# Passive Target Detection Based on GLRT Using Multi-satellite Illumination

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**Abstract.** This paper proposes a novel passive location parameter estimator using multiple satellites for moving aerial targets. Specifically, we consider target detection in the ground dual receiver system, which is divided into reference channel and monitoring channel. The reference channel receives the direct wave signals from multiple external emitters, and the monitoring channel receives the echo signals reflected by the target. We propose a target detection method based on a generalized likelihood ratio (GLRT). This detection algorithm is robust to the reference channel noise. Extensive simulations are conducted to evaluate the performance of the algorithm under various network settings.

**Keywords:** Generalized likelihood ratio test · Multi-satellite illumination · Passive detection · Target detection

## 1 Introduction

Passive detection systems utilize the reflected echo signals from a moving target generated by a non-cooperative radiation source and realize the target's passive detection [1–5]. As such, an external radiation source for obtaining the moving target detection has been extensively studied with its superiority on concealment and reliability [6].

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The traditional method for passive detection is sensitive to the noise in the reference channel based on the reference channel's direct wave signal and the monitoring channel's echo signal [7–10]. Thus, the detection performance is readily affected by the noise of the reference channel [11,12]. At present, most works assume that the signal-to-noise ratio of the reference channel is high. On this basis, the echo signal is detected, which is inconsistent with the actual scene. The current works use the generalized likelihood ratio method to achieve the purpose of weak echo detection. However, these detection methods are related to the modeling and analysis in a single emitter scene. For effectively detecting the weak echo from multiple satellites in the multi-satellite joint scene, a multi-satellite collaborative target detection method based on the generalized likelihood ratio method is introduced as a more suitable alternative.

## 2 System Model

In the actual receiving scene, the ground receiving system consists of two antennas. One is called the monitoring channel antenna, and the other is the reference channel antenna. The reference antenna is placed in the satellite direction, and the monitoring antenna is an antenna with a wide beam pointing to the monitoring area. When the satellite emitter radiates signals, we expect to receive a direct wave signal and an echo signal reflected by the target. The time delay of the echo signal is $\tau$, and the Doppler frequency shift is $f_d$. In this process, some clutter will inevitably be received. When there are multiple satellites as external radiation sources, the signal received by the monitoring channel is represented by $x(t)$, which can be given by

$$\begin{aligned} x(t) &= \sum_{\eta=1}^{M} a_\eta s_\eta\left(t-\tau_\eta\right) e^{j2\pi f_{d\eta} t} \\ &+ \sum_{\eta=1}^{M} \phi_{\eta,0} s_\eta(t) + \sum_{\eta=1}^{M} \sum_{k=1}^{N_c} \phi_{\eta,k}(t) s_\eta\left(t-\tau_{\eta,k}^{(c)}\right) \\ &+ \sum_{m=1}^{K} \gamma_m^{(t)} s_m\left(t-\tau_m^{(t)}\right) e^{j2\pi f_{dm}^{(t)} t} + n_s(t) \quad 0<t<T, \end{aligned} \tag{1}$$

where $M$ is the number of satellite radiation sources; $f_{d\eta}$, $\tau_\eta$ and $\alpha_\eta$ are Doppler frequency shift, delay and amplitude of target echo respectively; $n_s(t)$ is Gaussian white noise; $K$ is the number of jamming targets; $\beta_m^{(t)}$, $\tau_m^{(t)}$, $f_{dm}^{(t)}$ are amplitude, delay and Doppler frequency shift of the target echo; $\tau_{\eta,k}^{(c)}$ and $\phi_{\eta,k}(t)$ are the delay and amplitude of multipath signal respectively; $T$is the observation time; $N_C$ is the number of multipath of a single satellite signal; the amplitude $\phi_{\eta,k}(t)$ of multipath is a random process with special power spectral density. In the signal model of this paper,we assume that the amplitude of multipath signal has special power spectral density,which can be expressed by multiple frequencies $f_i^{(c)}$, we can get the following results

$$\phi_{\eta,k}(t) = \sum_{i=-Q}^{Q} \phi_{\eta,k,i} e^{j2\pi f_i^{(c)} t}, \tag{2}$$

where $f_i\left(c\right) = (i - 1 - Q)\Delta f_c, i = 1, 2, \cdots, 2Q + 1$, and $Q$ and $\Delta f_c$ are two suitable parameters selected according to the actual scene. In this case, $\phi_{\eta,k,i}$ is the amplitude of the multipath signal at frequency $f_i^{(c)}$ and time delay $\tau_{\eta,k}^{(c)}$ at $k = 1, 2, \cdots, N_c$. In the actual scene, the reference signal in the reference channel may also be affected by multipath and target echo. Therefore, it will affect the performance of subsequent target detection. Therefore, before detecting the target in the monitoring channel, it is necessary to use some signal processing algorithms such as beamforming or channel equalization to suppress multipath and echo signals in the reference channel. The space-time constant modulus algorithm can recover the degradation of detection performance caused by the presence of echo and multipath signals in the reference channel. Therefore, after a proper signal processing algorithm and band-pass filter separation, the signal in the reference channel can be given by:

$$y_\eta\left(t\right) = b_\eta s_\eta\left(t\right) + n_\eta\left(t\right)\ 0 \le t < T\eta = 1, 2 \cdots, M, \tag{3}$$

where $b_\eta$ is the amplitude of the direct wave signal received by the reference antenna; $n_\eta\left(t\right)$ is the sum of the noise of the reference channel receiver and the interference signal after signal processing. The signals $x(t)$ and $y_\eta(t)$ are sampled at time $t_n = {}^{n}\!/_{f_s} = nT_s, n = 1, 2, \cdots, N$. At the $n$th sampling interval, the sampling signals of the received signals in the monitoring channel and the reference channel are represented as $x(n)$ and $y_\eta\left(n\right)$, respectively, as follows

$$\begin{aligned} x[n] = &\sum_{\eta=1}^{M} a_\eta s_\eta\left[n - n_\eta\right] e^{j2\pi\Omega_n} + \sum_{\eta=1}^{M} \phi_{\eta,0} s_\eta[n] \\ &+ \sum_{\eta=1}^{M}\sum_{k=1}^{N_c}\sum_{i=-Q}^{Q} \phi_{\eta,k,i} s_\eta\left[n - n_{\eta,k}^{(c)}\right] e^{j2\pi\Omega_m^{(t)}} + n_s[n] \\ &+ \sum_{m=1}^{K} \gamma_m^{(t)} s_\eta\left[n - n_m^{(t)}\right] e^{j2\pi\Omega_m^{(t)}} \quad n = 1, 2, \cdots, N, \end{aligned} \tag{4}$$

$$y_\eta\left(n\right) = b_\eta s_\eta\left(n\right) + n_\eta\left(n\right) \quad n = 1, 2, \cdots, N, \tag{5}$$

where $\Omega_\eta = 2\pi f_{d\eta} T_s$ and $\Omega_m^{(t)} = 2\pi f_{dm}^{(t)} T_s$ represent the normalized Doppler frequency of target echo and jamming target echo respectively. And $\tau_\eta = n_\eta T_s$, $\tau_m^{(t)} = n_m^{(t)} T_s$, $\tau_{\eta,k}^{(c)} = n_{\eta,k}^{(c)} T_s$, where $n_\eta$, $n_m^{(t)}$ and $n_{\eta,k}^{(c)}$ represent the echo signal of received signal in the monitoring channel, jamming target echo and multipath delay respectively. Substituting Eq. (5) into Eq. (4), the results can be given by

$$
\begin{aligned}
x(n) = & \sum_{\eta=1}^{M} \tfrac{a_\eta}{b_\eta} y_\eta \left[n-n_\eta\right] e^{jn\Omega_\eta} + \sum_{\eta=1}^{M} \tfrac{\phi_{\eta,0}}{b_\eta} y_\eta[n] \\
& + \sum_{\eta=1}^{M} \sum_{k=1}^{N_c} \sum_{i=-Q}^{Q} \tfrac{\phi_{\eta,k,i}}{b_\eta} y_\eta \left[n-n_{\eta,k}^{(c)}\right] e^{j2\pi f_i^{(c)} T_s n} \\
& + \sum_{m=1}^{K} \tfrac{\gamma_m^{(t)}}{b_\eta} y_m \left[n-n_m^{(t)}\right] e^{jn\Omega_m^{(t)}} + n_s(n) \\
& + \sum_{\eta=1}^{M} \tfrac{a_\eta}{b_\eta} n_\eta \left[n-n_\eta\right] e^{jn\Omega_\eta} + \sum_{\eta=1}^{M} \tfrac{\phi_{\eta,0}}{b_\eta} n_\eta[n] \\
& + \sum_{\eta=1}^{M} \sum_{k=1}^{N_c} \sum_{i=-Q}^{Q} \tfrac{\phi_{\eta,k,i}}{b_\eta} y_\eta \left[n-n_{\eta,k}^{(c)}\right] e^{j2\pi f_i^{(c)} T_s n} \\
& + \sum_{m=1}^{K} \tfrac{\gamma_m^{(t)}}{b_\eta} n_m \left[n-n_m^{(t)}\right] e^{jn\Omega_m^{(t)}},
\end{aligned}
\tag{6}
$$

By integrating the last four noise terms in Eq. (6) into $n_s[n]$, the above equation can be rewritten as follows

$$
\begin{aligned}
x(n) = & \sum_{\eta=1}^{M} \alpha_\eta y_\eta [n-n_\eta]^{jn\Omega_\eta} + \sum_{\eta=1}^{M} c_{\eta,0} y_\eta[n] \\
& + \sum_{\eta=1}^{M} \sum_{k=1}^{N_c} \sum_{i=-Q}^{Q} c_{\eta,k,i} y_\eta \left[n-n_{\eta,k}^{(c)}\right] e^{j2\pi f_i^{(c)} T_s n} \\
& + \sum_{m=1}^{K} \beta_m^{(t)} y_m \left[n-n_m^{(t)}\right] e^{jn\Omega_m^{(t)}} + n_s[n],
\end{aligned}
\tag{7}
$$

where $\alpha_\eta \triangleq {\alpha_\eta}/{b_\eta}$, $c_{\eta,0} \triangleq {\alpha_\eta}/{b_\eta}$, $c_{\eta,k,i} \triangleq {\phi_{\eta,k,i}}/{b_\eta}$, $\beta_m^{(t)} \triangleq \gamma_m^{(t)} \big/ b_\eta$. By constructing the matrix $\Lambda$ and matrix $\Gamma$ as follows

$$
\left[\Lambda^{(c)}\right]_{kj} = 2\pi f_k^{(c)} T_s \quad k = 1, 2, \cdots, 2\mathrm{Q}+1, \mathrm{j} = 1, 2, \cdots, \mathrm{N_c}, \tag{8}
$$

$$
\left|D^{(c)}\right|_{kj} = n_j \quad k = 1, 2, \cdots, 2Q+1,\ j = 1, 2, \cdots, N_c, \tag{9}
$$

By defining vectors $\Omega \triangleq [0, vec(\Lambda)]^T$ and $n \triangleq [0, vec(D)]^T$, Eq. (6) can be given by

$$
\begin{aligned}
x[n] = & \sum_{\eta=1}^{M} a_\eta y_\eta \left[n-n_\eta\right] e^{j2\pi\Omega_n} + \sum_{\eta=1}^{M} \phi_{\eta,0} y_\eta[n] \\
& + \sum_{\eta=1}^{M} \sum_{k=1}^{N_c} c_{\eta,k} y_\eta \left[n-n_{\eta,k}^{(c)}\right] e^{jn\Omega_{\eta,k}^{(c)}} + n_s[n] \\
& + \sum_{m=1}^{K} \beta_m^{(t)} y_m \left[n-n_m^{(t)}\right] e^{j2\pi\Omega_m^{(t)}} \quad n = 1, 2, \cdots, N,
\end{aligned}
\tag{10}
$$

where $P = (2Q+1)\,N_c + 1$; $\Omega^{(c)}_{\eta,k}$ and $n^{(c)}_{\eta,k}$ are the $k$th elements of $\Omega^{(c)}$ and $n^{(c)}$, respectively. As shown in the above formula, $\Omega^{(c)}_{\eta,k}$ and $n^{(c)}_{\eta,k}$ at $k = 2, 3, \cdots, P$ correspond to the Doppler frequency shift and time delay of multipath signal in the monitoring channel; $c_1$ at $k = 1$ corresponds to the amplitude of direct wave signal.

## 3 Multi-satellite Weak Target Echo Detection Based on GLRT

The general problem of detecting targets is characterized in that when there are $K$ interfering target echoes, direct waves and multipaths, and the amplitude of the target echo signal $\alpha_\eta$ is unknown, the target is detected. The problem can be described by the following assumptions

$$\begin{aligned} H_0 : x(n) = & \sum_{\eta=1}^{M}\sum_{k=1}^{P} c_{\eta,k} y_\eta \left[n - n^{(c)}_{\eta,k}\right] e^{jn\Omega^{(c)}_{\eta,k}} \\ & + \sum_{m=1}^{K} \beta^{(t)}_m y_m \left[n - n^{(t)}_m\right] e^{jn\Omega^{(t)}_m} + n_s\,[n]\,, \end{aligned} \tag{11}$$

$$\begin{aligned} H_1 : x(n) = & \sum_{\eta=1}^{M} \alpha_\eta y_\eta\,[n - n_\eta] e^{jn\Omega_\eta} \\ & + \sum_{\eta=1}^{M}\sum_{k=1}^{P} c_{\eta,k} y_\eta \left[n - n^{(c)}_{\eta,k}\right] e^{jn\Omega^{(c)}_{\eta,k}} \\ & + \sum_{m=1}^{K} \beta^{(t)}_m y_m \left[n - n^{(t)}_m\right] e^{jn\Omega^{(t)}_m} + n_s\,[n]\,, \end{aligned} \tag{12}$$

where $n = 0, 1, \cdot\cdot\cdot, N-1$, the first term is target echo, the second term is multipath and direct wave, the third term is interference echo, and the fourth term is Gaussian noise. $M$ represents the number of satellites, $P$ is the number of multipaths, $n_\eta$, $\Omega_\eta$, $\alpha_\eta$ are the time delay, Doppler shift, and amplitude of the echo signal respectively. When $k = 1$, $c_{\eta,k}$ is the amplitude of the direct wave. When $k = 2, 3, \cdot\cdot\cdot, P$, $\Omega^{(c)}_{\eta,k}$, $n^{(c)}_{\eta,k}$, $c_{\eta,k}$ are Doppler shift, time delay, amplitude of multipath signals respectively, $K$ represents the number of interference targets, $\Omega^{(t)}_m$, $n^{(t)}_m$, $\beta^{(t)}_m$ respectively represent the Doppler shift, time delay, and amplitude of the echo signals of the interference target, $n_s$ represents white Gaussian noise. To solve this detection problem, the assumptions are as following:

(1) When $m = 1, 2, \cdot\cdot\cdot, M$, the amplitude of the echo signals of the interference target $\beta^{(t)}_m$ is a certain unknown variable;
(2) The number of interference targets $K$ is unknown, when $m = 1, 2, \cdot\cdot\cdot, K$, the time delay $n^{(t)}_m$ and Doppler shift $\Omega^{(t)}_m$ of each interference echo are known;

(3) When $\eta = 1, 2, \cdots, M$, $k = 2, \cdots, P$, the amplitude of the multipath signal $c_{\eta,k}$ is a certain unknown variable, and when $k = 1$, $c_{\eta,1}$ is the amplitude of the direct wave signal, which is a certain unknown variable;
(4) When $\eta = 1, 2, \cdots, M$, $k = 2, \cdots, P$, the time delay $n_{\eta,k}^{(c)}$ and the Doppler shift of the multipath signal are certain known variables;
(5) $n_s$ represents Additive white Gaussian noise with unknown variance $\sigma^2$;
(6) The subsequent detection needs to construct the probability density function of the sampled signal $x = [x(0), x(2), \cdots x(N-1)]$ under two hypotheses.

According to formulas (11) and (12), the probability density function of $x[n]$ under the assumption $H_0$ can be given by

$$\begin{aligned} & p\left(x; \alpha_\eta, \sigma_2, c_{\eta,k}, \beta_m^{(t)}, H_0\right) \\ & = \tfrac{1}{(\pi\sigma^2)^N} \exp[-\tfrac{1}{\sigma^2} \sum_{n=0}^{N-1} |x[n] - \sum_{\eta=1}^{M} \sum_{k=1}^{P} c_{\eta,k} y_\eta \\ & \left[n - n_{\eta,k}^{(c)}\right] e^{jn\Omega_{\eta,k}^{(c)}} - \sum_{m=1}^{K} \beta_m^{(t)} y_m \left[n - n_m^{(t)}\right] e^{jn\Omega_m^{(t)}} |^2]. \end{aligned} \tag{13}$$

The probability density function of $x[n]$ under the assumption $H_1$ is

$$\begin{aligned} & p\left(x; \alpha_\eta, \sigma_2, c_{\eta,k}, \beta_m^{(t)}, H_1\right) \\ & = \tfrac{1}{(\pi\sigma^2)^N} \exp[-\tfrac{1}{\sigma^2} \sum_{n=0}^{N-1} |x[n] - \sum_{\eta=1}^{M} \alpha_\eta y_\eta [n - n_\eta] e^{jn\Omega_\eta} \\ & - \sum_{\eta=1}^{M} \sum_{k=1}^{P} c_{\eta,k} y_\eta \left[n - n_{\eta,k}^{(c)}\right] e^{jn\Omega_{\eta,k}^{(c)}} \\ & - \sum_{m=1}^{K} \beta_m^{(t)} y_m \left[n - n_m^{(t)}\right] e^{jn\Omega_m^{(t)}} |^2]. \end{aligned} \tag{14}$$

According to the Newman Pearson criterion, the optimal detection method for the hypothesis testing problems (11) and (12) is the likelihood ratio test, because the likelihood ratio test requires knowing the parameters, $\alpha = [\alpha_1, \alpha_2, \cdots, \alpha_M]^T$, $\overset{\wedge}{c} = \begin{matrix} \overset{\wedge}{c}_{1,1} & \cdots & \overset{\wedge}{c}_{M,1} \\ \vdots & \ddots & \vdots \\ \overset{\wedge}{c}_{1,P} & \cdots & \overset{\wedge}{c}_{M,P} \end{matrix}$, $\sigma^2$, $\beta = \left[\beta_1^{(t)}, \beta_2^{(t)}, \cdots, \beta_K^{(t)}\right]^T$, in actual situations, all parameters are unknown. One possible way to avoid this problem is to use GLRT, which is equivalent to replacing these unknown parameters with the maximum likelihood estimation of these parameters in the likelihood ratio test. In the following, the GLRT-based detector is derived in three cases: the noise variance $\sigma^2$ is known and there is no interference target, that is $K = 0$; the noise variance $\sigma^2$ is unknown and there is no interference target, that is $K = 0$; the noise variance $\sigma^2$ is unknown and there are interference targets, that is $K \neq 0$.

This part contains the derivation of GLRT when the noise variance $\sigma^2$ is known and there is no interference target. GLRT can be obtained by replacing the maximum likelihood estimation of unknown parameters under each assumption. GLRT can be written as

$$L_{GLR}(x) = \frac{\max_{\alpha,c} f(x;\alpha,c,H_1)}{\max_{\alpha,c} f(x;\alpha,c,H_0)} \mathop{\gtrless}_{H_0}^{H_1} \xi. \tag{15}$$

The selected threshold $\xi$ should be determined according to the false alarm probability. In order to construct GLRT, first use MLE to estimate the unknown parameter, the parameters $\overset{\wedge}{c_1} = \begin{bmatrix} \overset{\wedge}{c}_{1,11} & \cdots & \overset{\wedge}{c}_{M,11} \\ \vdots & \ddots & \vdots \\ \overset{\wedge}{c}_{1,P1} & \cdots & \overset{\wedge}{c}_{M,P1} \end{bmatrix}$, $\alpha = [\alpha_1, \alpha_2, \cdots, \alpha_M]^T$ under the assumption $H_1$ can be obtained by differentiating the unknown parameters $\overset{\wedge}{c_1}$ and $\alpha$ in Eq. (14) and setting the derivative equal to zero. That is to say, under the assumption $H_1$, $f(x;H_1)$ finds the partial derivative of $\alpha_\eta$ and $c_{\eta,k}$ respectively, then we can get

$$\begin{aligned}
&\sum_{n=0}^{N-1} x[n] y_r^*[n-n_r] e^{-jn\Omega_r} \\
&= \sum_{\eta=1}^{M} \overset{\wedge}{\alpha}_\eta \sum_{n=0}^{N-1} y_\eta[n-n_\eta] y_r^*[n-n_r] e^{-jn(\Omega_r-\Omega_\eta)} \\
&+ \sum_{\eta=1}^{M}\sum_{k=1}^{P} \overset{\wedge}{c}_{\eta,k1} \sum_{n=0}^{N-1} y_\eta\left[n-n_{\eta,k}^{(c)}\right] y_r^*[n-n_r] e^{-jn\left(\Omega_r-\Omega_{\eta,k}^{(c)}\right)},
\end{aligned} \tag{16}$$

$$\begin{aligned}
&\sum_{n=0}^{N-1} x[n] y_q^*\left[n-n_{q,s}^{(c)}\right] e^{-jn\Omega_{q,s}^{(c)}} \\
&= \sum_{\eta=1}^{M} \overset{\wedge}{\alpha}_\eta \sum_{n=0}^{N-1} y_\eta[n-n_\eta] y_q^*\left[n-n_{q,s}^{(c)}\right] e^{-jn\left(\Omega_{q,s}^{(c)}-\Omega_\eta\right)} \\
&+ \sum_{\eta=1}^{M}\sum_{k=1}^{P} \overset{\wedge}{c}_{\eta,k1} \sum_{n=0}^{N-1} y_\eta\left[n-n_{\eta,k}^{(c)}\right] y_q^*\left[n-n_{q,s}^{(c)}\right] e^{-jn\left(\Omega_{q,s}^{(c)}-\Omega_{\eta,k}^{(c)}\right)},
\end{aligned} \tag{17}$$

where $q, r = 1, 2, \cdots, M$, $s = 1, 2, \cdots, P$, the insertion symbol represents the estimated value of the unknown parameter, and the third parameter in $\overset{\wedge}{c}_{\eta,k1}$ represents the hypothesis $H_1$. In the same way, the unknown parameter $\overset{\wedge}{c_0} = \begin{bmatrix} \overset{\wedge}{c}_{1,10} & \cdots & \overset{\wedge}{c}_{M,10} \\ \vdots & \ddots & \vdots \\ \overset{\wedge}{c}_{1,P0} & \cdots & \overset{\wedge}{c}_{M,P0} \end{bmatrix}$ under the assumption $H_0$ can be obtained by differentiating the unknown parameter $\overset{\wedge}{c_0}$ in Eq. (13) and setting the derivative equal to zero. That is, under the assumption $H_0$, $f(x;H_0)$ finds the partial derivative of $c_{\eta,k}$ respectively, then we can get

$$\begin{aligned}&\sum_{n=0}^{N-1} x[n] y_q^*\left[n-n_{q,s}^{(c)}\right] e^{-jn\Omega_{q,s}^{(c)}} \\ &= \sum_{\eta=1}^{M}\sum_{k=1}^{P} \overset{\wedge}{c}_{\eta,k0} \sum_{n=0}^{N-1} y_\eta\left[n-n_{\eta,k}^{(c)}\right] y_q^*\left[n-n_{q,s}^{(c)}\right] e^{-jn\left(\Omega_{q,s}^{(c)}-\Omega_{\eta,k}^{(c)}\right)}.\end{aligned} \tag{18}$$

Write (16) and (17) in matrix form

$$\begin{bmatrix} R_c & r_{sc} \\ r_{sc}^H & r_{ss} \end{bmatrix} \begin{bmatrix} \overset{\wedge}{c_1} \\ \overset{\wedge}{\alpha} \end{bmatrix} = \begin{bmatrix} r_{xc} \\ r_{xs} \end{bmatrix}, \tag{19}$$

where $\overset{\wedge}{c_1} = \begin{matrix} \overset{\wedge}{c}_{1,11} & \cdots & \overset{\wedge}{c}_{M,11} \\ \vdots & \ddots & \vdots \\ \overset{\wedge}{c}_{1,P1} & \cdots & \overset{\wedge}{c}_{M,P1} \end{matrix}$, $\overset{\wedge}{\alpha} = \left[\overset{\wedge}{\alpha}_1 \cdots \overset{\wedge}{\alpha}_M\right]^T$. $\overset{\wedge}{c_1}$ represents the amplitude of the direct wave and multipath signal of each satellite under the assumption $H_1$, $\overset{\wedge}{c}_{M,P1}$ is the $P$th multipath amplitude of the $M$-th satellite signal under the assumption $H_0$, and $p = 0$ represents the amplitude of the direct wave corresponding to the satellite. $\overset{\wedge}{\alpha}$ shows the amplitude of the echo signal of each satellite, $\overset{\wedge}{\alpha}_M$ is the amplitude of the echo signal of the $M$-th satellite signal.

Under the assumption $H_0$, write (18) in matrix form

$$R_c \overset{\wedge}{c_0} = r_{xc}. \tag{20}$$

In (20), $\overset{\wedge}{c_0}$ represents the maximum likelihood estimation of $C$ under the hypothesis $H_0$, $R_c$ represents the correlation of multipath signals, $R_c$ is the matrix of $P*P$, and $[R_c]_{sk}$ is the element of $R_c$ which can be expressed as

$$[R_c]_{sk} = \sum_{\eta=1}^{M}\sum_{q=1}^{M}\sum_{n=0}^{N-1} y_\eta\left[n-n_{\eta,k}^{(c)}\right] y_q^*\left[n-n_{q,s}^{(c)}\right] e^{-jn\left(\Omega_{q,s}^{(c)}-\Omega_{\eta,k}^{(c)}\right)}, \tag{21}$$

where $r_{ss}$ represents the auto-correlation between echoes, $[r_{ss}]$ is a matrix of $M*M$, $[r_{ss}]_{r\eta}$ represents the elements of $r_{ss}$, expressed as

$$[r_{ss}]_{r\eta} = \sum_{n=0}^{N-1} y_\eta[n-n_\eta]\, y_r^*[n-n_r]\, e^{-jn(\Omega_r-\Omega_\eta)}, \tag{22}$$

where $r_{sc}$ represents the correlation between multipath and echo, $[r_{sc}]$ is a matrix of $M*M$, $[r_{sc}]_{q\eta}$ represents the elements of $r_{sc}$, expressed as

$$[r_{sc}]_{q\eta} = \sum_{s=1}^{P}\sum_{n=0}^{N-1} y_\eta[n-n_\eta]\, y_q^*\left[n-n_{q,s}^{(c)}\right] e^{-jn\left(\Omega_{q,s}^{(c)}-\Omega_\eta\right)}, \tag{23}$$

where $r_{xc}$ indicates the correlation between the received signal,the direct wave and multipath signal at $\left(n_r^{(c)}, \Omega_r^{(c)}\right)$ in the monitoring channel. $[r_{sc}]$ is a matrix of $P * M$, which represents the elements of $[r_{xc}]_{qs}$, expressed as

$$[r_{xc}]_{qs} = \sum_{n=0}^{N-1} x[n] y_q^* \left[n - n_{q,s}^{(c)}\right] e^{-jn\Omega_{q,s}^{(c)}}, \tag{24}$$

where $r_{xs}$ represents the cross-correlation of each target echo signal of the signal received by the monitoring channel at $(n_r, \Omega_r)$, $[r_{xs}]$ is the vector of $M * 1$, and $[r_{xs}]_r$ is the elements of $r_{xs}$, expressed as

$$[r_{xs}]_r = \sum_{n=0}^{N-1} x[n] y_r^* [n - n_r] e^{-jn\Omega_r}. \tag{25}$$

In order to obtain the maximum likelihood estimation of unknown parameters from (19), the inverse matrices of matrices $R_c$, $r_{sc}$, $r_{ss}$, $r_{xc}$ exist, and they are all positive semi-definite matrices, first calculate their inverse matrices, and then, unknown parameters under each assumption can be calculated

$$\begin{bmatrix} \hat{c}_1 \\ \hat{\alpha} \end{bmatrix} = \begin{bmatrix} R_c & r_{sc} \\ r_{sc}^H & r_{ss} \end{bmatrix}^{-1} \begin{bmatrix} r_{xc} \\ r_{xs} \end{bmatrix}, \tag{26}$$

$$\hat{c}_0 = R_c^{-1} r_{xc}. \tag{27}$$

In (26), we use the principle of inversion of the segmentation matrix, the maximum likelihood estimation of the unknown parameters under hypothesis $H_1$ is

$$\begin{bmatrix} R_c & r_{sc} \\ r_{sc}^H & r_{ss} \end{bmatrix}^{-1} = \begin{bmatrix} R_c^{-1} + R_c^{-1} r_{sc} g r_{sc}^H R_c^{-1} & -R_c^{-1} r_{sc} g \\ -g r_{sc}^H R_c^{-1} & g \end{bmatrix}, \tag{28}$$

where $g = \left(r_{ss} - r_{sc}^H R_c^{-1} r_{sc}\right)^{-1}$. From (27) and (28), we can get

$$\begin{bmatrix} \hat{c}_1 \\ \hat{\alpha} \end{bmatrix} = \begin{bmatrix} \hat{c}_0 + g R_c^{-1} r_{sc} r_{sc}^H \hat{c}_0 - g r_{xs} R_c^{-1} r_{sc} \\ -g r_{sc}^H \hat{c}_0 + g r_{xs}. \end{bmatrix}. \tag{29}$$

Substitute the maximum likelihood estimates of the unknown parameters under the two assumptions into the two probability density functions (13) and (14), construct the likelihood ratio, take the logarithm, and obtain the detection statistics after simplification

$$\ln L_{GLR} = \frac{r_{xc}^H \left(\hat{c}_1 - \hat{c}_0\right) + \hat{c}_1^H \left(r_{xc} - R_c \hat{c}_1\right) + r_{ss} \hat{\alpha}}{\sigma^2} \underset{H_0}{\overset{H_1}{\gtrless}} \xi. \tag{30}$$

For (30), use (27) and (29) to further simplify, then we can get

$$\frac{1}{\sigma^2}\frac{\left|r_{xs}-r_{sc}^H\overset{\wedge}{c}_0\right|}{r_{ss}-r_{sc}^H R_c^{-1} r_{sc}}\underset{H_0}{\overset{H_1}{\gtrless}}\xi. \tag{31}$$

In order to obtain a more intuitive representation of the detection statistics, the detection statistics can be expressed as

$$\frac{1}{\sigma^2}\frac{\sum_{q=1}^{M}\left|\sum_{n=0}^{N-1}\left(x\left[n\right]-\sum_{\eta=1}^{M}\sum_{k=1}^{P}\left[\overset{\wedge}{c}_0\left(:,\eta\right)\right]_k y\left[n-n_{\eta,k}^{(c)}\right]e^{-jn\Omega_k^{(c)}}\right)y_q^*\left[n-n_q\right]e^{-jn\Omega_q}\right|^2}{\left|r_{ss}-r_{sc}^H R_c^{-1} r_{sc}\right|}\underset{H_0}{\overset{H_1}{\gtrless}}\xi. \tag{32}$$

It can be seen from (32) that the numerator of the detection statistics represents the two-dimensional cross-correlation between the pure monitoring channel signal and the echo signal at $[n_q, \Omega_q]$.

## 4 Simulation Results and Discussion

In order to verify the performance of the algorithm, matlab is used for simulation. This article uses GPS, DVB-S, and inmarsat satellite signal models. The noise is Gaussian white noise. The definition of SNR can be expressed as

$$SNR = 10\lg(P_s/P_N), \tag{33}$$

where $P_S$ and $P_N$ are the power of the satellite signal and Gaussian white noise respectively. The intensity ratio SDR of direct wave and echo is defined as

$$SDR = \frac{P_s}{P_D}, \tag{34}$$

where $P_D$ represents the power of a single direct wave, and $P_S$ represents the power of a single echo. The evaluation standard of the detection of algorithm is the detection accuracy rate, which can be expressed by the following formula

$$\delta_H = \frac{N_R}{N}\times 100\%, \tag{35}$$

where $N_R$ and $N$ are the number of correct detection and the total number of simulations respectively.

The direct wave and multipath in the monitoring channel are suppressed by the ECA method, and three satellite signals, i.e., GPS, DVB-S, Inmarsat, are used for simulation experiments. The carrier frequencies of the three signals are $f_G = 1.57\,\text{GHz}$, $f_D = 12.38\,\text{GHz}$ and $f_i = 4.2\,\text{GHz}$. Suppose the three echo signals' time delays are 1 µs, 2 µs, 3 µs respectively, and the Doppler frequency shifts are 100 Hz, 150 Hz, 200 Hz respectively. The strengths of the three direct waves of signals are −130.1 dBw, −111.83 dBw, −120.61 dBw in order.

The difference between the direct wave power and its corresponding echo power is 40 dB. The distances of the aircraft are 10 km, 15 km, 20 km and the speeds are 300 km/h (83 m/s), 350 km/h (97 m/s), 800 km/h (220 m/s), respectively. The number of sampling points is $10^5$. We utilize Matlab for simulation and carry out 2000 Monte Carlos experiments.

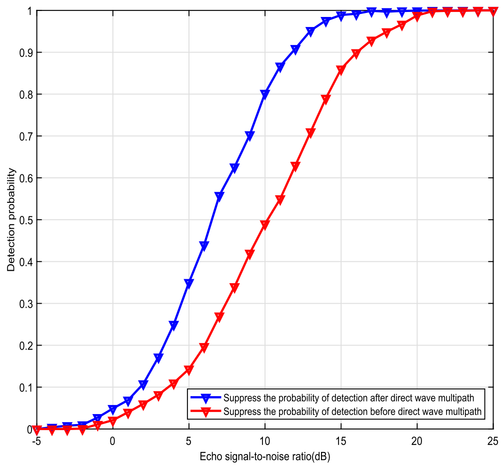


**Fig. 1.** Comparison of detection probability based on GLRT before and after direct wave and multipath are suppressed

To verify the influence of the direct wave and echo in the monitoring channel on the detection performance of GLRT, the simulation result is shown in Fig. 1. It can be seen from Fig. 1 that after suppressing the direct wave and multipath in the monitoring channel, the detection performance is improved. This is because when the direct wave and multipath are in the echo signal, the maximum likelihood estimation is used to estimate the amplitude of the direct wave and multipath. Although the maximum likelihood estimation is the best parameter estimation method, there will inevitably be errors. The existence of the direct wave and multipath itself is a substantial interference to the echo signal, so it will cause the echo signal's detection performance to decrease. After the direct wave and multipath of the monitoring channel are suppressed by the suppression method of the direct wave and multipath, the algorithm's performance will be improved to a certain extent.

To verify the influence of the number of satellite signals on the detection performance of GLRT, we compare the detection performance, using GPS, GPS+DVB-S, GPS+DVB-S +inmarsat satellite signals. It can be seen from Fig. 2 that under the same signal-to-noise ratio, as the number of satellite signals increases, the detection performance gradually decreases. This is because increasing the number of signals raises the direct ratio and multipath interference in the monitoring channel. As the multipath interference increases, mul-

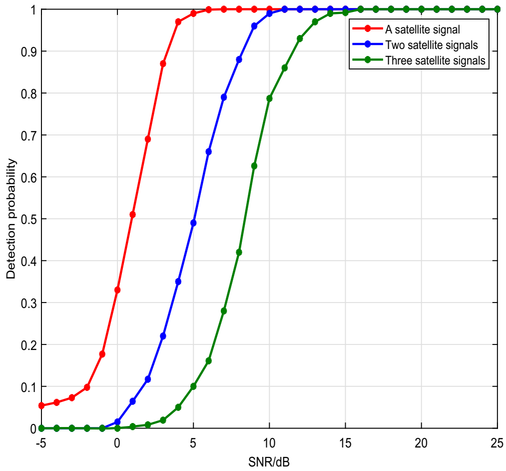


**Fig. 2.** Comparison of detection probability based on GLRT with different number of satelites

tiple echo signals affect each other, which affects the detection performance of echo signals. However, the joint detection of echo signals by multiple satellites will improve the detection results' reliability. This sacrifices a certain detection probability but improves detection reliability, which is of great significance in practical applications.

We evaluate the impacts of the distance between the target and the receiver and the speed of the target on the detection performance of GLRT. We conduct simulation experiments with three satellite signals, i.e., GPS, DVB-S, and Inmarsat, at different distances and speeds, and the simulation result is shown in Fig. 3. It can be seen from Fig. 3 that as the target speed increases, the detection performance of the echo signal gradually decreases. This is because when the target speed is different, the Doppler frequency shifts of the corresponding echo signals are becoming various. If the target's speed increases, one will affect the receiver's correct reception of the signal. The other will increase the corresponding Doppler shift, which will change the echo signal's detection. This indicates that the proposed method is suitable for detecting low-speed and low-altitude flying targets.

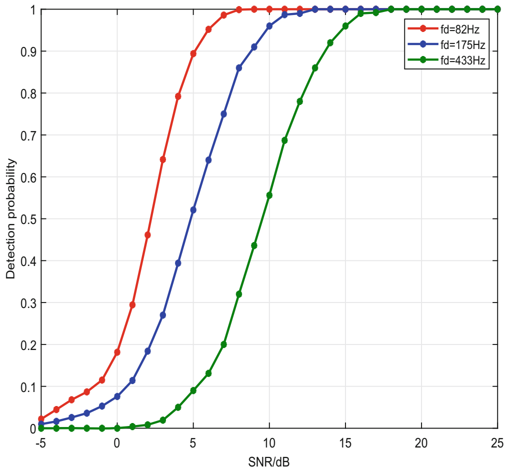


**Fig. 3.** Comparison of detection probability based on GLRT at different speeds

## 5 Conclusion

This paper developed a passive target detection method using multi-satellite illumination based on the generalized likelihood ratio. We derived the detector based on the generalized likelihood ratio. Furthermore, we established the adaptive detection threshold on this basis to complete the detection of the echo signal. Finally, various aspects that may affect the detection algorithm's performance were simulated respectively. The results indicate that the detection algorithm can be used to detect target using multi-satellite illumination.

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