Smart Beamformer Based on Artificial Intelligence

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Abstract. A decentralized smart beamformer based on artificial intelligence is proposed. Transmission weights adjustment of different users are described as a multi-users game. The existence and uniqueness of the Nash equilibrium in the adaptive beamforming algorithm based on artificial intelligence are proved. Convergent transmission weights update algorithm is designed.

Keywords: Smart beamformer · Artificial intelligence · Game theory

1 Introduction

The smart antenna [1] can focus the transmitted signal only on the desired user, making spatial nulls in the direction to the undesired users. Moreover it can adjust its transmission weight according to condition change of users. Game theory in artificial intelligence algorithm has become popular recently to analyze distributed problem. A game theoretic framework for greedy interference avoidance algorithm is presented in [2]. This model provides insight into development of algorithms that are fairer than the greedy interference avoidance algorithms. A distributed power control algorithm for wireless data systems is proposed in [3]. The QoS a wireless terminal receives is referred to as the utility and distributed power control where users maximize their utilities is a non-cooperative power control game.

In this paper we present a decentralized smart beamformer based on game theory. The main contributions of this paper are: 1) beamformer game model is constructed and transmission weights adjustment of different users are described as multi-users game; 2) the existence and uniqueness of the Nash equilibrium in the decentralized adaptive beamformer based on game theory are proved; 3) Convergent transmission weights update algorithm is designed.

2 System Model

A multiple-input multiple-output (MIMO) system using M transmit and receive antennas is characterized by:

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$$\mathbf{y}_l = \mathbf{H}_l \mathbf{v}_l \mathbf{x}_l + \mathbf{n}_l \tag{1}$$

where y is the M dimensional receive vector of user $l, l = 1, 2, \ldots, L$. We assume that totally L users are distributed in the system. $\mathbf{n}$ is the M dimensional white Gaussian noise vector, $\mathbf{H}_l$ describes the channel matrix and the elements of $\mathbf{H}_l$ are modeled as zero mean complex Gaussian random variables, and $\mathbf{v}_l$ is transmission weight of user l. We suppose $E[\mathbf{nn}^H] = \mathbf{I}$, $\|\mathbf{v}_l\| = 1$ and $E\{x_l x_i^H\} = \begin{cases} 1 & l = i \\ 0 & l \neq i \end{cases}$. The resulting received SINR of user l is:

$$\boldsymbol{\Gamma}_l = \mathbf{v}_l^H \mathbf{R}_l \mathbf{v}_l \Big/ \left(\sum_{i=1, i \neq l}^{L} \mathbf{v}_i^H \mathbf{R}_l \mathbf{v}_i + 1 \right) \tag{2}$$

where $\mathbf{v}_i$ is transmission weight of user i and $\mathbf{R}_l$ is the covariance of channel and $\mathbf{R}_l = E\{\mathbf{HH}^H\}$. The transmission weight vectors can be split up into [4]:

$$\mathbf{v}_l = \sqrt{P_l}\mathbf{u}_l \tag{3}$$

where P_l and $\mathbf{u}_l$ are the transmit power and the orientation of the transmission weight vector respectively. In this letter, we suppose that $\mathbf{u}_l$ is known by the transmitter. So the SINR of user l is a function of transmit power P_l.

3 Smart Beamformer Based on Game Theory

Game theory can be used to predict the outcome of these interactions and to identify optimal strategies and deleterious ones. The fundamental component of game theory is the notion of a game, expressed in normal form as $\Lambda = \{L, \{S_l\}_{l \in L}, \{U_l\}_{l \in L}\}$, where Λ is a particular game. L is a finite set of players, S_l is the set of the action available to players and U_l is the set of pure utility. In this letter we define U_l as:

$$U_l = U_u - U_{\cos t} = \Gamma_l/(\Gamma_l + \alpha) - \lambda P_l \tag{4}$$

where $U_u = \Gamma_l/(\Gamma_l + \alpha)$ is the whole utility function and represents the function of the SINR of user l and $U_{\cos t} = \lambda P_l$ is the cost function. α and λ are constant. α is an adjustable parameter and shows craggedness degree of U_u, which is set by the same value for all users. λ is the cost factor and defines the user's cost when it is interfered by other users.

We consider a best-response dynamics defined below:

$$\arg\max U_l = \arg\max(\Gamma_l/(\Gamma_l + \alpha) - \lambda P_l) \tag{5}$$

We suppose $k = \mathbf{u}_l^H \mathbf{R}_l \mathbf{u}_l \bigg/ (\sum_{i=1, i \neq l}^{L} P_i \mathbf{u}_i^H \mathbf{R}_l \mathbf{u}_i + 1)$, so the SINR $\mathbf{\Gamma}_l$ of user l in the Eq. (2) can also be expressed:

$$\mathbf{\Gamma}_l = kP_l \tag{6}$$

Differentiating the Eq. (5) yields:

$$\begin{aligned} &\frac{\partial U_l}{\partial P_l} = \frac{\partial U_l}{\partial \Gamma_l} \cdot \frac{\partial \Gamma_l}{\partial P_l} - \lambda = \frac{\alpha}{(\Gamma + \alpha)^2} \frac{\partial \Gamma_l}{\partial P_l} - \lambda \\ &= \frac{k\alpha}{(\Gamma + \alpha)^2} - \lambda = 0 \end{aligned} \tag{7}$$

Through the Eq. (7) transmit power P_l can be expressed as:

$$\begin{aligned} &\frac{k\alpha}{(\Gamma + \alpha)^2} = \lambda \Rightarrow \frac{k\alpha}{\lambda} = (\Gamma + \alpha)^2 \\ &\Rightarrow kP_l = \sqrt{k\alpha / \lambda} - \alpha \\ &\Rightarrow P_l = \sqrt{\alpha / \lambda k} - \alpha / k \end{aligned} \tag{8}$$

From the Eq. (8) we can find that the transmit power P_l of user l includes the interference of other users. So the transmit power of different users will affect each other. Next we will demonstrate that the power allocation for different users can converge to a Nash Equilibrium (NE). At a NE, given the power levels of other players, no users can improve its utility level by making individual changes in its power. In [3] the theorem of existence and uniqueness of NE is presented. According to these theorems we will prove the existence and uniqueness of NE of the proposed algorithm.

Theorem 1: The existence of NE of the proposed algorithm
① P_l is a nonempty, convex and compact subset of some Euclidean space. ② U_l is continuous in P and quasi-concave in P_l.

The first condition is satisfied because each user has a strategy space that is defined by $[0, P_l^{\max}]$ and all the power values in between.

Differentiating the Eq. (4) twice yields:

$$\frac{\partial^2 U_l}{\partial P_l^2} = \frac{\partial \left| k\alpha \big/ (\Gamma + \alpha)^2 - \lambda \right|}{\partial P_l} = \frac{-2k^2\alpha}{(kP_l + \alpha)^3} < 0 \tag{9}$$

From the above equation we can know that the pure utility function U_l is concave in P_l for all l. A concave function is quasi-concave so the pure utility function U_l is quasi-concave in P_l. This completes the proof of the Theorem 1.

Theorem 2: The uniqueness of NE of the proposed algorithm
By Theorem 1, we know that there exists a NE P_l and define $r(P_l) = \sqrt{\alpha / \lambda k} - \alpha / k$. The key aspect of the uniqueness proof is that $r(P_l)$ is a standard function [5]. A function is said to be standard if it satisfied the following properties.

① *Positivity*: $r(P_l) > 0$
This property can be implied by a nonzero background receiver noise or by admission control.
② Monotonicity: If $P_l \le P'_l$, then $r(P_l) \ge r(P'_l)$
The system is available so $U_l \ge 0$ and $k \ge k'$ for $P_l \le P'_l$. Then we can obtain:

$$\Gamma_l/(\Gamma_l + \alpha) - \lambda P_l \ge 0 \Rightarrow \alpha/\lambda \ge \alpha^2 \big/ k \tag{10}$$

So

$$\begin{aligned} r(P_l) - r(P'_l) &= \left(\sqrt{\alpha/\lambda k} - \alpha/k\right) - \left(\sqrt{\alpha/\lambda k'} - \alpha/k'\right) \\ &\ge \left(\alpha k' - \alpha\sqrt{kk'} - \alpha k' + \alpha k\right)\Big/ kk' = (k - kk')\alpha/kk' \ge 0 \end{aligned} \tag{11}$$

So the monotonicity is satisfied.
③ *Scalability:* For all $\eta > 1$, $\eta r(P_l) \ge r(\eta P_l)$
We define $k_\eta = \mathbf{u}_l^H \mathbf{R}_l \mathbf{u}_l \Big/ (\sum\limits_{i=1, i \ne l}^{L} \eta P_i \mathbf{u}_i^H \mathbf{R}_l \mathbf{u}_i + 1)$ for all $\eta > 1$ so $k_\eta < k$. $r(\eta P_l)$ can be expressed $r(\eta P_l) = \sqrt{\alpha/\lambda k_\eta} - \alpha/k_\eta$ so

$$\begin{aligned} &\eta r(P_l) - r(\eta P_l) = \eta\left(\sqrt{\alpha/\lambda k} - \alpha/k\right) - \left(\sqrt{\alpha/\lambda k_\eta} - \alpha/k_\eta\right) \\ &\ge \alpha/\sqrt{k}\left(\eta/\sqrt{k} - 1/\sqrt{k_\eta}\right) - \eta\alpha/k + \alpha/k_\eta \\ &= \alpha\left(\eta/k - 1/\sqrt{kk_\eta} - \eta/k + 1/k_\eta\right) \\ &= \alpha\left((k - \sqrt{kk_\eta})/kk_\eta\right) > 0 \end{aligned} \tag{12}$$

So the scalability property is satisfied. Thus we prove that $r(P_l)$ is a standard function. It is shown in [5] that the NE P_l is unique for a standard function. Therefore, the NE of the proposed algorithm is unique.

According to the above conclusions we present a convergent transmission weight update algorithm to reach NE, which can be expressed as:

① Initialization: Set transmission weight $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_L]$.
② Weight update:

$$\mathbf{v}_l = \arg\max(\Gamma_l/(\Gamma_l + \alpha) - \lambda P_l)$$

$$P_l = \sqrt{\alpha/\lambda k} - \alpha/k,\ k = \mathbf{u}_l^H \mathbf{R}_l \mathbf{u}_l \Big/ (\sum_{i=1, i \ne l}^{L} P_i \mathbf{u}_i^H \mathbf{R}_l \mathbf{u}_i + 1).$$

③ Iterative process:

$$|U_{n+1} - U_n| \leq \varepsilon$$

where ε is convergent precision. If the above equation is satisfied, the iteration is over.

4 Conclusions

Decentralized smart beamforming algorithm based on game theory is proposed. We construct beamforming game algorithm mathematics model. Transmission weights adjustment of different users are described as multi-users game. The existence and uniqueness of the Nash equilibrium in the smart beamforming algorithm based on game theory are proved. Convergent transmission weights update algorithm is designed.

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