





Accurate Frequency Estimator of Real Sinusoid Based on Maximum Sidelobe Decay Windows

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Abstract. The estimation of real sinusoid frequency is a significant problem in many scientific fields. The positive- and negative-frequency components of a real sinusoid interact with each other in the frequency spectrum. This leads to estimation bias. In this paper, we proposed an algorithm which is based on maximum sidelobe decay (MSD) windows. Firstly, the coarse frequency estimate is obtained by using Discrete Fourier Transform (DFT) and MSD windows. Then the negative-frequency component is removed by frequency shift. At last, the fine frequency estimation is performed by a high-precision frequency estimation algorithm. Simulation results show that the proposed algorithm has higher accuracy and better frequency estimation performance than AM algorithm, Candan algorithm, and Djukanovic algorithm.

Keywords: Frequency estimation · Real sinusoid · Windows · DFT

1 Introduction

The problem of sinusoid signal frequency estimation is quite significant in many scientific fields, such as communication, audio system, radar, power system, sonar, measurement and instrument. For example, in communication field, there may be a deviation between the oscillation frequency generated by the crystal oscillator and the nominal frequency, and there is generally a relative motion between the transmitting end and the receiving end of communication, resulting in the Doppler frequency shift. Carrier frequency offset is quite common in communication systems. Therefore, the carrier frequency offset must be estimated correctly. In music signal processing, standard music signals including human songs and musical instrument sounds are usually modeled as the sum of multiple time-varying sinusoidal signals, and the number of these sinusoidal signals is usually unknown. Next, multiple sinusoidal signals in music signals need to be detected. The common method is to continuously search and estimate the frequency of a single sinusoidal signal in the short-time Fourier transform domain. When using linear frequency modulated continuous wave radar for ranging, it is important to exactly estimate the

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frequency of linear frequency modulated continuous wave signal. Because the accuracy of frequency estimation directly determines the ranging accuracy. In power system, a significant parameter of power quality is frequency. And the variation of frequency is the result of dynamic imbalance between the generation and load, which needs accurate estimation. Accurate frequency estimation is the premise of power grid stability and normal operation of electrical equipment.

As we all know, frequency estimation algorithms are mainly divided into two categories in the case of additive white Gaussian noise, time domain algorithms [1–9] and frequency domain algorithms [10–20]. Time domain algorithms include the least mean square algorithm, the autocorrelation algorithm and the maximum likelihood algorithm. The accuracy of the maximum likelihood algorithms is very high, reaching the Cramer Rao lower bound (CRLB). Nevertheless, due to the large amount of computation, they are difficult to be used in real-time applications [1]. The frequency domain algorithms are primarily based on DFT. These algorithms have many advantages, such as fairly little computation and important signal-to-noise ratio (SNR) gain [18–22]. Therefore, they are suitable for real-time applications. For the DFT based algorithms, in additive broadband Gaussian noise, the peak position of the Discrete Time Fourier Transform (DTFT) of the whole signal represents the maximum likelihood frequency estimation of a sinusoid [10]. The location of the peak can be located through a two-step search process. The first step can be named as coarse-search, is to determine the maximum amplitude of DFT samples by a simple maximum search procedure. The second step is the fine-search which obtains the relative frequency deviation between the true frequency and the rough estimate by means of certain interpolation methods. The difference between different interpolation algorithms lies only in the second step. Aboutanios and Mulgrew (A&M) algorithm uses two spectral lines located exactly halfway between the maximum spectral line and its two neighbors for accurate estimation [18]. Candan algorithm achieves accurate estimation by using the maximum spectral line and two spectral lines on the left and right of the maximum spectral line [19].

All the above algorithms are used to estimate the frequency of complex sinusoid. There are also many practical situations and applications related to the real sinusoid model. For the real sinusoid, estimation bias is caused by the spectral superposition of its positive-frequency and negative-frequency components. As we can see in [23], multiplying the received signal by a window function is a simple and effective method to reduce the spectrum leakage. In [24], by adding Kaiser window to suppress the sidelobe of complex sinusoid, the estimation bias is reduced. However, there are still significant deviations in the windowed data. The time domain frequency estimators based on autocorrelation are the most accurate [9]. Although the algorithm in [9] is more precise than other time-domain estimation algorithms, this algorithm has root-mean-square error (RMSE) saturation under high SNR. Candan obtains the fine frequency estimation with arbitrary window functions [25]. In [26], Djukanovic derived a method based on the Candan method [25] and AM method [18], and the negative-frequency component is shifted via modulation before the fine estimation.

The MSD window is a kind of commonly used window function, which belongs to cosine windows. The sidelobe decay rate of MSD window is as high as $6(2H - 1)$

dB/octave, when its number of terms H is constant. Thus, a frequency estimation algorithm based on MSD windows in this paper is proposed to accurately estimate the frequency of real sinusoid. The coarse frequency estimate is obtained with MSD windows. Then the estimation bias is reduced by removal of the negative frequency component which is done by frequency shift. Finally, the fine frequency estimation is performed by a high-precision frequency estimation algorithm which uses two DTFT spectrum lines and the maximum DFT spectrum line. Computer simulations are conducted, and the results of simulation experiments show that the algorithm we proposed in this paper has higher accuracy and better frequency estimation performance than AM algorithm [18], Candan algorithm [25], and Djukanovic algorithm [26].

2 Proposed Algorithm

An accurate real sinusoid frequency estimation algorithm based on MSE windows is mainly described in this part. In the background of additive white Gaussian noise, the following signal model is considered

$$x(n) = A \cos(2\pi f_0 n + \phi) + z(n) \quad n = 0, 1, 2, \dots, N - 1 \quad (1)$$

In the above formula A represents the amplitude of the sinusoid, and f_0 is the frequency, ϕ is the initial phase. $z(n)$ represents zero-mean additive white Gaussian noise with variance σ_z^2 . In addition, $A > 0$ and $0 < f_0 < 1/2$. The SNR can be defined as $SNR = \frac{A^2}{2\sigma_z^2}$. The frequency estimation of the CRLB is [27]

$$\text{var}(\hat{f}_0) \geq \frac{12}{(2\pi)^2 SNR \cdot N(N^2 - 1)} \quad (2)$$

We know that the spectrum of a real sinusoid contains both positive and negative frequency components, and the estimation bias is caused by spectral superposition of the positive- and negative-frequency complex components. A known strategy to cut down frequency deviation is to multiply the real sinusoid by a window function. This is a simple method to reduce spectrum leakage [23].

We propose an algorithm by removing the negative-frequency component to reduce the estimation bias. The windowing method shown in Table 1 is used to acquire a rough frequency estimation for the received signal. Then the negative-frequency component can be moved to the low-pass band, and the negative-frequency component accounts for only the DC (direct-current) component of the modulated signal. After the removal of DC component, in this paper, the high-precision algorithm proposed is used to accurately estimate the positive-frequency component of complex sinusoid.

Step1: Coarse frequency estimation

Cosine window has been widely used in many documents [9, 23–25, 28]. Cosine windows include maximum sidelobe decay (MSD) window. The method proposed in this paper multiply the signal by MSD windows. The expressions of H -term MSD window are as follows [29]

$$w(n) = \sum_{h=0}^{H-1} (-1)^h a_h \cos\left(2\pi \frac{h}{N} n\right) \quad n = 0, 1, 2, \dots, N - 1 \quad (3)$$

The number of window coefficients a_h is H , $H \geq 1$. The coefficients expression of the H -term MSD window can be shown as [29]

$$a_h = \frac{C^{H-h-1}}{2^{2H-3}} \quad h = 0, 1, \dots, N-1 \quad (4)$$

Multiplying $x(n)$ by a cosine window, we have

$$x_w(n) = x(n) \cdot w(n) \quad n = 0, 1, \dots, N-1 \quad (5)$$

Then we perform DFT on $x_w(n)$, and search the index number k_m of the maximum DFT spectral line.

We use Δf to represent the DFT frequency resolution. When $p = \pm 0.1$, the DFT samples $X_w(0.1)$ and $X_w(-0.1)$ which are $\pm 0.1\Delta f$ away from the maximum DFT spectral line can be calculated as

$$X_w(p) = \sum_{n=0}^{N-1} x_w(n) e^{-j2\pi \frac{k_m+p}{N} n} \quad (6)$$

In [30], the estimation formula of the normalized frequency shift δ is shown as follows:

$$\hat{\delta} = r \cdot Y(\delta) \quad (7)$$

where $Y(\delta)$ is expressed as

$$Y(\delta) = \frac{N}{\pi} \tan^{-1} \left[\frac{(|X_w(i) - X_w(-i)|) \cdot \sin(\pi i/N)}{|X_w(i) - X_w(-i)| \cdot \cos(\frac{\pi i}{N}) - 2 \cos(\pi i) \cdot |X_w(0)|} \right] \quad (8)$$

and r is calculated as

$$r = \frac{\cos(\pi i) \cdot a_0 - \cos(\frac{\pi i}{N}) \cdot \sin(\pi i) \cdot \frac{1}{\pi} \cdot \sum_{h=0}^{H-1} (-1)^h \frac{a_h \cdot i}{i^2 - h^2}}{\left\{ \left[\frac{1}{\pi} \sin(\pi i) + i \cos(\pi i) \right] \sum_{h=0}^{H-1} \frac{(-1)^h a_h}{i^2 - h^2} - \frac{2i^2}{\pi} \sin(\pi i) \sum_{h=0}^{H-1} \frac{(-1)^h a_h}{(i^2 - h^2)^2} \right\} \cdot \sin(\frac{\pi i}{N})} \quad (9)$$

For two term MSD window, we have $r_{2MSD} = 134.97$. We show the iterative process of obtaining the coarse frequency estimate \hat{f}_0^f based on MSD windows in Table 1.

Step 2: Removal of the negative frequency component

Move the frequency of the received signal $x(n)$ as follows

$$x_m(n) = x(n) e^{j2\pi f_0^c n} \quad (10)$$

The results of (11) in a circular shift of the spectrum of $x(n)$ by f_0^c Hz. Because f_0^c is pretty near to the signal's real frequency f_0 , the negative frequency component can be shifted to the low-pass band. However, the most important part of energy located at the

Table 1. Coarse frequency estimation algorithm.

Algorithm: Coarse frequency estimation with MSD windows	
1	Get $x_w(n) = x(n) \cdot w(n)$ $n = 0, 1, \dots, N-1$
2	Perform N-point DFT of $x_w(n)$
3	Search the index number k_m of the maximum spectral line
4	Calculate $X_w(0.1)$ and $X_w(-0.1)$, via $X_w(p) = \sum_{n=0}^{N-1} x_w(n) e^{-j2\pi \frac{k_m+p}{N}}$, $p = \pm 0.1$
5	Calculate $\hat{\delta}_1$ with $X_w(0)$, $X_w(0.1)$, $X_w(-0.1)$, via (7)
6	Calculate $X_w(\hat{\delta}_1)$, $X_w(\hat{\delta}_1 + 0.1)$, and $X_w(\hat{\delta}_1 - 0.1)$, via $X_w(p) = \sum_{n=0}^{N-1} x_w(n) e^{-j2\pi \frac{k_m+p}{N}}$, $p = \hat{\delta}_1, \hat{\delta}_1 \pm 0.1$
7	Calculate $\hat{\delta}_2$ with $X_w(\hat{\delta}_1)$, $X_w(\hat{\delta}_1 + 0.1)$ and $X_w(\hat{\delta}_1 - 0.1)$, via (7)
8	The coarse frequency estimate is $f_0^c = (k_m + \hat{\delta}_1 + \hat{\delta}_2) \Delta f$

DC component of $x_m(n)$ [25]. Then the negative-frequency component can be removed from the received signal as

$$x_r(n) = x_m(n) e^{-j2\pi f_0^c n} - A_n e^{-j2\pi f_0^c n} \quad (11)$$

where A_n represents the amplitude of the negative frequency component of the real sinusoid, and is expressed as follows

$$A_n = \overline{x_m(n)} \quad (12)$$

where the $\overline{x_m(n)}$ represents the mean value of $x_m(n)$. And from the derivation of (10), the final expression of $x_r(n)$ is as follows

$$x_r(n) = x(n) - A_n e^{-j2\pi f_0^c n} \quad (13)$$

Step 3: Fine frequency estimation

We perform DFT on the reconstructed received signal $x_r(n)$, and then search the index number k_r of the maximum spectral line. When $l = \pm 0.1$, we have

$$X_r(l) = \sum_{n=0}^{N-1} x_r(n) e^{-j2\pi \frac{k_r+l}{N}} \quad l = \pm 0.1 \quad (14)$$

In [20], the estimation formula of the normalized frequency offset δ can be shown as follows

$$\hat{\delta} = \frac{N}{\pi} \tan^{-1} \left[\frac{(|X_r(i) - X_r(-i)|) \cdot \sin(\pi i/N)}{(|X_r(i) - X_r(-i)|) \cdot \cos(\frac{\pi i}{N}) - 2 \cos(\pi i) \cdot |X_r(0)|} \right] \quad (15)$$

We show the iterative process of obtaining fine frequency estimate f_0^f in Table 2.

Table 2. Fine frequency estimation algorithm.

Algorithm: Fine frequency estimation algorithm	
1	Perform N-point DFT of $x_r(n)$
2	Search the maximum spectral line k_r
3	Calculate $X_r(0.1)$ and $X_r(-0.1)$, via $X_r(l) = \sum_{n=0}^{N-1} x_r(n)e^{-j2\pi\frac{k_r+l}{N}}$ $l = \pm 0.1, l = \pm 0.1$
4	Calculate $\hat{\delta}_3$ with $X_r(0), X_r(0.1), X_r(-0.1)$, via (15)
5	Calculate $X_r(\hat{\delta}_3), X_r(\hat{\delta}_3+0.1)$, and $X_r(\hat{\delta}_3-0.1)$, via $X_r(l) = \sum_{n=0}^{N-1} x_r(n)e^{-j2\pi\frac{k_r+l}{N}}$, $l = \hat{\delta}_3, \hat{\delta}_3 \pm 0.1$
6	Calculate $\hat{\delta}_4$ with $X_r(\hat{\delta}_3), X_r(\hat{\delta}_3+0.1)$ and $X_r(\hat{\delta}_3-0.1)$, via (15)
7	The fine frequency estimate is $\hat{f}_0^f = (k_r + \hat{\delta}_3 + \hat{\delta}_4)\Delta f$

3 Simulation Results

For this section, we test and verify the performance of the proposed algorithm, and carry out simulation analysis in the presence of additive white Gaussian noise. At the same time, in order to find out the performance difference between the proposed algorithm and the competitive algorithm, this part of the experiments we compare the proposed algorithm with the AM algorithm [18], Candan algorithm [25] with Kaiser window, and the Djukanovic algorithm [26]. The simulation experiments are mainly divided into three categories: RMSE of frequency estimation versus SNR, RMSE versus the signal frequency f_0 and RMSE versus the initial phase ϕ .

The first category: When $f_0 = 0.1917$, $\phi = \pi/7$, and $N = 128$, we evaluate the RMSE of the proposed algorithm versus SNR, varied from -10 to 50 dB, in steps of 2 dB. Figure 1 shows the RMSE of the algorithm we proposed, AM algorithm, Candan algorithm with Kaiser window ($\beta = 5$), and the Djukanovic algorithm under variable SNR. The RMSE of the proposed algorithm based on 2-term MSD window is quite near to that of Djukanovic algorithm. Meanwhile, the proposed algorithm and Djukanovic algorithm surpass the other algorithms in terms of accuracy and RMSE saturation. The proposed algorithm is closer to CRLB than Djukanovic algorithm from the local enlarged view. AM algorithm shows RMSE saturation when SNR is higher than 10 dB. Although Candan algorithm with Kaiser window ($\beta = 5$) conforms to the CRLB trend in the whole range of SNR, the RMSE of this algorithm is relatively large, and its curve is not close to CRLB. Simulation experiments of the proposed algorithm with 3-term MSD window are conducted and the results are very similar to those when the signal is multiplied by 2-term MSD window. Therefore, only the results of the proposed method with 2-term MSD window are shown in the figure.

From the above simulation image, it can be seen that when SNR is variable the performance of the proposed algorithm is better than that of AM algorithm, Candan algorithm with Kaiser window ($\beta = 5$), and Djukanovic algorithm.

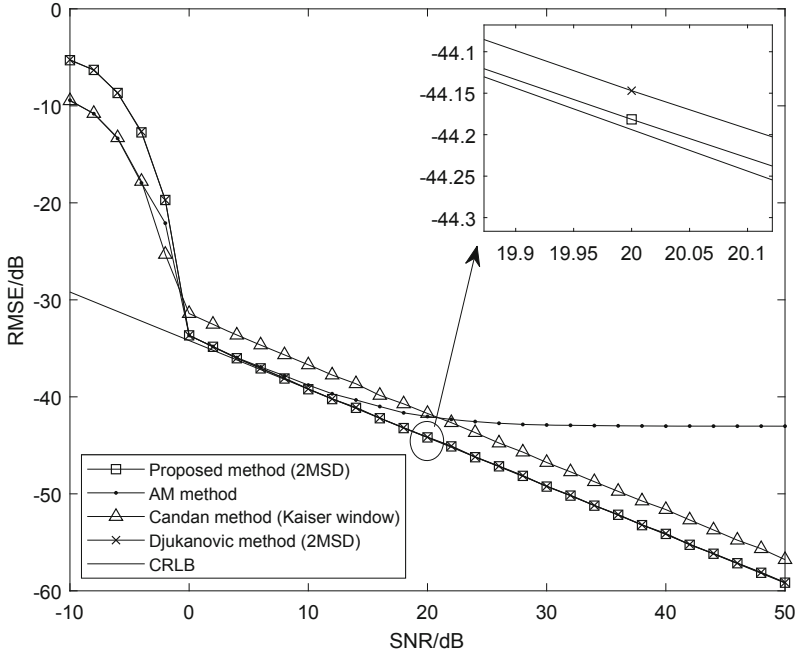


Fig. 1. RMSE versus SNR at $f_0 = 0.1917$, $\phi = \pi/7$ and $N = 128$

The second category: When $\phi = \pi/7$ and $N = 128$, we evaluate the RMSE of different algorithms versus the sinusoid frequency f_0 . And f_0 is variable, taken in interval $(0, 1/2)$ in 50 points.

Figure 2 shows the RMSE of the algorithm we proposed, AM algorithm, Candan algorithm with Kaiser window ($\beta = 5$), and the Djukanovic algorithm under variable sinusoid frequency f_0 for $SNR = 30$ dB. We can see that the RMSE of the algorithm we proposed in this paper based on 2-term MSD window is pretty near to that of Djukanovic algorithm. Meanwhile, the algorithm proposed in this paper and the Djukanovic algorithm are still closer to CRLB than the other estimators and their curves are relatively flat at high SNR. But the proposed algorithm is closer to CRLB than Djukanovic algorithm from the local enlarged view. The error of AM algorithm is large at high SNR. That is because AM algorithm is a complex sinusoid frequency estimation algorithm, and it can't effectively deal with the deviation caused by the superposition of positive- and negative-frequency components of real sinusoid. Although Candan algorithm reduces the deviation caused by the superposition of positive- and negative-frequency components of a real sinusoidal signal, the RMSE of this algorithm is about 2.5 dB higher than CRLB.

From the above simulation image, it can be seen that when f_0 is variable the performance of the proposed algorithm is better than that of AM algorithm, Candan algorithm with Kaiser window ($\beta = 5$), and Djukanovic algorithm. And the proposed algorithm is not sensitive to sinusoid frequency.

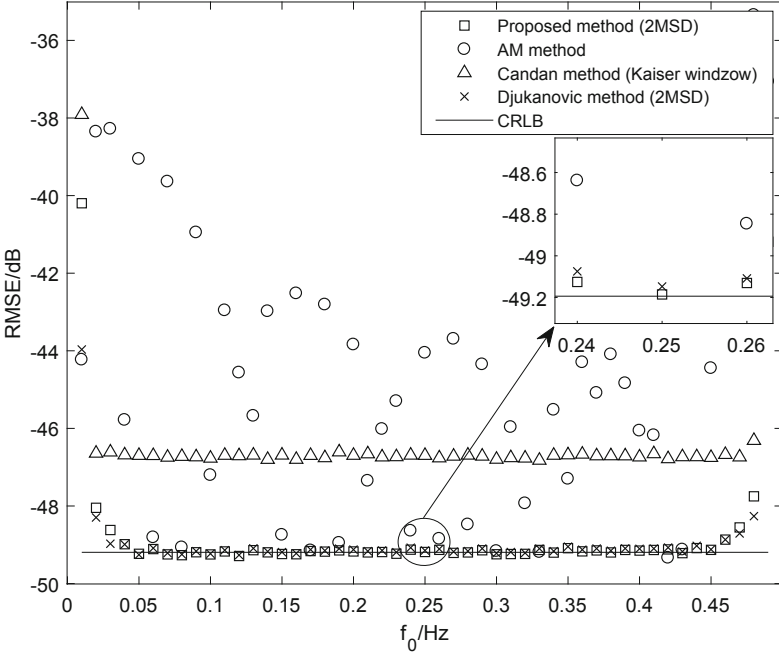


Fig. 2. RMSE versus f_0 with $N = 128$, $\phi = \pi/7$ and $SNR = 30$ dB

The third category: when $f_0 = 0.1917$, $N = 128$, and $SNR = 10$ dB and 50 dB are considered, we evaluate the RMSE of different algorithm versus initial phase ϕ . And ϕ is varied from 0 to 2π . The RMSE curves are calculated for 50 ϕ values.

Figure 3 shows the RMSE of the proposed algorithm, AM algorithm, Candan algorithm with Kaiser window ($\beta = 5$), and the Djukanovic algorithm under variable initial phase ϕ for $SNR = 10$ dB. We can see that Candan algorithm with Kaiser window ($\beta = 5$) has the largest RMSE in the four algorithms, although the algorithm reduces the error caused by the superposition of positive and negative frequency components of real sinusoidal signal, and the curve is not close to CRLB. The RMSE curve of the AM algorithm is also higher than CRLB. Because it is a complex sinusoid frequency estimation algorithm, it cannot effectively deal with the deviation caused by the superposition of positive frequency and negative frequency components of real sinusoid and has a large error in estimating the real signal frequency. The proposed algorithm and Djukanovic algorithm are closer to CRLB than the other two estimators.

When $SNR = 50$ dB and other conditions remain unchanged, Fig. 4 shows the RMSE of the algorithm we proposed and other three algorithms versus initial phase ϕ . We can see that the RMSE curve of AM method changes periodically, but it still has a large RMSE. The RMSE curve of Candan algorithm with Kaiser window ($\beta = 5$) is relatively flat, but it is about 2 dB higher than the CRLB, and the estimation bias is also large. However, even under the condition of higher SNR, the RMSE curves of the proposed algorithm and Djukanovic algorithm are very flat and close to CRLB.

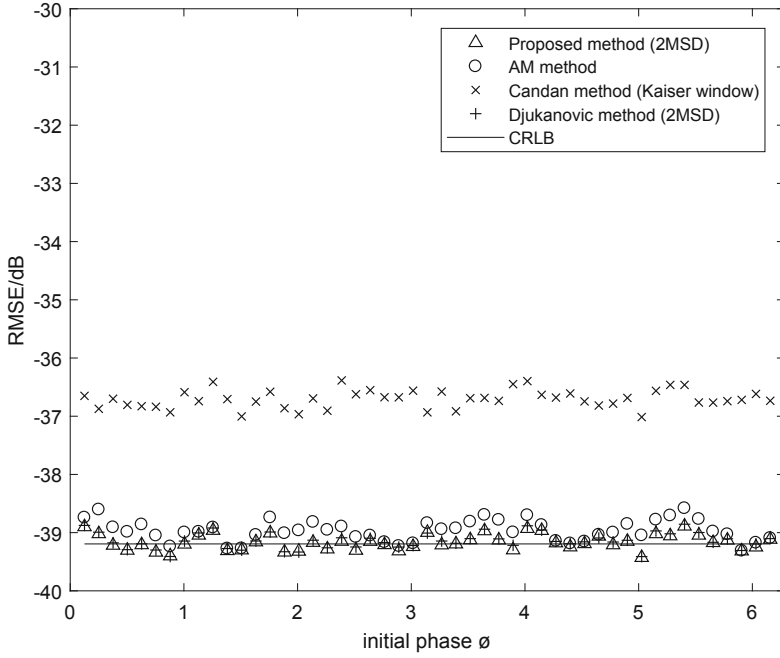


Fig. 3. RMSE versus ϕ with $N = 128, f_0 = 0.1917$ and $SNR = 10$ dB

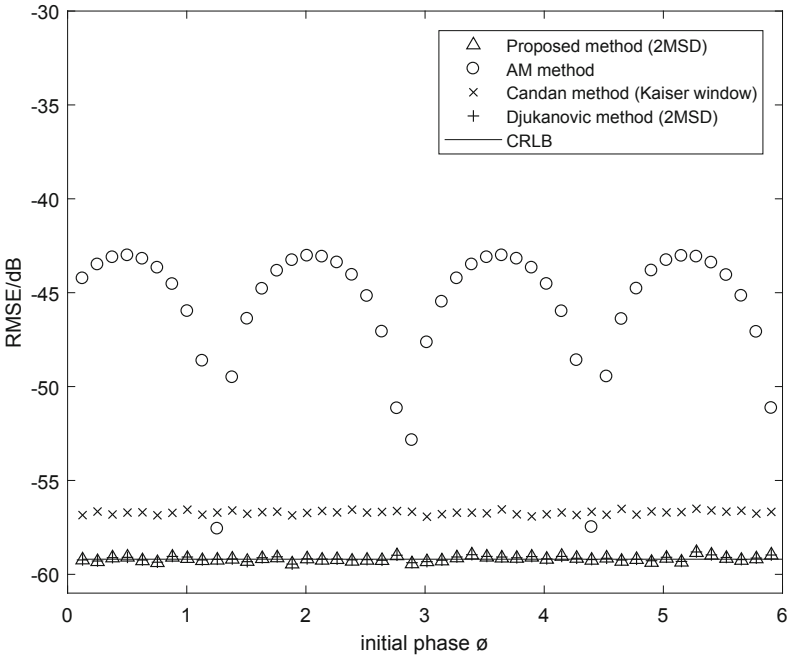


Fig. 4. RMSE versus ϕ with $N = 128, f_0 = 0.1917$ and $SNR = 50$ dB

From the above two simulation images, it can be seen that when initial phase ϕ is variable, the performance of the algorithm we proposed is better than AM algorithm, Candan algorithm with Kaiser window ($\beta = 5$), and its performance is similar to that of the Djukanovic algorithm. And the proposed algorithm is not susceptible to initial phase.

Table 3 shows the computational complexity of different algorithms. It can be seen that the proposed algorithm has the same complexity as the other three algorithms. However, according to the simulation results of Fig. 1 and Fig. 2 show that the algorithm proposed in this paper has better frequency estimation performance and higher accuracy.

Table 3. Computational complexity of different methods

Method	Computational complexity
AM [18]	$O(N \log_2 N)$
Candan [25]	$O(N \log_2 N)$
Djukanović [26]	$O(N \log_2 N)$
Proposed	$O(N \log_2 N)$

4 Conclusion

Owing to the spectrum superposition of positive- and negative-frequency components, in the process of real sinusoid frequency estimation, the estimation result will produce estimation error. In order to deal with this problem, a high-precision estimation algorithm based on MSD windows is proposed. Through coarse estimation with MSD window, removal of the negative frequency component with frequency shift and fine estimation, an accurate frequency estimate can be obtained. Computer simulations are conducted, and from the results we can see that the RMSE of the proposed algorithm is closer to CRLB than AM algorithm, Candan algorithm and Djukanovic algorithm. The proposed algorithm and the other three algorithms have the same computational complexity. And the proposed algorithm is not susceptible to the sinusoid frequency and the initial phase. The proposed algorithm can reduce the estimation bias due to the frequency spectrum superposition of real sinusoid and can be used in practical applications.

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