



# Multi Beam Forming Algorithms of LEO Constellation Satellite

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**Abstract.** Since the 21st century, the development of mobile Internet and Internet of things technology has introduced new development opportunities for the field of communication satellites. A large number of researchers use multi beam technology in low earth orbit high-throughput satellite constellation to improve the performance of Internet of things services. In this paper, the development prospect of low orbit high-throughput communication satellite and the research significance of multibeam technology are summarized firstly, then some key technologies involved in the beamforming process are discussed, and the mathematical model of multibeam forming network is established with digital beamforming as the background. Then, the theory of beamforming criterion and algorithm is analyzed, and the least mean square algorithm and recursive least square algorithm are studied in detail. Finally, a variable step size least mean square algorithm is proposed and the simulation results are compared with those before the improvement, which can guide the selection of multi beam forming algorithm on the satellite.

**Keywords:** LEO satellite · Beamforming technology · Adaptive algorithm · LMS algorithm · Variable step-size

## 1 Introduction

Since the first international commercial communication satellite between North America and Europe, named “morning bird”, was successfully launched by the United States in 1965, the satellite communication industry has experienced full development in recent decades. Since the 1990s, the rapid development of optical fiber and mobile cellular communication infrastructure on the ground has greatly squeezed the development space of commercial satellite communication industry. Although there is a certain momentum of growth, the development of its industrial scale is extremely difficult [1]. With the coming of the 21st century, the development of mobile Internet and Internet of things technology has brought new opportunities for the development of communication satellite industry, and a new development direction with multi-functional, comprehensive and Internet of things as the application goal has emerged.

The mobile broadband Internet satellite system based on LEO satellite constellation will realize the following services worldwide, including personal terminal wireless communication, global Internet access function, Internet of things access service,

timely information flow push service, stronger navigation ability and navigation, aviation monitoring.

The multi beam antenna technology is indispensable in the realization of the above system and each sub beam is responsible for covering a certain range of target areas. Compared with the traditional single beam antenna, the multi beam antenna has many advantages (Fig. 1).

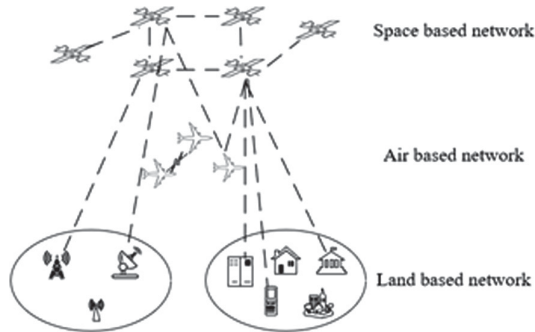


Fig. 1. The idea of the integration network of heaven and earth

## 2 Satellite Multi-beamforming Algorithm

### 2.1 Beamforming Based on Least Mean Square Algorithm

The least mean square (LMS) algorithm is an adaptive beamforming algorithm based on MMSE criteria in Table 3–1 above, which is the most widely used beamforming algorithm. It constructs a performance surface between the actual output and the desired signal through the square error under the condition that the desired signal  $D$  is known, and uses the random gradient method to minimize the error.

The basic principle of LMS algorithm is analyzed below:

At a specific time  $k$ , we sample the signal, and the error between the actual output of the array and the desired signal can be expressed as:

$$\varepsilon(k) = d(k) - w^H(k)x(k) \tag{1}$$

The expression of square error can be obtained by square it:

$$|\varepsilon(k)|^2 = |d(k) - w^H(k)x(k)|^2 \tag{2}$$

The cost function of LMS algorithm can be obtained by expanding the right expression of the above formula:

$$J(w) = D - 2w^H r + w^H R_{xx} w \tag{3}$$

Where,  $R_{xx}$  is the correlation matrix of antenna array,  $r$  is the correlation vector of the signal.

Because we don't know the statistical data of the signal, we can get the correlation matrix by sampling the snapshot data to estimate the signal. The estimated value is:

$$R_{xx}(k) = x(k)x^H(k) \tag{4}$$

$$r(k) = d^*(k)x(k) \tag{5}$$

The former formula is the cost function of the algorithm. The LMS algorithm approximates the optimal solution of the algorithm performance surface by finding its gradient and iterating in the opposite direction through the random gradient method. LMS algorithm is widely used in adaptive field because of its simple principle and small computation.

The following formula gives the iterative formula of LMS algorithm:

$$w_N(k + 1) = w_N(k) - \frac{1}{2} \mu \nabla J(w_N(k)) \tag{6}$$

The updated formula of the weight vector is:

$$e_N(k) = d_N(k) - w_N^H(k)x_N(k) \tag{7}$$

$$w_N(k + 1) = w_N(k) + \mu x_N(k)e_N^*(k) \quad 0 < \mu < \text{Trace}(R) \tag{8}$$

The parameter  $\mu$  is called step factor, which is used to control the convergence rate and the stability of the algorithm.

How to choose the value of parameter  $\mu$  will have a great influence on the convergence performance of LMS algorithm. If the step size factor is too small, the algorithm will appear over damping. If the step size factor is too large, the algorithm will not converge and will not get the optimal value. So the selection of parameter  $\mu$  is very important.

The implementation steps of LMS algorithm are summarized as follows:

- (1) Initialize the weight vector and record the weight vector of the array as  $w(k)$ ;
- (2) The sampling signal,  $x(t)$ , and  $d(t)$ , are recorded as  $x(n)$ ,  $d(n)$ , respectively;
- (3) Sampling time error estimation:

$$e(n) = d(n) - y(n) = d(n) - w^H x(n) \tag{9}$$

(4) Weight vector update:

$$w(n+1) = w(n) + \mu x(n)e^*(n) \tag{10}$$

LMS algorithm has the advantages of low computation complexity and small total computation. Each iteration of LMS algorithm only needs  $4N$  multiplication operations and  $6N$  addition operations. The main disadvantage of the algorithm is that it is difficult to choose a suitable step size to realize the trade-off between convergence speed and algorithm error.

The following is the implementation flow chart of LMS algorithm (Fig. 2).

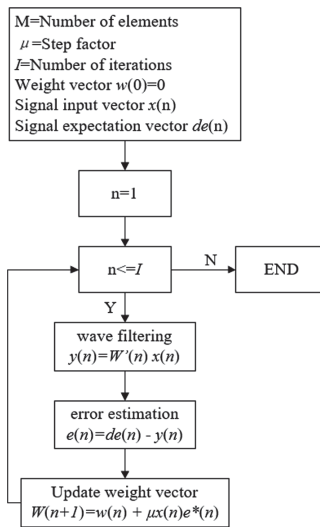
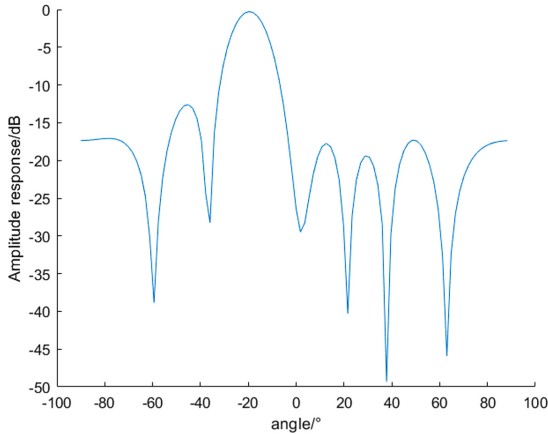


Fig. 2. LMS algorithm flow chart

### 2.2 Simulation Analysis of the LMS Algorithm

In the simulation stage, the number  $N$  of elements of the linear array is 8, the distance between elements is  $\lambda/2$ , the direction of the expected signal is  $-20^\circ$ , the expected signal is cosine signal, and the direction of interference signal is  $0^\circ$ . Set the noise as Gaussian white noise with power of 0dbW; the signal-to-noise ratio and dry noise ratio are both 10 dB. When the number of iterations is set to 500 and the step size is chosen as the gradient, it is found that when the step size is set to 0.001, an ideal result can be obtained (Fig. 3):



**Fig. 3.** LMS algorithm single beam forming

The LMS algorithm can form the main lobe of the beam in the desired direction, the algorithm achieves initial convergence after about 30 iterations, and the output signal of the array can roughly follow the desired signal stably. With the increase of the number of elements, it can achieve faster convergence speed and better stability of convergence error, but it will increase the amount of calculation accordingly.

Considering that the computation of LMS algorithm is very small and the complexity of the algorithm is low, using this algorithm on the satellite can reduce the demand for the satellite load calculation ability and achieve the purpose of saving hardware expenses.

### 3 LMS Algorithm with Variable Step Size

#### 3.1 Sigmoid-based Variable Step-Size LMS

In the classical LMS algorithm, the step factor  $\mu$  of the algorithm is constant, and the range of its error reaching the convergence requirement is  $0 < \mu < \text{Trace}(R)$ , Where,  $\text{Trace}(R)$  is the trace of the autocorrelation matrix of the signal received by the array antenna. The smaller the step size is, the smaller the error is, but the longer the convergence time constant is. Obviously, the fixed value of  $\mu$  leads to the contradiction between the convergence speed and stability of LMS algorithm.

The LMS can be avoided to some extent by setting the convergence completion condition. The convergence speed and its steady-state performance should be considered at the same time. Therefore, the method of changing step factor with time in algorithm iteration is a simple and effective strategy. In order to achieve better steady-state performance in the later stage of the algorithm, the step factor of the algorithm should be reduced synchronously with the decrease of the mean square error. Therefore, it is an effective way to change the step factor of LMS algorithm to the dynamic value of error level with time.

Scholars at home and abroad do not have little research on variable step size LMS algorithm, among which the classical variable step size LMS algorithm is an algorithm proposed by Kwong. R. H. To dynamically change the step factor in the iterative process according to the instantaneous error of the algorithm. The expression of this algorithm can be written as:

$$\mu(n) = \alpha\mu(n - 1) + be(n)^2 \tag{11}$$

Where:  $0 < \alpha < 1$ ,  $b > 0$ , and the step factor shall meet the following requirements:

$$\mu(n) = \begin{cases} \mu_{max}, \mu(n) > \mu_{max} \\ \mu_{min}, \mu(n) < \mu_{min} \\ \mu(n), others \end{cases} \tag{12}$$

Intuitively speaking, when the algorithm error increases, the step factor of the control algorithm is increased, so as to achieve faster return to the convergence error level. In the initial stage of the algorithm, the step factor can be preset to a larger value to achieve fast convergence, which is consistent with the control idea of the synchronous long factor. The parameters  $\alpha$  and  $b$  in the algorithm are fixed values. In practice, a large number of experiments are needed to select the ideal  $\alpha$  and  $b$ . similar to the fixed step size LMS algorithm, the step size is also obtained according to a large number of experimental experience, and the practicability is not improved obviously. Therefore, the algorithm has become a classic algorithm of variable step size LMS.

In the practical application scenario, another variable step-size lead mean square (SVSLMS) algorithm based on the sigmoid function has also been widely used. It is proposed by Qin Jingfan and others. The guiding idea is to map the error  $e(n)$  through the sigmoid function as the value of step factor  $\mu(n)$ :

$$\mu(n) = \beta \left( \frac{1}{1 + e^{-\alpha|e(n)|}} - 0.5 \right) \tag{13}$$

The parameter  $\alpha$  controls the shape of the sigmoid function and the rising speed of the curve, and the parameter  $\beta$  controls the value range of the sigmoid function.

Similarly, in the initial stage of the algorithm, due to the large instantaneous error, the algorithm step factor is at a relatively large level, and the algorithm can achieve a faster convergence speed in the early stage of the iteration. With the decrease of the error, the step factor will decrease rapidly to improve the stability of the algorithm.

Considering that the modulus  $|e(n)|$  of error must be close to 0 when converging, we can increase the number of  $|e(n)|$  terms in Eq. (4-3) to make the step factor curve more nonlinear in the region less than 1 and close to 0:

$$\mu(n) = \beta \left( \frac{1}{1 + e^{-\alpha|e(n)|^3}} - 0.5 \right) \tag{14}$$

To sum up, the iterative process of the SVSLMS algorithm is summarized as follows:

- (1) Initialize the weight vector, and record the weight vector of the array as  $w(k)$ ;
- (2) Signal sampling, sampling for element signal  $x(t)$ , expected signal  $d(t)$ , recorded as  $x(n)$ ,  $d(n)$ ;
- (3) Sampling time error estimation:

$$e(n) = d(n) - y(n) = d(n) - w^H x(n) \tag{15}$$

- (4) Step factor update:

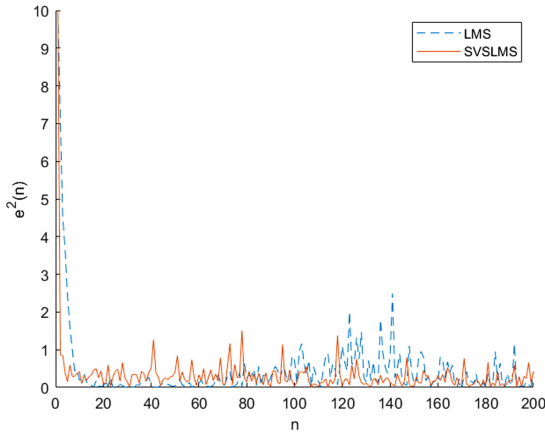
$$\mu(n) = \beta \left( \frac{1}{1 + e^{-\alpha|e(n)|^3}} - 0.5 \right) \tag{16}$$

- (5) Weight vector update:

$$w(n+1) = w(n) + \mu x(n)e^*(n) \tag{17}$$

### 3.2 Simulation Analysis of Variable Step Size LMS Algorithm and Classical Algorithm

Keep the simulation conditions of single beam forming unchanged, compare the effect of SVSLMS algorithm and classic LMS algorithm, and fix the number of iterations 200 times (Fig. 4):



**Fig. 4.** Comparison of convergence error between SVSLMS and classical LMS

From the simulation results, it can be seen that after the initial convergence, the algorithm error of SVSLMS is generally more stable than that of classic LMS due to the reduction of step factor. At the same time, the convergence speed of SVSLMS algorithm in the early stage is also slightly improved compared with the classical algorithm.

However, the performance of SVSLMS algorithm in the graph still depends on the fixed values of  $\alpha$  and  $\beta$ . How to continue to improve the flexibility of step factor adjustment and reduce the steady-state error at the same time will be an improvement direction worth studying.

Table 1 shows the calculation amount comparison of LMS, RLS and SVSLMS in each iteration when the array element number of array antenna is  $N$ :

**Table 1.** Complexity Comparison of LMS, RLS and SVSLMS

Algorithm name	Multiplication	Addition operation
LMS	$4N$	$6N$
RLS	$4N^2 + 7N$	$3N^2 + 4N$
SVSLMS	$4N + 6$	$6N + 2$

It can be seen from the table that the computational complexity of SVSLMS algorithm is slightly higher than that of classical LMS algorithm, and it is also much lower than RLS algorithm when the number of array elements  $N$  is large. Compared with the classical LMS algorithm, it has a significant advantage in improving the convergence stability. It can realize the relatively stable and fast convergence of the weight when the calculation complexity is not high, which makes the modified algorithm more applicable in the application scenario where the calculation complexity of the low orbit constellation satellite is not too high at the same time for the large-scale array antenna with many point beams.

## 4 Conclusion

The simulation model of the adaptive beamforming algorithm is established based on the mathematical model. The characteristics and advantages of different algorithms are compared from the convergence speed, convergence effect, steady-state error and so on. In view of the shortcomings of the convergence stability of the classical LMS algorithm, this paper explores the improvement feasibility of the variable step size LMS algorithm, and verifies the considerable improvement benefit and the acceptable increase cost of the operation amount through the simulation comparison. Based on the application requirements of multibeam in LEO constellation satellite, the factors of selecting the algorithm are compared.

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