}$$

$\widehat{X}_k^j$ is the $\boldsymbol{j^{th}}$ particle and $\widehat{V}_k^j$ $\boldsymbol{j^{th}}$ particle velocity. The coefficients $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ are learning terms, coefficients $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are random numbers following a uniform distribution and the coefficient $\boldsymbol{W}$ is the inertia factor. Where P_b^j is personal best and G_b global best. The detailed selection of the value of the coefficients can be found in (7). Here the adaptive scheme that decreases $\boldsymbol{W}$ linearly with iteration (9) has been used.

$$\boldsymbol{W} = \mathbf{0.96} * \boldsymbol{W}. \tag{9}$$

MATLAB simulation software package has been used to identify nominal model parameter. The following values: variable size(parameters) = 3, Number of population = 15, $C_1 = 1.821$, $C_2 = 2.6$, $W = 1$ are selected. The resulting estimated nominal model parametrs are illustrated in Table 1.

Table 1. Estimated nominal model parameters

T^*	K^*	τ^*
11.1914	1.9718	11.8531

Substituting the obtained parameters in (1), the nominal model becomes

$$\mathrm{G_{nom}(s)} = \frac{1.9718\mathrm{e}^{-11.1914\mathrm{s}}}{11.8531\mathrm{s} + 1} \tag{10}$$

3 Frequency Analysis of Uncertain System

Further analysis is needed to investigate model uncertainties of the system before designing a controller. This model uncertainty arises from the time delay, the gain and poles of the transfer function. In this section, within the framework of FOPTD model structure, multiplicative uncertainty of the real perturbed system is examined explicitly. According to Doyle et al. [9] multiplicative uncertainty of the transfer function of the real (perturbed) plant $G_P(s)$ is defined as:

$$G_P(s) = G_{nom}(s)(1 + W_I(s)\Delta_I(s)); \qquad |\Delta_I(jw)| \leq 1, \ \forall w \tag{11}$$

$W_I(s)$ is weighting function that represents an upper bound on the multiplicative uncertainty, $\Delta_I(s)$ is disturbance (perturbation) acting on the system. In [9, 10], the relative uncertainty can be given as (12)

$$l_I = max\left|\frac{G_P(jw) - G_{nom}(jw)}{G_{nom}(jw)}\right|, \quad with \ \ |W_I(jw)| \geq l_I(jw), \ \forall w \tag{12}$$

From Eqs. (2)–(5), it is clear to verify that time delay, gain and time constants are within the following boundaries: $10.64 \leq T \leq 19, 1.36 \leq K \leq 2.74, 4.24 \leq \tau \leq 15.5$ respectively. Taking this in to consideration, the parameter intervals are widening to capture a large number of multiplicative uncertainties. The intervals are selected as: $8 \leq T \leq 20, \ 1.2 \leq K \leq 4, 4 \leq \tau \leq 20$.

To determine the relative error of perturbed system with a different combination of τ, T and K., simulation is carried out using MATLAB software. The multiplicative error of the pre-compensated frequency responses from the nominal model is shown in Fig. 1. Consequently, first order weighting function in (13) has been obtained through trial and error approximation that limits the upper bound of the multiplicative uncertainty.

$$w_I(jw) \cong \frac{0.89\left(\frac{s}{0.04}+1\right)}{\left(\frac{s}{0.3335}+1\right)} \tag{13}$$

As shown in Fig. 1, the frequency responses of the relative error of the perturbed system remain below the frequency response of the weighting function ($w_I(jw)$) almost in all frequencies. Also, it is shown how the relative error increases with frequency.

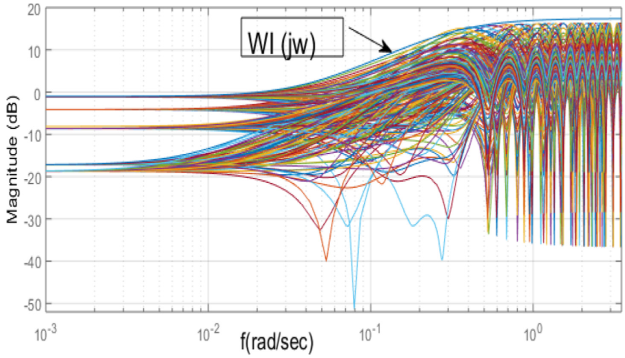

Fig. 1. Relative error of perturbed system

For this reason, the weighting function is increased with frequency to account the undesirable high-frequency unmodeled dynamics.

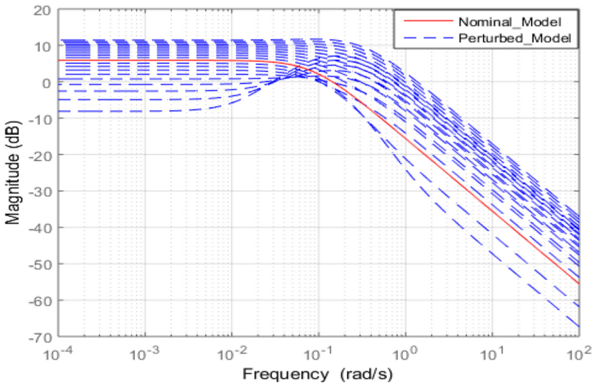

Fig. 2. Frequency response of nominal and perturbed model

Furthermore, the bode plot of the uncertain system with respect to weighting function is shown in Fig. 2. Obviously, most of the perturbed models show significant variation from the nominal model particularly at high frequencies.

4 PSO-PID Controller Design

Figure 3 demonstrates the designed implementation structure of PID-PSO controller. The optimal values of the controller parameters (Kc, T_I, and T_D) are tuned using PSO algorithm based on Integral absolute error (IAE) in (15) as a performance index.

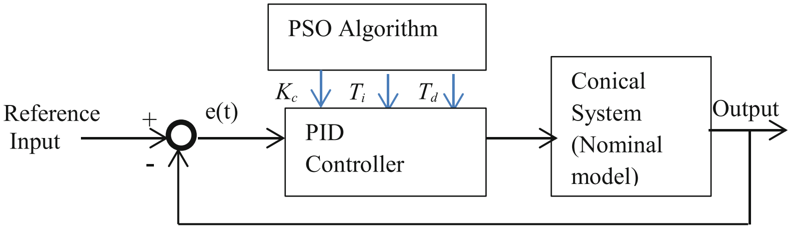

Fig. 3. PSO -PID Control Structure

$$I_{IAE} = \int_0^T |e(t)| dt \tag{15}$$

In most cases the structure of the PID controller is pre-determined for a specific application. Many variations of PID controller structure have been proposed in [3]. The initial investigation was made using PID with the filtered derivative structure described by Eq. (14).

$$G_c = K_c\left[1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s}\right] \tag{14}$$

The error, e(t) can be defined as,$e(t) = Y_{nom} - 1$. To run the PSO algorithm, different coefficient values and population sizes were chosen and investigated. The final selected values can be summarized as: variable size (parameters) = 3, number of population = 15, $C_1 = 2$, $C_2 = 2$, $W = 1$. The parameter Kc, Ti and Td are constrained with in the interval [0,1 to search for their upper and lower. Since at the beginning of each iteration, the difference between the actual output from the nominal model and the set point is large. Therefore, if the proportional part of the controller gain is too small during the PSO algorithm run time, the PID controller unable to respond adequately to the set point changes. To overcome this problem, an attempt was made by modifying the proportional part of the structure of the PID controller as shown in (16).

$$G_c = K_c\left[\left(1 + \frac{0.15}{K_c}\right) + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s}\right] \tag{16}$$

5 Result and discussion

5.1 PSO-PID Controller

The PID controller coefficients that minimizes the integral absolute error are identified and given in Table 2 for N = 50. The performance of the PSO based PID controller was evaluated by computing the step response of the nominal model, model1, model2, model3 and model4, as shown Fig. 4.

Table 2. PSO based PID parameter values

Kc	Ti	Td
0.016	0.9996	0.9817

In [7], the parameters of PI controller for the four models were obtained by ACO technique. The global best parameters of PI controller are given in Table 3. Using this data, the step response of PSO based PID controller is compared with ACO based PI controller for different levels and the simulation results are depicted in Fig. 4, Fig. 5, Fig. 6, and Fig. 7. The results show that the nominal model demonstrates fast and smooth (nonoscillatory) response at level of conical tank system.

Table 3. ACO based PI parameter values [7]

Parameters	Model1	Model2	Model3	Model4
Kp	0.21805	0.31813	0.4797	0.5502
Ki	0.02606	0.0227	0.0264	0.0252

Table 4. Comparison of time domain specifications

Controller	Model 1		Model2		Model3		Model4	
	Over shoot (%)	Settling time (s)	Over shoot (%)	Settling time (s)	Over shoot (%)	Settling time (s	Over shoot (%)	Settling time (s)
ACO-PI	23.28	73	19.44	102.4	16.2	95.6	15.36	120.2
PSO-PID	0.49	81	0.65	52.3	1.45	78.9	5.1	162

As can be seen from Table 4, PSO based PID controller shows better performance compared to ACO based PI controller regarding to overshoot for all four models. Similarly, settling time result shows model2 and model3 have been improved.

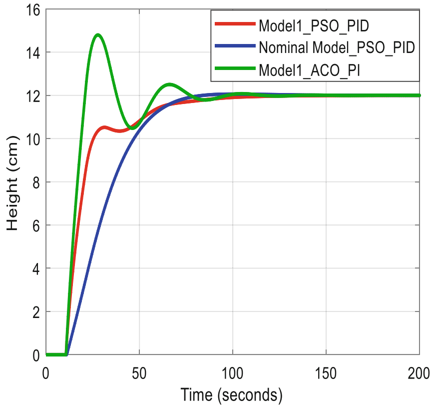

Fig. 4. ACO-PI and PSO-PID step response for model1

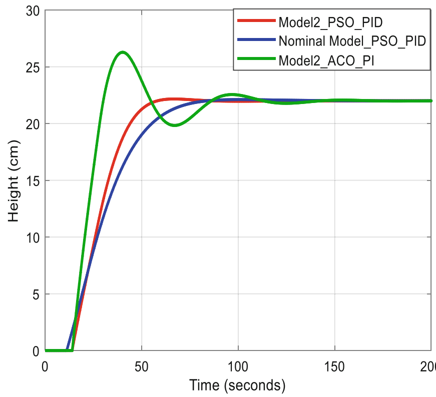

Fig. 5. ACO-PI and PSO-PID step response for model2

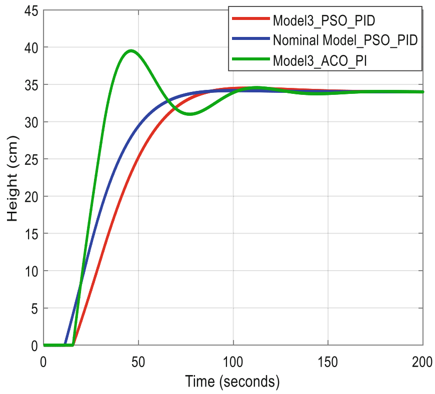

Fig. 6. ACO-PI and PSO-PID step response for model3

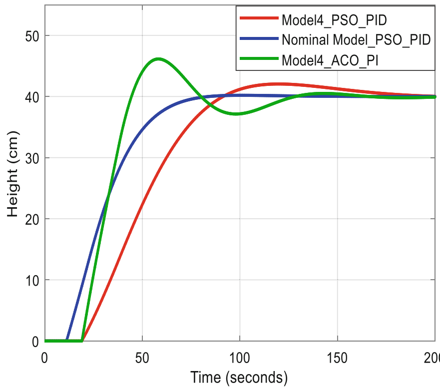

Fig. 7. ACO-PI and PSO-PID step response for model4

This section also investigates the degree of robustness of the PSO optimized PID controller to model uncertainty. These step response simulations are depicted in Fig. 8, Fig. 9, Fig. 10, and Fig. 11. The simulation results show that the designed PSO optimized PID controller is robust to the given uncertainties, since all the models and their uncertainties exhibit well behaved step responses.

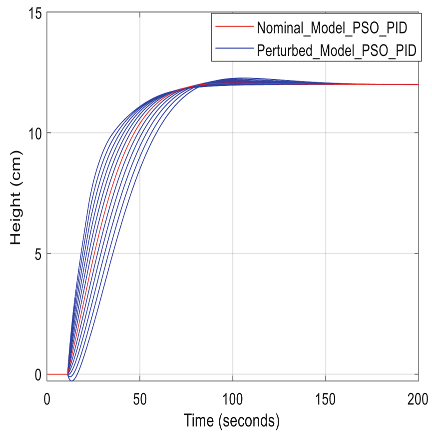

Fig. 8. Nominal Model and 30% uncertainty with step input of 12 cm

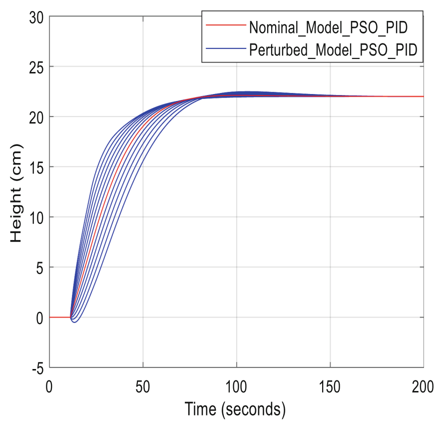

Fig. 9. Nominal Model and 30% uncertainty with step input of 22 cm

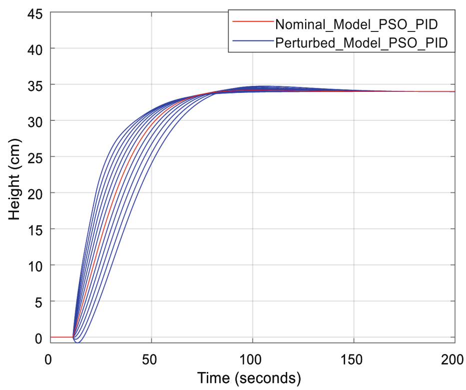

Fig. 10. Nominal Model and 30% uncertainty with step input of 34 cm

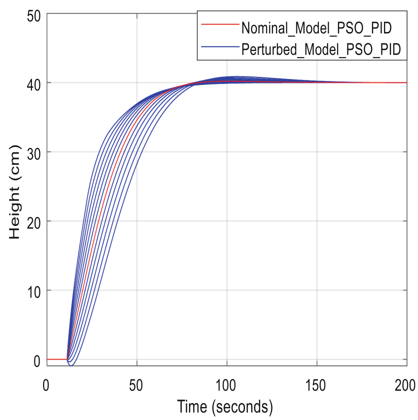

Fig. 11. Nominal Model and 30% uncertainty with step input of 40 cm

6 Conclusion

This paper has presented the PSO algorithm to solve the optimal parameters of nominal model that sufficiently enough to describe the overall height of conical tank system. In the proposed approach, a 2-norm is adopted as the performance index to minimize the error between the nominal model and the given four models. The unknown nominal model parameters can be obtained using the proposed PSO algorithm. The first order weighting function that limits the upper bound of the multiplicative uncertainty has been found. Moreover, an optimal value of PID controller parameters also obtained by using PSO algorithm. From the simulation results, one can to observe that PSO optimized PID controller has shown good performance. Additionally, the PID controller has shown robustness that able to reject model uncertainty up to 30%.

References

1. Pushpaveni, T., Srinivasulu Raju, S., Archana, N., Chandana, M.: Modeling and Controlling of Conical tank system using adaptive controllers and performance comparison with conventional PID. IJSER **2013**, 629–635 (2013)
2. Ziegler, J.G., Nichols, N.B.: Optimum settings for automatic controllers. Trans. Asme **64**, 759–765 (1942)
3. O'Dwyer, A.:"Handbook of PI and PID Controller Tuning Rules. 2nd Edition, Dublin Institute of Technology, Ireland (2006)
4. Asan, K., Saravanakumar, G., Valarmathi, K., Devaraj, D., Radhakrishnan, T.K.: Real-coded Genetic Algorithm for system identification and tuning of a modified Model Reference Adaptive Controller for a hybrid tank system. Appl. Math. Mod. **37**(6), 3829–3847 (2013). https://doi.org/10.1016/j.apm.2012.08.019
5. Mercy. D., Girirajkumar, S.M.: Real time implementation of improved PSO tuning for a conical tank process used in biodiesel production. Int. J. Pure Appl. Math. **118**(18), 2051–2062 (2018)
6. Chang, W.-D., Cheng, J.-P., Hsu, M.-C., Tsai, L.-C.: Parameter identification of nonlinear systems using a particle swarm optimization approach. In: IEEE International Conference on Networking and Computing. https://doi.org/10.1109/ICNC.2012
7. GiriRajkumar, S.M., Ramkumar, K., Sanjay Sarma, O.V.: Real time application of ants colony optimization. Int. J.Comput . Appl. **3**(8), 3446 (2010). https://doi.org/10.5120/753-1054
8. Li, C.: Particle Swarm Optimization in Stationary and Dynamic Environments, PhD thesis, University of Leicester, Department of Computer Science (2010)
9. Doyle, J.C., Francis, B.A., Tannenbaum, A.R.: Feedback Control Theory. Macmillan Publishing, New York (1992)
10. Skogestad, S., Postlethwaite, I.: Multivariable Feedback Control Analysis and Design. Norwagian University of Science and Technology, Jhone Wiley & Sons