Detection Algorithm Based on Eigenvalues of Sampling Covariance Matrix for Satellite Cognitive Network

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Abstract. Satellite cognitive network is currently facing a lot of complex spectrum environment with a lot of interference, and the required user signal strength will change with a variety of external factors, which directly affects the above series of results obtained through the decision mechanism and it can not be well applied to satellite cognitive network. So a new blind detection algorithm based on maximum and minimum eigenvalues of sampling covariance matrix is proposed. In this algorithm, the ratio of the difference and sum of the maximum and minimum eigenvalues of the sampling covariance matrix is used as the perceptual decision quantity. Then, by introducing the latest results of the distribution of the maximum and minimum eigenvalues of the sampling covariance matrix in large dimensional random matrix, an effective decision threshold calculation method is designed. Compared with the classical eigenvalue detection algorithm, the new algorithm has the advantage of accurate calculation of perceptual decision threshold, and can effectively improve the detection performance and the reliability of decision results.

Keywords: Satellite cognitive network · Spectrum sensing · Detection based on sampling covariance eigenvalues

1 Introduction

Satellite cognitive network [1] in today's society is an important part as a space of cognitive network, not only can share resources of rational utilization and efficient spatial spectrum, but also can improve the network intelligent level, so as to efficiently promote the spatial heterogeneous network integration. Satellite cognitive network is currently the hot areas of satellite communication technology research and direction. However, spectrum sensing technology [2] is the foundation of satellite cognitive network construction, and spectrum sensing technology also determines its development. Because the satellite communication system has the characteristics of wide frequency range, complex node type, large space propagation loss and dynamic spectrum environment change, the traditional

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spectrum sensing technology based on the ground wireless network environment is faced with many problems when it is applied to the satellite cognitive wireless network.

Traditional main user signal detection algorithms include matched filtering algorithm [3] and energy detection algorithm [4]. Although matched filtering algorithm has the best detection performance, it requires to obtain the prior statistics of the main user and channel and has limited application scenarios. ED algorithm does not need the prior information of the main user and channel, so it is simple to implement and generally has good detection performance. However, the biggest problem of the algorithm [5] is the scattering loss caused by the light of the satellite in the CN environment, the shadowing and signal weakness [6] will make the signal power sent to the low orbit satellite by the user on the high orbit satellite continuously decrease, and the signal strength to be received will not be stable all the time. Therefore, a Detection Algorithm based on Sampling Covariance Matrix (DASCM) is designed. The change of the maximum and minimum eigenvalues of the sampling covariance matrix of the received signal can well reflect the change of the main user's signal energy and related characteristics. The algorithm takes the ratio of the difference and sum of the maximum and minimum eigenvalues of the sample covariance matrix as the perceptual decision quantity, and uses the latest results of the asymptotic distribution of eigenvalues in the large dimensional random matrix to design an effective decision threshold calculation method.

2 Mathematical Model and Cognitive Scenario

2.1 Cognitive Scenario

Geo satellite and Leo satellite is to use the consistent range of frequencies, when high-orbit satellites are working, in order to prevent certain interference from low-orbit satellites. At this time, the detection algorithm based on sampling covariance matrix is considered, which can detect the transmission frequency of low orbit satellite to the main user uplink on the ground, this detection algorithm can also be used to set up a sensing link between the high orbit satellite and the base station on the ground. When working in high orbit satellite uplink, based on the sampling covariance matrix of the detection algorithm can get high orbit satellite signals emitted by base stations on the ground, when this occurs, should stop take up Leo satellite uplink real-time communication, so as to maximum avoid Leo satellite in high orbit satellite communication series of interference.

The spectrum of high orbit satellite (GEO) network is the authorization network, while that of low orbit satellite (LEO) network is the cognitive network.

How to reasonably allocate the frequency band resources of high orbit satellite and low orbit satellite is always a key research problem As shown in the figure below, the two satellites are involved in roughly the same scope. The system model is shown in the figure below (Fig. 1):

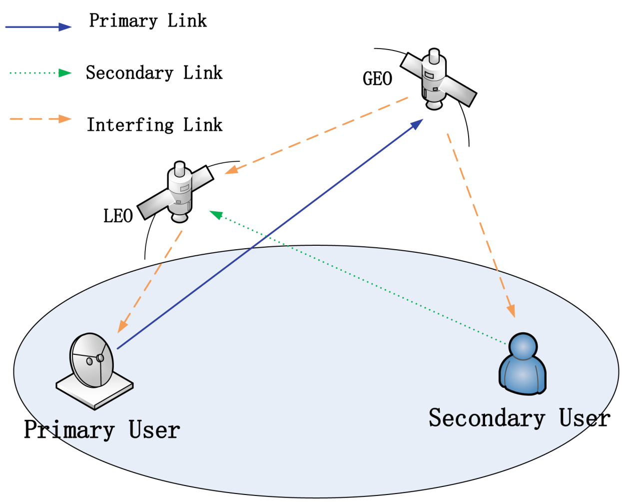

Fig. 1. GEO-LEO cognitive scenario.

2.2 Mathematical Model

Assume that the number of antennas required is M. Let the noise vector to be detected at n be $\mathbf{w}(n)$, the primary user vector $\mathbf{s}(n)$ and the acquired vector $\mathbf{x}(n)$ can be expressed as

$$\begin{cases} \mathbf{w}(n) = [w_1(n)w_2(n)\cdots w_M(n)]^{\mathrm{T}} \\ \mathbf{s}(n) = [s_1(n)s_2(n)\cdots s_M(n)]^{\mathrm{T}} \\ \mathbf{x}(n) = [x_1(n)x_2(n)\cdots x_M(n)]^{\mathrm{T}} \end{cases} \tag{1}$$

In the formula T is the transpose operation of the matrix. We assume that the main user signal and noise are independent of each other. According to the knowledge of signal detection theory, the following formula can be derived:

$$\begin{cases} H_0 : \mathbf{x}(n) = \mathbf{w}(n) \\ H_1 : \mathbf{x}(n) = \mathbf{s}(n) + \mathbf{w}(n) \end{cases} \tag{2}$$

Among them H_0 means the channel is not occupied at this time, when the channel is idle, only noise signals can be detected, H_1 means that the channel has both noise signals and master user signals, and the channel is occupied.$\mathbf{w}\,(n)$ is additive White Gaussian noise, its covariance matrix is $\sigma^2\mathbf{I}_M$. The above formula can be written in another way:

$$\begin{cases} H_0 : \mathbf{R}_x = E\left\{\mathbf{x}(n)\mathbf{x}^{\mathrm{T}}(n)\right\} = \sigma^2\mathbf{I}_M \\ H_1 : \mathbf{R}_x = E\left\{\mathbf{x}(n)\mathbf{x}^{\mathrm{T}}(n)\right\} = \mathbf{R}_s + \sigma^2\mathbf{I}_M \end{cases} \tag{3}$$

The covariance matrix of the received signals in the above equation are $\mathbf{R}_{\mathrm{x}}$, $\mathbf{R}_s = E\left\{\mathbf{s}(n)\mathbf{s}^{\mathrm{T}}(n)\right\}$ is the covariance matrix of the master user signal. As can be seen from the above two formulas, assuming that the channel is occupied and idle, because of the generation of the master user signals, the covariance matrix of the received signals have different expressions. When we do this process, it is often difficult to get the covariance matrix information of the target signals. So in reality we replace the covariance matrix with the sample covariance matrix, it can be expressed as:

$$\hat{\mathbf{R}}_{x(n)} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}(n)\mathbf{x}^{\mathrm{T}}(n) \tag{4}$$

When the number M of samples receiving signals in the sampling covariance matrix is larger, then it can be concluded that $\mathbf{R}_{x(n)} = \lim_{N\to\infty}\hat{\mathbf{R}}_{x(n)}$. When the primary user signals do not exist, the sample covariance matrix of the received signals are consistent with that of the noise, that is

$$\hat{\mathbf{R}}_{w(n)} \triangleq \hat{\mathbf{R}}_x \mid H_0 = \frac{1}{N}\sum_{n=1}^{N}\mathbf{w}(n)\mathbf{w}^{\mathrm{T}}(n) \tag{5}$$

3 Signal Detection Algorithm Based on Matrix Eigenvalue

3.1 Theoretical Analysis of DASCM Algorithm

We make the eigenvalue of the master user signals sampling covariance matrix are $\rho_i(i = 1, 2, \cdots M)$,$\rho_1 \geq \rho_2 \geq \cdots \geq \rho_M$. It can be seen that when the channel is idle, the eigenvalues of the sampling covariance matrix of the received signals meet $\lambda_i = \sigma^2$, when the channel is occupied, eigenvalues meet $\lambda_i = \rho_i + \sigma^2(i = 1, \cdots, M)$. Therefore, it is easy to see that the generation of the main user signals will make the maximum eigenvalue and the minimum eigenvalue significantly different under the condition of channel idle and occupied. Let $\Delta = \lambda_1 - \lambda_2$ be the difference between them, when the channel is idle, $\lambda_1 \to \lambda_2$, we can draw $\Delta \to 0$. Again, when the channel is occupied, $\lambda_1 > \lambda_2$, we can draw $\Delta > 0$. As the power and relevance of the signals needed to be received increases, the fluctuation of the maximum and minimum eigenvalues increases when the channel is occupied and idle, therefore, we can further improve the detection algorithm according to this difference.

In addition, we found that if we only take Δ as the detection decision quantity we need, then, in the case of idle channel, the specific condition of detection decision depends on the quantity and integrity of noise. At the same time, it may cause the result to be excessively influenced by the prior information of noise variance. So, the performance of the detection algorithm constructed with Δ as the detection decision will still be disturbed by the uncertainty of noise, to prevent this, we need to make certain corrections:

$$\begin{aligned} T_{DASCM} &= \frac{\Delta}{\lambda_1 + \lambda_2} \\ &= \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \end{aligned} \tag{6}$$

e saw from the previous description that when the channel is idle, we can draw $T \rightarrow 0$, when the channel is occupied, we can draw $\Delta > 0$, $T > 0$. So, we can judge whether the channel is free by the state of T. In addition, when $\lambda_1 + \lambda_2$ detection result is idle, noise variance information is not needed to judge. On the other hand, the above formula can determine that T only depends on the received data, without relying on the prior information of the master users' signals. Therefore, we can obtain a completely blind detection algorithm according to the state of T.

3.2 Specific Flow of DASCM Algorithm

In general, the flow chart of sampling covariance matrix for decision taking T as a test decision quantity is shown below

Algorithm 1. DASCM master user signals detection flow chart

Calculate the sample covariance matrix
Calculate the maximum and minimum eigenvalues of the sampling covariance matrix
Calculate the detection decision T
Calculate the decision threshold φ according to P_f
if $T > \varphi$ **then**
    H_1
else
    H_0
end if

We start by taking signals from M antennas, So we get the M-dimensional vector of the received signals, and then we start counting the sample covariance matrix $\hat{\mathbf{R}}_{x(n)} = \frac{1}{N}\sum_{i=1}^{N} \mathbf{x}(n)\mathbf{x}^{\mathrm{T}}(n)$. The next step is to factor $\hat{\mathbf{R}}_x$into the smallest and largest eigenvalues of λ_2 and λ_1, and then we can figure out $T_{DASCM} = \frac{\lambda_1-\lambda_2}{\lambda_1+\lambda_2}$. Then through the false alarm probability, the value of the threshold can be obtained, and finally the corresponding judgment results: when $T > \varphi$, the channel is occupied, when $T < \varphi$, the channel is free.

3.3 Calculation of Threshold of Decision Based on Sampling Covariance Matrix

One of the most important things when we do testing is how do we calculate the threshold φ. Through observation, it is found that there is a certain relationship between false alarm probability and judgment threshold:

$$\begin{aligned} P_{f\,DASCM} &= P(T > \varphi) \\ &= P(\frac{\lambda_1 - \lambda_M}{\lambda_1 + \lambda_M} > \varphi) \end{aligned} \tag{7}$$

Conversely, the above can also be expressed as:

$$P_{f\,DASCM} = P\left(\frac{\frac{\lambda_1}{\lambda_2} - 1}{\frac{\lambda_1}{\lambda_2} + 1} > \varphi\right) \tag{8}$$

It can be assumed that: $D = \frac{\lambda_1}{\lambda_2}$, substitute it into the above expression to obtain:

$$\begin{aligned} P_{f\,DASCM} &= P(\frac{D-1}{D+1} > \varphi) \\ &= P(D > \frac{1+\varphi}{1-\varphi}) \end{aligned} \tag{9}$$

Suppose $d = \frac{1+\varphi}{1-\varphi}$, substitute in again to obtain:

$$\begin{aligned} P_{f\,DASCM} &= P(D > d) \\ &= P(\frac{\lambda_1}{\lambda_2} > d) \\ &= P(\lambda_1 > d\lambda_2) \end{aligned} \tag{10}$$

So in summary, if we set a certain false alarm probability in advance $P_{PRE} = P(T > \varphi)$, when the channel is idle, we can derive d from the probability density function of D, and then from the expression above for d and φ, finally, we can get the value of the threshold. Therefore, the most critical problem is the probability density function of D in the idle state of the channel. Some random matrix theory related knowledge is given below:

A: Assuming that $\lim\limits_{M,N\to\infty} \frac{M}{N} \to k \quad (0 < k < 1)$, then the maximum and minimum eigenvalues λ_2 and λ_1 of the sampling covariance matrix $\hat{\mathbf{R}}_{w(n)}$ of noise converge respectively to $\frac{\sigma^2}{N}(\sqrt{N}+\sqrt{M})^2$ and $\frac{\sigma^2}{N}(\sqrt{N}-\sqrt{M})^2$.

B: Assuming that $\lim\limits_{M,N\to\infty}\frac{M}{N}\to k \quad (0<k<1)$, set $\mu_1=(\sqrt{N-1}+\sqrt{M})^2$, $v_1=(\sqrt{N-1}+\sqrt{M})\left(\frac{1}{\sqrt{N-1}}+\frac{1}{\sqrt{M}}\right)^{1/3}$, $X=\frac{N\hat{R}_w}{\sigma^2}$. if $\lambda_1(X)$ is the largest eigenvalue of X, so $\frac{\lambda_1(X)-\mu_1}{v_1}$ obeys a first order TW distribution.

C: Assuming that $\lim\limits_{M,N\to\infty}\frac{M}{N}\to k \quad (0<k<1)$, set $\mu_2=(\sqrt{N}-\sqrt{M})^2$, $v_2=-(\sqrt{N}-\sqrt{M})\left(\frac{1}{\sqrt{M}}-\frac{1}{\sqrt{N}}\right)^{1/3}$, $X=\frac{N\hat{R}_w}{\sigma^2}$, if $\lambda_2(X)$ is the minimum eigenvalue of X, so $\frac{\lambda_2(X)-\mu_2}{v_2}$ obeys a first order TW distribution.

The distribution function of TW(Tracy-widow) can be defined as:

$$F_{DASCM}(t)=\exp\left(-\frac{1}{2}\int\limits_t^\infty \left(q(u)+(u-t)q^2(u)\right)du\right) \tag{11}$$

The $q\left(u\right)$ that appears in the formula above, we have the following equation:

$$q''(u)_{DASCM}=uq(u)+2q^3(u) \tag{12}$$

According to the above conclusions, when the channel is idle, the limits of λ_1 and λ_2 obtained by sample covariance matrix can be expressed as follows:

$$\begin{gathered}\lambda_2\to\frac{\sigma^2}{N}(\sqrt{N}-\sqrt{M})^2\\ \lambda_1\to\frac{\sigma^2}{N}(\sqrt{N}+\sqrt{M})^2\end{gathered} \tag{13}$$

$$\begin{gathered}\lambda_1(X)=\frac{N}{\sigma^2}\lambda_1\\ \lambda_2(X)=\frac{N}{\sigma^2}\lambda_2\end{gathered} \tag{14}$$

Combined with the above results, the false alarm probability of λ_2 can be obtained as follows:

$$\begin{aligned}P_{fDASCM}&=P(\lambda_1>d_0\lambda_2)\\ &=P(\lambda_1>d_0\frac{\sigma^2}{N}(\sqrt{N}-\sqrt{M})^2)\\ &=P(\frac{N}{\sigma^2}\lambda_1>d_0(\sqrt{N}-\sqrt{M})^2)\\ &=P(\lambda_1(\hat{A})>d_0(\sqrt{N}-\sqrt{M})^2)\\ &=1-F_1(\frac{d_0(\sqrt{N}-\sqrt{M})^2)-\mu_1}{v_1})\end{aligned} \tag{15}$$

From the expression of $F\left(.\right)$, it can be obtained:

$$d_0=\frac{v_1F_1^{-1}\left(1-P_f\right)+\mu_1}{(\sqrt{N}-\sqrt{M})^2} \tag{16}$$

where $F_1^{-1}(.)$ is the inverse of $F_1(.)$.

Similarly, the false alarm probability of λ_1 is:

$$P_{f\,DASCM} \approx P(\lambda_1 > d_1 \frac{\sigma^2}{N} \lambda_2(X)) \tag{17}$$

We can also conclude that

$$d_1 = \frac{(\sqrt{N} + \sqrt{M})^2}{v_2 F_1^{-1}(P_f) + \mu_2} \tag{18}$$

We take the average value of the two threshold values obtained above, and finally obtain a more accurate discriminant threshold value, that is:

$$d_{DASCM} = \frac{1}{2}[\frac{v_1 F_1^{-1}(1 - P_f) + \mu_1}{(\sqrt{N} - \sqrt{M})^2} + \frac{(\sqrt{N} - \sqrt{M})^2}{v_2 F_1^{-1}(1 - P_f) + \mu_2}] \tag{19}$$

We had $d = \frac{1+\varphi}{1-\varphi}$ before, and if we substitute that in, we can get:

$$\varphi_{DASCM} = 1 - \frac{4}{[\frac{v_1 F_1^{-1}(1-P_f)+\mu_1}{(\sqrt{N}-\sqrt{M})^2} + \frac{(\sqrt{N}+\sqrt{M})^2}{v_2 F_1^{-1}(P_f)+\mu_2}] + 2} \tag{20}$$

According to the derivation, when the master user signals are not accurately understood, the detection decision quantity of the sampling covariance matrix can be obtained, and then a relatively accurate decision threshold can be obtained to judge the existence of PU signal. So this algorithm can be widely used.

4 The Simulation Analysis

Our goal is to demonstrate the advantages of detection algorithms based on the ratio of the difference between the maximum and minimum eigenvalues of the sample covariance matrix, the traditional detection algorithm should be added in the simulation comparison. We assume that the preset false alarm probability value is $P_{\text{FA}} = 0.1$. The value of N is 200, the number of antennas M required is 8, after 5000 Monte Carlo simulations, the changes of P_d and P_f along with SNR of the two algorithms mentioned above are shown in the figure below, In the simulation figure, DASCM represents the simulation result of the improved algorithm, while PRE DASCM represents the simulation result of the previous classical and traditional detection algorithm (Fig. 2).

As can be seen from the above simulation results, there is a positive correlation between P_d and SNR of the two detection algorithms, P_d increases as SNR increases, when the SNR growth reaches a certain value, the detection

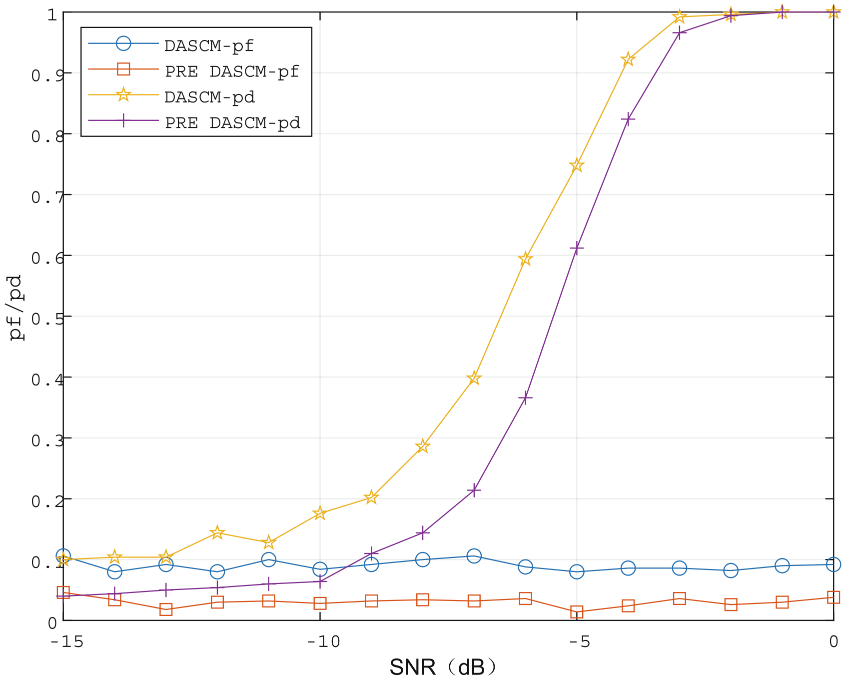

Fig. 2. P_d and P_f at different SNR ($N = 200$, $M = 8$).

algorithm can obtain 100 detection probability. However, the detection probability of DASCM can reach 100% earlier than that of PRE DASCM, this fully shows that the detection performance of DASCM algorithm studied above is better than that of PRE DASCM. Secondly, P_f of DASCM detection algorithm is closer to the pre-set false alarm probability value numerically than P_f of PRE DASCM detection algorithm. From this perspective, it can be seen that DASCM detection algorithm can obtain more accurate judgment threshold value compared with traditional detection algorithm, which also proves that the calculation method of judgment threshold studied in this section is accurate, and further reflects the superiority of DASCM detection algorithm.

We further want to explore the impact on the detection performance of this algorithm, We assume that the preset false alarm probability value is $P_{\mathrm{FA}} = 0.1$. The value of N is 800, the number of antennas M required is 8, after 5000 Monte Carlo simulations, the simulation results that can be obtained are shown below. By comparing the obtained simulation graph with the above one, we can find that when the number of samples increases, the detection probability of these two detection algorithms also keeps increasing, forming a positive correlation trend. It can also be seen that the detection performance of DASCM detection algorithm is still better than that of traditional DASCM detection algorithm (Fig. 3).

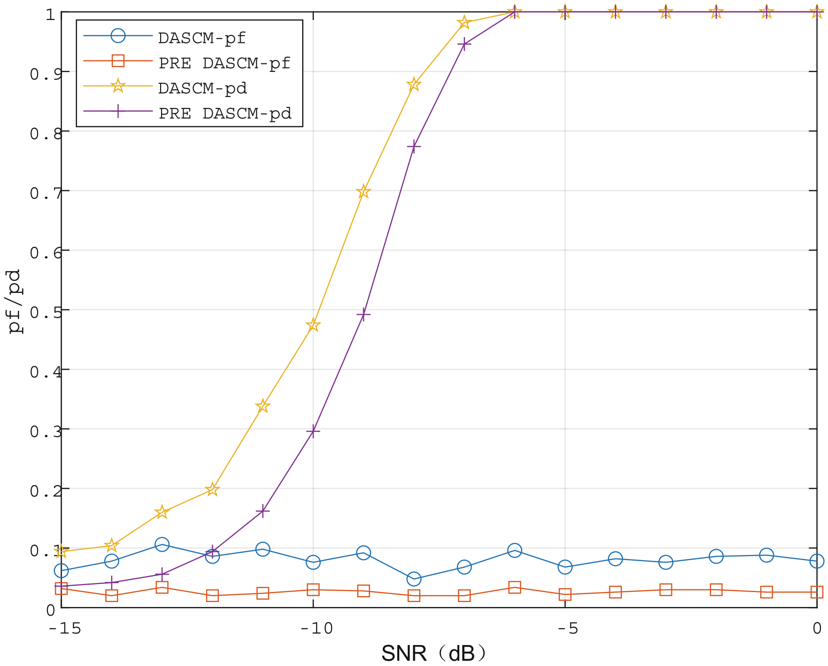

Fig. 3. P_d and P_f at different SNR ($N = 800$, $M = 8$).

Next, we want to explore what kind of interference the number of antennas M will bring to the detection performance of this algorithm. We assume that the preset false alarm probability value is $P_{\mathrm{FA}} = 0.1$. The value of N is 200, the number of antennas M required is 10, after 5000 Monte Carlo simulations, the simulation results that can be obtained are shown below. By comparing the obtained simulation diagram with the one above, we can find that, when the number of antennas keeps increasing, the detection probability of these two detection algorithms also keeps increasing, forming a positive correlation trend, however, the effect of the new DASCM algorithm is better than that of the traditional algorithm even when the SNR values are different. The false alarm probability of the new DASCM algorithm is still approximately equal to 0.1, while the traditional detection method is far less than 0.1. Therefore, it is clear that the new DASCM algorithm is better for performance improvement (Fig. 4).

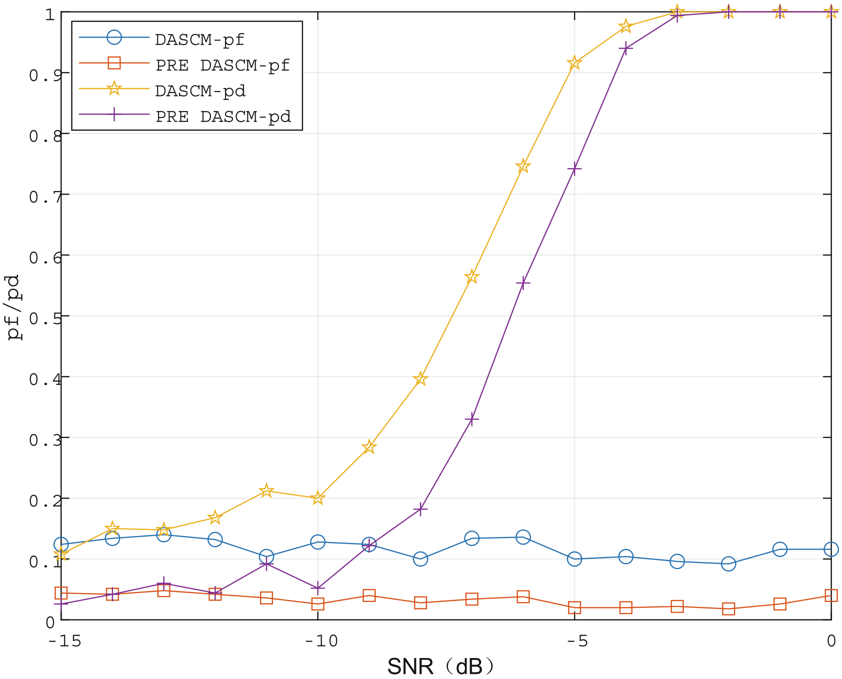

Fig. 4. P_d and P_f at different SNR ($N = 200$, $M = 10$).

5 Conclusion

This paper proposes a channel state information based on GEO-LEO cognitive scenarios. The current spectrum state can be sensed by building a channel state sensing model. Adopting a through the sampling covariance matrix of maximum and minimum eigenvalue of the numerical relationship of detection algorithm (DASCM), it is not need to know the PU signal of a priori knowledge, is a kind of blind detection, don't need to know the condition of second order moment of signal information, and can be a very accurate decision is obtained by calculating the threshold, the performance of the algorithm is greatly improved.

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