On Exponential Stability for Delayed Inertial BAM Neural Networks via Non-reduced Order Approach

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Abstract. The present paper is studying a class of inertial BAM neural networks with general activations and delays. With the help of the non-reduced order method and designing some useful Lyapunov functions, criterions ensuring the exponential stability of the investigated network system are proposed, the obtained conditions are essentially new and complement previously stability results. Moreover, a simulated example is also presented in order to support the established fruits.

Keywords: General BAM neural network · Stability

1 Introduction

In recent decades, numerous neural networks (NNs) have attracted lots of attention of researchers in view of their wide applications in many engineering fields. As an significant kind of NNs, the known bidirectional associative memory (BAM) NNs, firstly introduced by Kosko [1,2], have a wide application prospect in all kinds of fields, for example, pattern recognition, intelligent information processing, optimization problem calculation and complex control, see [3–10]. It be universally known that the limited signal propagation time and switching time interval are inevitable in nature, the time delay is ubiquitous and inescapable in the real world applications. Thus, the kinetic study of delayed NNs has been extensively noticed and discussed during these years, particularly, dynamical behaviors including stability [11], periodic oscillation [12,13], synchronization

B. Li—This work was supported by Social science fund project of Jiangsu Institute of Technology (NO: KYY17504), Guizhou University of Finance and Economics (NO: 2018YJ19, 2019XYB21).

H. Song and D. Jiang (Eds.): SIMUtools 2020, LNICST 369, pp. 225–237, 2021.
https://doi.org/10.1007/978-3-030-72792-5_21

problem [14,15], and bifurcation [16,17], and so on, many kinds of delayed BAM NNs have been widely investigated.

It is worth highlighting that the aforementioned theoretical results are mainly with regard to neural networks modeling by first order DDEs (namely, delay differential equations). Actually, inertial effects are inescapable in many practical systems, for instance, power electronics, GRNs, NNs, etc., see such as [18–20]. Consequently, it is important and meaningful to research the dynamical behaviors of NNs with inertia characteristics. As a result of the existence of inertial factors, the model stemming from the real word is usually expressed by two order functional differential equations, which is having nothing in common with the traditional ones and brings many theoretical and technical obstacles when investigating their dynamical behaviors. In addition, in many existing literatures, many researchers applied the reduced-order technique to consider the inertial NNs, although it is an effective method but there are still some problems, for example, when by applying appropriate variable transformations, the two order DEs are equivalent to two sub-systems, and considering BAM neural networks are binary systems, which unquestionably raises the dimension of the system and tremendously causes difficulty on theoretical analysis and the computational complexity of the established outcomes. Consequently, it is badly in need of new approach to investigate the exponential stability of delayed inertial BAM NNs with general activations.

On account of the above discussions and some recent references [21–23], the exponential stability of delayed inertial BAM NNs with activations. By employing the non-reduced order methods, a up-to-date Lyapunov-Kraiiovskii function is constructed, efficient criterions are established for the investigated NNs. Distinct from existing theoretical results with regard to the exponential stability of delayed inertial neural network where the reduced-order methods is adopted, the obtained main results improve some of the latest research results.

This followings are the arrangements of this paper. Section 2 presents some preliminaries. In part 3, we concretely research the exponential stability. Section 4 provides a simulated example. Ultimately, a conclusion is obtained in Sect. 5.

2 Preliminaries

In this part, we will study the delayed inertial general BAM neural networks with general activations:

$$\begin{cases} m_i''(t) = -a_i m_i'(t) - d_i m_i(t) + \sum\limits_{j=1}^{n} p_{ji} f_j(m_j(t-\sigma_{ji}), z_j(t-\tau_{ji})) + I_i, \\ z_j''(t) = -b_i z_j'(t) - h_j z_j(t) + \sum\limits_{i=1}^{n} q_{ij} g_i(m_i(t-\delta_{ij}), z_i(t-\eta_{ij})) + J_j, \end{cases} \tag{1}$$

the initial datas concerning (1) are equipped as

$$\begin{cases} m_i(t) = \vartheta_i(t), \quad s \in [-r_1, 0], \quad r_1 = \max\{\max\limits_{1\leq i\leq n, 1\leq j\leq n} \sigma_{ji}, \max\limits_{1\leq i\leq n, 1\leq j\leq n} \delta_{ij}\}, \\ z_j(t) = \psi_j(t), \quad s \in [-r_2, 0], \quad r_2 = \max\{\max\limits_{1\leq i\leq n, 1\leq j\leq n} \tau_{ji}, \max\limits_{1\leq i\leq n, 1\leq j\leq n} \eta_{ij}\}, \end{cases} \tag{2}$$

In system (1), the second derivative terms are the so-called inertial terms, m_i and z_j stand for the states of the ith neuron and the jth neuron, respectively; $f_j(\cdot,\cdot)$ and $g_i(\cdot,\cdot)$ on behalf of the activation functions of the jth and ith unit, respectively; d_i and h_j delegate the rate with which the i-th nerve cell and j-th nerve cell will reset its potential to the resting state in isolation when they are not related to the system and external input; a_i, b_j are positive constants, p_{ji} and q_{ij} mean the connection weights of the neuron i and j, and $i, j = 1, 2, ..., n$.

In order to establish the theoretical results, we impose the Lipschitz conditions on the activation functions.
($\mathbf{H}_1$) There are positive constants $\alpha_j, \beta_j, \omega_i, \gamma_i$, such that

$$|f_j(p_1, q_1) - f_j(p_2, q_2)| \leq \alpha_j|p_1 - p_2| + \beta_j|q_1 - q_2|,$$

$$|g_i(p_1, q_1) - g_i(p_2, q_2)| \leq \omega_i|q_1 - q_2| + \gamma_i|q_1 - q_2|,$$

for any $p_1, p_2, q_1, q_2 \in R$, $i, j = 1, 2, ..., n$.

3 Main Results

Theorem 1. *Let Hypothesis* ($\mathbf{H}_1$) *hold. Then system (1) is globally exponentially stable if the coefficients satisfy the following conditions:*

$$\mathfrak{J}_i < 0, \quad \mathcal{J}_i < 0, \tag{3}$$

and

$$4\mathfrak{J}_i\mathfrak{K}_i > (\mathfrak{L}_i)^2, \quad 4\mathcal{J}_i\mathcal{K}_i > (\mathcal{L}_i)^2, \tag{4}$$

in which

$$\begin{cases} \mathfrak{J}_i = \xi_{1i}\zeta_{1i} - a_i\xi_{1i}^2 + \frac{1}{2}\sum\limits_{j=1}^{n} \xi_{1i}^2|p_{ji}|(\alpha_j + \beta_j), \\ \mathfrak{K}_i = \frac{1}{2}\sum\limits_{j=1}^{n} |\xi_{1i}||\zeta_{1i}||p_{ji}|(\alpha_j + \beta_j) + \frac{1}{2}\sum\limits_{j=1}^{n} \left(\xi_{1i}^2|p_{ji}|\alpha_j + |\xi_{1i}||\zeta_{1i}||p_{ji}|\alpha_j\right) \\ \quad +\frac{1}{2}\sum\limits_{j=1}^{n} \left(\xi_{2i}^2|q_{ij}|\omega_i + |\xi_{2i}||\zeta_{2i}||q_{ij}|\omega_i\right) - d_i\xi_{1i}\zeta_{1i}, \\ \mathfrak{L}_i = \lambda_{1i} + \zeta_{1i}^2 - d_i\xi_{1i}^2 - a_i\xi_{1i}\zeta_{1i}, \end{cases}$$

and

$$\begin{cases} \mathcal{J}_i = \xi_{2i}\zeta_{2i} - b_j\xi_{2i}^2 + \frac{1}{2}\sum\limits_{j=1}^{n} \xi_{2i}^2|q_{ij}|(\omega_i + \gamma_i), \\ \mathcal{K}_i = \frac{1}{2}\sum\limits_{j=1}^{n} |\xi_{2i}||\zeta_{2i}||q_{ij}|(\omega_i + \gamma_i) + \frac{1}{2}\sum\limits_{j=1}^{n} \left(\xi_{1i}^2|p_{ji}|\beta_j + |\xi_{1i}||\zeta_{1i}||p_{ji}|\beta_j\right) \\ \quad +\frac{1}{2}\sum\limits_{j=1}^{n} \left(\xi_{2i}^2|q_{ij}|\gamma_i + |\xi_{2i}||\zeta_{2i}||q_{ij}|\gamma_i\right) - h_j\xi_{2i}\zeta_{2i}, \\ \mathcal{L}_i = \lambda_{2i} + \zeta_{2i}^2 - h_j\xi_{2i}^2 - b_j\xi_{2i}\zeta_{2i}. \end{cases}$$

Proof. Let $\mathbf{m(t)} = (m_1(t), m_2(t), \cdots, m_n(t)), \mathbf{z(t)} = (z_1(t), z_2(t), \cdots, z_n(t))$ and $\mathbf{m}^*(\mathbf{t}) = (m_1^*(t), m_2^*(t), \cdots, m_n^*(t)), \mathbf{z}^*(\mathbf{t}) = (z_1^*(t), z_2^*(t), \cdots, z_n^*(t))$ be two different solutions of system (1). Denote $x_i(t) = m_i(t) - m_i^*(t)$, $y_j(t) = z_j(t) - z_j^*(t)$, then

$$\begin{cases} x_i''(t) = -a_ix_i'(t) - d_ix_i(t) + \sum\limits_{j=1}^{n} p_{ji}\tilde{f}_j(x_j(t-\sigma_{ji}), y_j(t-\tau_{ji})), \\ y_j''(t) = -b_jy_j'(t) - h_jy_j(t) + \sum\limits_{i=1}^{n} q_{ij}\tilde{g}_i(x_i(t-\delta_{ij}), y_i(t-\eta_{ij})), \end{cases} \tag{5}$$

where

$$\tilde{f}_j(x_j, y_j) = f_j(m_j, z_j) - f_j(m_j^*, z_j^*),$$

and

$$\tilde{g}_i(x_i(t), y_i(t)) = g_i(m_i(t), z_i(t)) - g_j(m_i^*(t), z_i^*(t)).$$

We deduce from the continuity theory and (3)–(4) that there is a constant $\varepsilon > 0$ satisfying

$$\mathfrak{J}_i^\varepsilon < 0, \quad \mathcal{J}_i^\varepsilon < 0,$$

and

$$4\mathfrak{J}_i^\varepsilon\mathfrak{K}_i^\varepsilon > (\mathfrak{L}_i^\varepsilon)^2, \quad 4\mathcal{J}_i^\varepsilon\mathcal{K}_i^\varepsilon > (\mathcal{L}_i^\varepsilon)^2,$$

where

$$\begin{cases} \mathfrak{J}_i^\varepsilon = \xi_{1i}^2\varepsilon + \xi_{1i}\zeta_{1i} - a_i\xi_{1i}^2 + \frac{1}{2}\sum\limits_{j=1}^{n} \xi_{1i}^2|p_{ji}|(\alpha_j + \beta_j), \\ \mathfrak{K}_i^\varepsilon = \lambda_{1i}\varepsilon + \zeta_{1i}^2\varepsilon + \frac{1}{2}\sum\limits_{j=1}^{n} |\xi_{1i}||\zeta_{1i}||p_{ji}|(\alpha_j + \beta_j) \\ \quad +\frac{1}{2}\sum\limits_{j=1}^{n} \left(\xi_{1i}^2|p_{ji}|\alpha_j + |\xi_{1i}||\zeta_{1i}||p_{ji}|\alpha_j\right) \\ \quad +\frac{1}{2}\sum\limits_{j=1}^{n} \left(\xi_{2i}^2|q_{ij}|\omega_i + |\xi_{2i}||\zeta_{2i}||q_{ij}|\omega_i\right) - d_i\xi_{1i}\zeta_{1i}, \\ \mathfrak{L}_i^\varepsilon = \lambda_{1i} + \zeta_{1i}^2 + 2\varepsilon\xi_{1i}\zeta_{1i} - d_i\xi_{1i}^2 - a_i\xi_{1i}\zeta_{1i}, \end{cases} \tag{6}$$

and

$$\begin{cases} \mathcal{J}_i^{\varepsilon} = \xi_{2i}^2\varepsilon + \xi_{2i}\zeta_{2i} - b_j\xi_{2i}^2 + \frac{1}{2}\sum\limits_{j=1}^{n}\xi_{2i}^2|q_{ij}|(\omega_i+\gamma_i), \\ \mathcal{K}_i^{\varepsilon} = \lambda_{2i}\varepsilon + \zeta_{2i}^2\varepsilon + \frac{1}{2}\sum\limits_{j=1}^{n}|\xi_{2i}||\zeta_{2i}||q_{ij}|(\omega_i+\gamma_i) \\ \quad +\frac{1}{2}\sum\limits_{j=1}^{n}\left(\xi_{1i}^2|p_{ji}|\beta_j + |\xi_{1i}||\zeta_{1i}||p_{ji}|\beta_j\right) \\ \quad +\frac{1}{2}\sum\limits_{j=1}^{n}\left(\xi_{2i}^2|q_{ij}|\gamma_i + |\xi_{2i}||\zeta_{2i}||q_{ij}|\gamma_i\right) - h_j\xi_{2i}\zeta_{2i}, \\ \mathcal{L}_i^{\varepsilon} = \lambda_{2i} + \zeta_{2i}^2 + 2\varepsilon\xi_{2i}\zeta_{2i} - h_j\xi_{2i}^2 - b_j\xi_{2i}\zeta_{2i}. \end{cases} \tag{7}$$

Designing Lyapunov function with integral term as follows:

$$\begin{aligned} V(r) =& \underbrace{\frac{1}{2}\sum_{i=1}^{n}\lambda_{1i}x_i^2(r)e^{2\varepsilon t} + \frac{1}{2}\sum_{i=1}^{n}(\xi_{1i}x_i'(r)+\zeta_{1i}x_i(r))^2e^{2\varepsilon r}}_{V_1(r)} \\ &+ \underbrace{\frac{1}{2}\sum_{i=1}^{n}\lambda_{2i}y_i^2(r)e^{2\varepsilon r} + \frac{1}{2}\sum_{i=1}^{n}(\xi_{2i}y_i'(r)+\zeta_{2i}y_i(r))^2e^{2\varepsilon r}}_{V_2(r)} \\ &+ \underbrace{\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{1i}^2|p_{ji}|\alpha_j + |\xi_{1i}||\zeta_{1i}||p_{ji}|\alpha_j]\int_{r-\sigma_{ji}}^{r}e^{2\varepsilon(s+r_1)}(x_j(s))^2ds}_{V_3(r)} \\ &+ \underbrace{\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{1i}^2|p_{ji}|\beta_j + |\xi_{1i}||\zeta_{1i}||p_{ji}|\beta_j]\int_{r-\tau_{ji}}^{r}e^{2\varepsilon(s+r_2)}(y_j(s))^2ds}_{V_3(r)} \\ &+ \underbrace{\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{2i}^2|q_{ij}|\omega_i + |\xi_{2i}||\zeta_{2i}||q_{ij}|\omega_i]\int_{r-\delta_{ij}}^{r}e^{2\varepsilon(s+r_1)}(x_i(s))^2ds}_{V_4(r)} \\ &+ \underbrace{\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{2i}^2|q_{ij}|\gamma_i + |\xi_{2i}||\zeta_{2i}||q_{ij}|\gamma_i]\int_{r-\eta_{ij}}^{t}e^{2\varepsilon(s+r_2)}(y_i(s))^2ds}_{V_4(r)}. \end{aligned} \tag{8}$$

Compute the derivatives of (8), we deduce

$$\begin{aligned}
\frac{dV_1(r)}{dr} =&2\varepsilon\Big[\frac{1}{2}\sum_{i=1}^{n}\lambda_{1i}x_i^2(r)e^{2\varepsilon r}+\frac{1}{2}\sum_{i=1}^{n}(\xi_{1i}x_i'(r)+\zeta_{1i}x_i(r))^2e^{2\varepsilon r}\Big]\\
&+\sum_{i=1}^{n}\lambda_{1i}x_i(r)x_i'(r)e^{2\varepsilon r}+\sum_{i=1}^{n}(\xi_{1i}x_i'(r)+\zeta_{1i}x_i(r))(\xi_{1i}x_i''(r)+\zeta_{1i}x_i'(r))e^{2\varepsilon r}\\
=&2\varepsilon\Big[\frac{1}{2}\sum_{i=1}^{n}\lambda_{1i}x_i^2(r)e^{2\varepsilon r}+\frac{1}{2}\sum_{i=1}^{n}(\xi_{1i}x_i'(r)+\zeta_{1i}x_i(r))^2e^{2\varepsilon r}\Big]\\
&+\sum_{i=1}^{n}(\lambda_{1i}+\zeta_{1i}^2)x_i(r)x_i'(r)e^{2\varepsilon r}+\sum_{i=1}^{n}(\xi_{1i}\zeta_{1i})(x_i'(r))^2e^{2\varepsilon r}\\
&+\sum_{i=1}^{n}\xi_{1i}(\xi_{1i}x_i'(r)+\zeta_{1i}x_i(r))e^{2\varepsilon r}\Big[-a_ix_i'(r)-d_ix_i(r)\\
&+\sum_{j=1}^{n}p_{ji}(\alpha_j|x_j(r-\sigma_{ji})|+\beta_j|y_j(r-\tau_{ji})|)\Big]\\
\le& e^{2\varepsilon r}\Big[\sum_{i=1}^{n}(\lambda_{1i}+\zeta_{1i}^2+2\varepsilon\xi_{1i}\zeta_{1i}-d_i\xi_{1i}^2-a_i\xi_{1i}\zeta_{1i})x_i(r)x_i'(r)\\
&+\sum_{i=1}^{n}(\lambda_{1i}\varepsilon+\zeta_{1i}^2\varepsilon-d_i\xi_{1i}\zeta_{1i})(x_i(r))^2\\
&+\sum_{i=1}^{n}(\xi_{1i}^2\varepsilon+\xi_{1i}\zeta_{1i}-a_i\xi_{1i}^2)(x_i'(r))^2\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}(\xi_{1i}^2|x_i'(r)|+|\xi_{1i}||\zeta_{1i}||x_i(r)|)|p_{ji}|(\alpha_j|x_j(r-\sigma_{ji})|+\beta_j|y_j(r-\tau_{ji})|)\Big].
\end{aligned} \tag{9}$$

By the elementary inequalities, we have that

$$
\begin{aligned}
&\sum_{i,j=1}^{n}(\xi_{1i}^2|x_i'(r)|+|\xi_{1i}||\zeta_{1i}||x_i(r)|)|p_{ji}|(\alpha_j|x_j(t-\sigma_{ji})|+\beta_j|y_j(r-\tau_{ji})|)\\
=&\sum_{i,j=1}^{n}\xi_{1i}^2|p_{ji}||x_i'(r)|(\alpha_j|x_j(r-\sigma_{ji})|+\beta_j|y_j(r-\tau_{ji})|)\\
&+\sum_{i,j=1}^{n}|\xi_{1i}||\zeta_{1i}||p_{ji}||x_i(r)|(\alpha_j|x_j(r-\sigma_{ji})|+\beta_j|y_j(r-\tau_{ji})|)\\
\le&\frac{1}{2}\sum_{i,j=1}^{n}\xi_{1i}^2|p_{ji}|\alpha_j[(x_i'(r))^2+(x_j(r-\sigma_{ji}))^2]\\
&+\frac{1}{2}\sum_{i,j=1}^{n}\xi_{1i}^2|p_{ji}|\beta_j[(x_i'(r))^2+(y_j(r-\tau_{ji}))^2]\\
&+\frac{1}{2}\sum_{i,j=1}^{n}|\xi_{1i}||\zeta_{1i}||p_{ji}|\alpha_j[(x_i(r))^2+(x_j(r-\sigma_{ji}))^2]\\
&+\frac{1}{2}\sum_{i,j=1}^{n}|\xi_{1i}||\zeta_{1i}||p_{ji}|\beta_j[(x_i(r))^2+(y_j(r-\tau_{ji}))^2]\\
=&\frac{1}{2}\sum_{i,j=1}^{n}\xi_{1i}^2|p_{ji}|(\alpha_j+\beta_j)(x_i'(r))^2+\frac{1}{2}\sum_{i,j=1}^{n}|\xi_{1i}||\zeta_{1i}||p_{ji}|(\alpha_j+\beta_j)(x_i(r))^2\\
&+\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{1i}^2|p_{ji}|\alpha_j+|\xi_{1i}||\zeta_{1i}||p_{ji}|\alpha_j](x_j(r-\sigma_{ji}))^2\\
&+\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{1i}^2|p_{ji}|\beta_j+|\xi_{1i}||\zeta_{1i}||p_{ji}|\beta_j](y_j(r-\tau_{ji}))^2.
\end{aligned}
\tag{10}
$$

Putting (9) and (10) together, we have

$$
\begin{aligned}
\frac{dV_1(r)}{dr}\le&e^{2\varepsilon r}\Bigg[\sum_{i=1}^{n}(\lambda_{1i}+\zeta_{1i}^2+2\varepsilon\xi_{1i}\zeta_{1i}-d_i\xi_{1i}^2-a_i\xi_{1i}\zeta_{1i})x_i(r)x_i'(r)\\
&+\sum_{i=1}^{n}\big(\lambda_{1i}\varepsilon+\zeta_{1i}^2\varepsilon-d_i\xi_{1i}\zeta_{1i}+\frac{1}{2}\sum_{j=1}^{n}|\xi_{1i}||\zeta_{1i}||p_{ji}|(\alpha_j+\beta_j)\big)(x_i(r))^2\\
&+\sum_{i=1}^{n}\big(\xi_{1i}^2\varepsilon+\xi_{1i}\zeta_{1i}-a_i\xi_{1i}^2+\frac{1}{2}\sum_{j=1}^{n}\xi_{1i}^2|p_{ji}|(\alpha_j+\beta_j)\big)(x_i'(t))^2\\
&+\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{1i}^2|p_{ji}|\alpha_j+|\xi_{1i}||\zeta_{1i}||p_{ji}|\alpha_j](x_j(r-\sigma_{ji}))^2\\
&+\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{1i}^2|p_{ji}|\beta_j+|\xi_{1i}||\zeta_{1i}||p_{ji}|\beta_j](y_j(r-\tau_{ji}))^2\Bigg].
\end{aligned}
\tag{11}
$$

Analogously, we have

$$\begin{aligned}
\frac{dV_2(r)}{dr} \le & e^{2\varepsilon r}\Big[\sum_{i=1}^{n}(\lambda_{2i}+\zeta_{2i}^2+2\varepsilon\xi_{2i}\zeta_{2i}-h_j\xi_{2i}^2-b_j\xi_{2i}\zeta_{2i})y_i(r)y_i'(r) \\
& +\sum_{i=1}^{n}\big(\lambda_{2i}\varepsilon+\zeta_{2i}^2\varepsilon-h_j\xi_{2i}\zeta_{2i}+\frac{1}{2}\sum_{j=1}^{n}|\xi_{2i}||\zeta_{2i}||q_{ij}|(\omega_i+\gamma_i)\big)(y_i(r))^2 \\
& +\sum_{i=1}^{n}\big(\xi_{2i}^2\varepsilon+\xi_{2i}\zeta_{2i}-b_j\xi_{2i}^2+\frac{1}{2}\sum_{j=1}^{n}\xi_{2i}^2|q_{ij}|(\omega_i+\gamma_i)\big)(y_i'(t))^2 \\
& +\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{2i}^2|q_{ij}|\omega_i+|\xi_{2i}||\zeta_{2i}||q_{ij}|\omega_i](x_i(r-\delta_{ij}))^2 \\
& +\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{2i}^2|q_{ij}|\gamma_i+|\xi_{2i}||\zeta_{2i}||q_{ij}|\gamma_i](y_i(r-\eta_{ij}))^2\Big].
\end{aligned} \tag{12}$$

Through a simple manipulation, one yields

$$\begin{aligned}
\frac{dV_3(r)}{dt} \le & \frac{1}{2}\sum_{i,j=1}^{n}[\xi_{1i}^2|p_{ji}|\alpha_j+|\xi_{1i}||\zeta_{1i}||p_{ji}|\alpha_j]e^{2\varepsilon r}(x_j(r))^2 \\
& -\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{1i}^2|p_{ji}|\alpha_j+|\xi_{1i}||\zeta_{1i}||p_{ji}|\alpha_j]e^{2\varepsilon r}(x_j(r-\sigma_{ji}))^2 \\
& +\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{1i}^2|p_{ji}|\beta_j+|\xi_{1i}||\zeta_{1i}||p_{ji}|\beta_j]e^{2\varepsilon r}(y_j(r))^2 \\
& -\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{1i}^2|p_{ji}|\beta_j+|\xi_{1i}||\zeta_{1i}||p_{ji}|\beta_j]e^{2\varepsilon r}(y_j(r-\tau_{ji}))^2,
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
\frac{dV_4(r)}{dr} \le & \frac{1}{2}\sum_{i,j=1}^{n}[\xi_{2i}^2|q_{ij}|\omega_i+|\xi_{2i}||\zeta_{2i}||q_{ij}|\omega_i]e^{2\varepsilon r}(x_i(r))^2 \\
& -\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{2i}^2|q_{ij}|\omega_i+|\xi_{2i}||\zeta_{2i}||q_{ij}|\omega_i]e^{2\varepsilon r}(x_i(r-\delta_{ij}))^2 \\
& +\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{2i}^2|q_{ij}|\gamma_i+|\xi_{2i}||\zeta_{2i}||q_{ij}|\gamma_i]e^{2\varepsilon r}(y_i(r))^2 \\
& -\frac{1}{2}\sum_{i,j=1}^{n}[\xi_{2i}^2|q_{ij}|\gamma_i+|\xi_{2i}||\zeta_{2i}||q_{ij}|\gamma_i]e^{2\varepsilon r}(y_i(r-\eta_{ij}))^2.
\end{aligned} \tag{14}$$

With the help of (11)–(14) and (8) produces

$$\begin{aligned}
\frac{dV(r)}{dr} \leq & e^{2\varepsilon r}\Bigg\{\sum_{i=1}^{n}(\lambda_{1i}+\zeta_{1i}^2+2\varepsilon\xi_{1i}\zeta_{1i}-d_i\xi_{1i}^2-a_i\xi_{1i}\zeta_{1i})x_i(r)x_i'(r) \\
& +\sum_{i=1}^{n}\Bigg[\lambda_{1i}\varepsilon+\zeta_{1i}^2\varepsilon-d_i\xi_{1i}\zeta_{1i}+\frac{1}{2}\sum_{j=1}^{n}|\xi_{1i}||\zeta_{1i}||p_{ji}|(\alpha_j+\beta_j) \\
& +\frac{1}{2}\sum_{j=1}^{n}\big(\xi_{1i}^2|p_{ji}|\alpha_j+|\xi_{1i}||\zeta_{1i}||p_{ji}|\alpha_j\big) \\
& +\frac{1}{2}\sum_{j=1}^{n}\big(\xi_{2i}^2|q_{ij}|\omega_i+|\xi_{2i}||\zeta_{2i}||q_{ij}|\omega_i\big)\Bigg](x_i(r))^2 \\
& +\sum_{i=1}^{n}\big(\xi_{1i}^2\varepsilon+\xi_{1i}\zeta_{1i}-a_i\xi_{1i}^2+\frac{1}{2}\sum_{j=1}^{n}\xi_{1i}^2|p_{ji}|(\alpha_j+\beta_j)\big)(x_i'(r))^2\Bigg\} \\
& +e^{2\varepsilon r}\Bigg\{\sum_{i=1}^{n}(\lambda_{2i}+\zeta_{2i}^2+2\varepsilon\xi_{2i}\zeta_{2i}-h_j\xi_{2i}^2-b_j\xi_{2i}\zeta_{2i})y_i(r)y_i'(r) \\
& +\sum_{i=1}^{n}\Bigg[\lambda_{2i}\varepsilon+\zeta_{2i}^2\varepsilon-h_j\xi_{2i}\zeta_{2i}+\frac{1}{2}\sum_{j=1}^{n}|\xi_{2i}||\zeta_{2i}||q_{ij}|(\omega_i+\gamma_i) \\
& +\frac{1}{2}\sum_{j=1}^{n}\big(\xi_{1i}^2|p_{ji}|\beta_j+|\xi_{1i}||\zeta_{1i}||p_{ji}|\beta_j\big)+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\big(\xi_{2i}^2|q_{ij}|\gamma_i \\
& +|\xi_{2i}||\zeta_{2i}||q_{ij}|\gamma_i\big)\Bigg](y_i(t))^2 \\
& +\sum_{i=1}^{n}\big(\xi_{2i}^2\varepsilon+\xi_{2i}\zeta_{2i}-b_j\xi_{2i}^2+\frac{1}{2}\sum_{j=1}^{n}\xi_{2i}^2|q_{ij}|(\omega_i+\gamma_i)\big)(y_i'(t))^2\Bigg\} \\
= & e^{2\varepsilon t}\Bigg\{\sum_{i=1}^{n}\Big[\mathfrak{J}_i^\varepsilon(x_i'(r))^2+\mathfrak{K}_i^\varepsilon(x_i(r))^2+\mathfrak{L}_i^\varepsilon x_i(r)x_i'(r)\Big] \\
& +\sum_{i=1}^{n}\Big[\mathcal{J}_i^\varepsilon(y_i'(r))^2+\mathcal{K}_i^\varepsilon(y_i(r))^2+\mathcal{L}_i^\varepsilon y_i(r)y_i'(r)\Big]\Bigg\} \\
= & e^{2\varepsilon r}\Bigg\{\sum_{i=1}^{n}\mathfrak{J}_i^\varepsilon\big(x_i'(r)+\frac{\mathfrak{L}_i^\varepsilon}{2\mathfrak{J}_i^\varepsilon}x_i(r)\big)^2+\sum_{i=1}^{n}\big(\mathfrak{K}_i^\varepsilon-\frac{(\mathfrak{L}_i^\varepsilon)^2}{4\mathfrak{J}_i^\varepsilon}\big)(x_i(r))^2 \\
& +\sum_{i=1}^{n}\mathcal{J}_i^\varepsilon\big(y_i'(r)+\frac{\mathcal{L}_i^\varepsilon}{2\mathcal{J}_i^\varepsilon}y_i(r)\big)^2+\sum_{i=1}^{n}\big(\mathcal{K}_i^\varepsilon-\frac{(\mathcal{L}_i^\varepsilon)^2}{4\mathcal{J}_i^\varepsilon}\big)(y_i(r))^2\Bigg\} \\
\leq & 0, \quad \text{for all} \quad r\in[0,\infty).
\end{aligned} \tag{15}$$

Therefore,

$$V(r)\leq V(0), \quad \text{for all} \quad r\in[0,\infty),$$

we can deduce from (8) that

$$|x_i(r)|\leq Me^{-\varepsilon r}, \quad |y_i(r)|\leq Me^{-\varepsilon r},$$

where M is a constant, the above inequality means that $|x_i(r)|$, $|y_i(r)|$ exponentially converge to 0. The proof of Theorem 1 is ended.

4 Numerical Simulations

Example 1. Consider the inertial delayed general BAM NNs with a general binary activation function:

$$\begin{cases} m_i''(t) = -a_i m_i'(t) - d_i m_i(t) + \sum\limits_{j=1}^{2} p_{ji} f_j(m_j(t-0.4), z_j(t-0.4)) + I_i, \\ z_j''(t) = -b_j z_j'(t) - h_j z_j(t) + \sum\limits_{i=1}^{2} q_{ij} g_i(m_i(t-0.4), z_i(t-0.4)) + J_j, \end{cases} \tag{16}$$

where

$$f_j(u,v) = g_i(u,v) = 0.8|u| + 0.2|v|, \quad i,j = 1,2,$$

where $a_1 = 4.21$, $a_2 = 3.15$, $b_1 = 4.15$, $b_2 = 3.9$, $p_{11} = q_{11} = 1.46$, $p_{12} = q_{12} = -1.28$, $p_{21} = q_{21} = -2.1$, $p_{22} = q_{22} = 1.9$, $I_1 = J_1 = 1.5$, $I_2 = J_2 = 1.2$, $d_1 = 10$, $d_2 = 9$, $h_1 = 5.2$, $h_2 = 7.1$. By a simple calculation, we have $\alpha_j = \omega_i = 0.8$, $\beta_j = \gamma_i = 0.2$, $\xi_{1i} = \xi_{2i} = \zeta_{1i} = \zeta_{2i} = 1$, $\lambda_{11} = 11.36$, $\lambda_{12} = 10.15$, $\lambda_{21} = 8.2$, $\lambda_{22} = 8.5$, and

$$\mathfrak{J}_1 = -1.43, \mathfrak{K}_2 = -0.56, \mathfrak{K}_1 = -3.18, \mathfrak{K}_2 = -1.666, \mathfrak{L}_1 = -1.85, \mathfrak{L}_2 = -1,$$

and

$$\mathcal{J}_1 = -1.78, \mathcal{J}_2 = -0.9, \mathcal{K}_1 = -2.57, \mathcal{K}_2 = -3.664, \mathcal{L}_1 = -0.15, \mathcal{L}_2 = -1.5.$$

It is easy to see that

$$\mathfrak{J}_1 < 0, \quad 4\mathfrak{J}_1\mathfrak{K}_1 = 18.1896 > 3.4225 = \mathfrak{L}_1^2,$$

$$\mathfrak{J}_2 < 0, \quad 4\mathfrak{J}_2\mathfrak{K}_2 \approx 3.7318 > 1 = \mathfrak{L}_2^2,$$

and

$$\mathcal{J}_1 < 0, \quad 4\mathcal{J}_1\mathcal{K}_1 = 18.2984 > 0.0225 = \mathcal{L}_1^2,$$

$$\mathcal{J}_2 < 0, \quad 4\mathcal{J}_2\mathcal{K}_2 = 13.1904 > 2.25 = \mathcal{L}_2^2.$$

that the system (16) is exponentially stable. The performed numerical simulations by Matlab to strongly verify this fact, see Figs. 1–2.

Remark 1. In recent years, many nice theoretical results concerning the dynamics of BAM NNs have been reported, see, e.g., [6,9,24–26], nevertheless, the models studied in the aforementioned references do not include the inertial effect, and therefore the established results effectively complement and extend existing ones. On the other hand, by choosing proper variable transformation and reduced-order approach, the authors in [27–30] studied the dynamical behaviors of inertial BAM NNs, clearly, the non reduced-order methods employed in this paper enrich the analysis method for studying inertial BAM NNs.

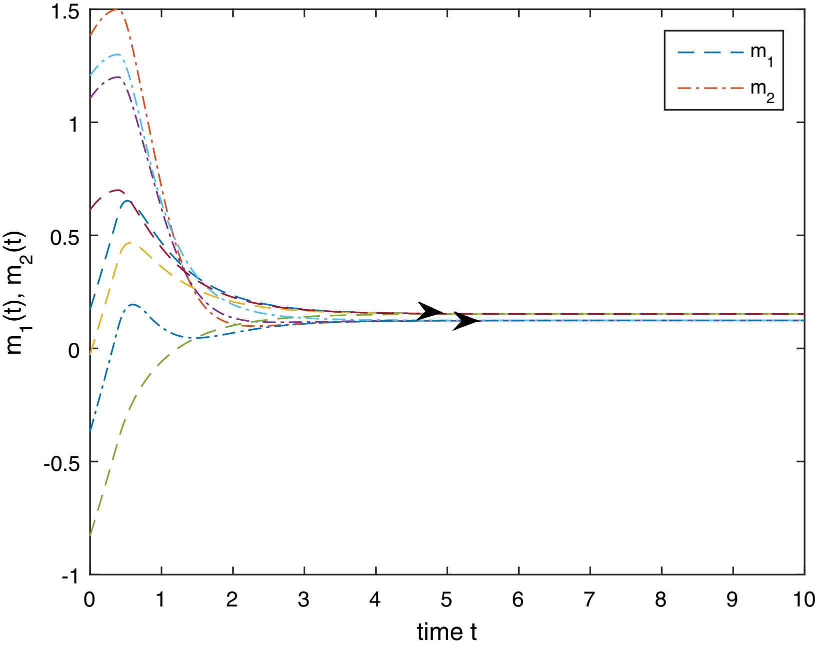

Fig. 1. The orbit graphics of variables $m_i(t)$ with distinct initial datas.

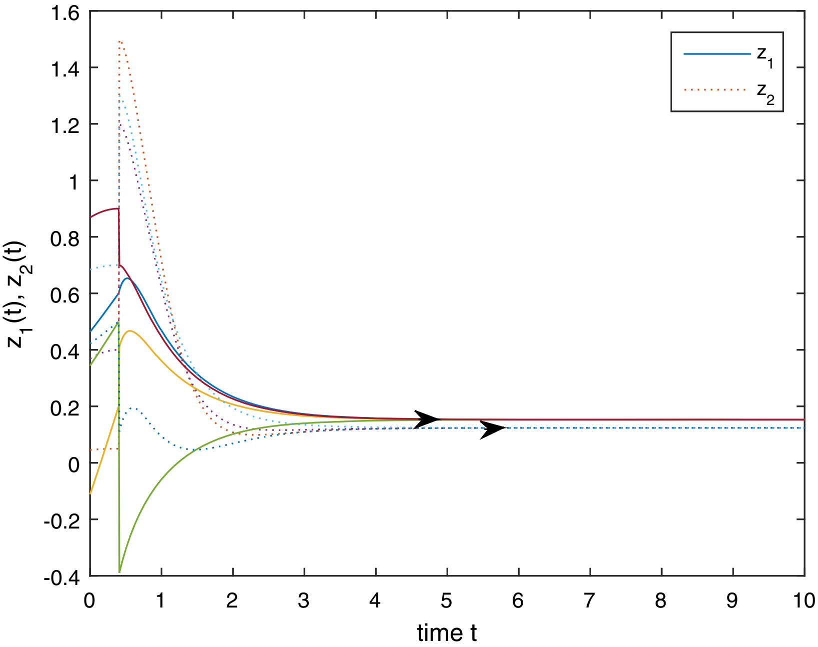

Fig. 2. The orbit graphics of variables $z_i(t)$ with distinct initial datas.

5 Conclusion

In this manuscript, without help of using the usual reduced order approach, we investigated the exponential stability issue for a general delayed inertial BAM NNs, new conditions are obtained by designing a novel Lyapunov function and inequality methods. Finally, a simulation example is also performed to support the established main interesting results. What is worth noting is that the proposed non-reduced order approach in this manuscript can be extended to inertial complex-valued NNs, we will study this problem in the foreseeable future.

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