



Autonomous Positioning Algorithm for UE in Cellular Networks

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Abstract. The positioning techniques in wireless cellular networks detect and receive signals transmitted by the user equipment (UE) through base stations (BS), then the network side uses the time difference of arrival (TDOA) values carried in the received signals to calculate UE's position. However, if the network side doesn't report the positioning information to the UE, UE will never know its positioning result. Besides, a BS can accurately locate the UE within its coverage area, the security issue of UE needs to be considered. In order to improve the autonomy and security of UE, in this paper, we propose an autonomous positioning algorithm for the UE in cellular networks. The UE uses TDOA measurements and multiple UE positioning results sent by the network side to inversely calculate the positions of the BS participating in the positioning. After that, the UE can use calculated BS position coordinates and TDOA values measured by UE itself to calculate its position independently. The simulation results show that the method is effective, and the error of the calculated BSs' coordinates is within the acceptable range.

Keywords: Positioning · TDOA · Genetic algorithm

1 Introduction

Since the promulgation of E-911 regulations in 1996 [1], positioning technology has received the attention of companies and institutions in various countries due to its huge commercial potential. With the rapid development of technology of mobile cellular communications, the number of users increased significantly.

According to the characteristics of measured value, the positioning methods of the UE in cellular network can be divided into three categories: field location algorithm, the positioning algorithm based on the incident angle of the signal arrival (AOA) and the positioning algorithm based on the electric wave arrival time (TOA) or time difference of arrival (TDOA). The field location algorithm is based on the proportional relationship between the energy loss of the signal

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propagating in the channel and the propagation distance [2]. By detecting the field strength values of the transmitted signal and the received signal, the position coordinates of the UE can be roughly estimated. AOA measures the angle of arrival between the UE and the BS. The ray formed from the BS will pass through UE, so that the intersection of two rays is the position of UE [3]. The traveling time of a radio signal between a BS and an UE is the fundamental parameter from which a distance between them is calculated. TOA takes a BS as the center and the distance between the BS and the UE as the radius to make a circle, the intersection of three circles is the position of UE [4]. However, compared with the time difference, the absolute time is more difficult to measure and the accuracy is low. TDOA takes the base stations as the focus and the distance difference between the BS and the UE as the long axis to make a hyperbola, the intersection of two hyperbolas is the signal position [5].

Among these positioning technologies, limited by positioning accuracy requirements and other conditions, the positioning algorithm research mainly focuses on TDOA measurement value estimation. Starting from 3GPP R9, LTE has proposed TDOA positioning technology standards. Later, R14 carried out research on the enhancement of 3G and 4G positioning technology, and enhanced the TDOA positioning technology in the shared physical cell identifier (PCI) scenario. Now 5G uses low-latency and high-precision synchronization technology, which can further improve the accuracy of TDOA [6]. TDOA is still the main technology for 5G positioning [7].

These processes of positioning techniques for wireless cellular networks are basically similar. The UE reports the information required for positioning (such as TDOA measurements) to the network side, and the network side calculates the positioning results. However, these positioning techniques based on the network side still have some problems to be solved. The deficiency is that the UE positioning autonomy is too low, which means if the network side doesn't report the positioning information to the UE, the UE will never know its positioning result. In addition, the network side has the positioning control right and it can accurately locate the positioning information of an UE within its coverage area at any time. The security issue of the UE also needs to be considered. Therefore, an autonomous positioning algorithm for the UE in cellular networks is needed. Without relying on network notifications, the UE can calculate the positioning result by itself and the security of the UE will also be improved.

In order to improve the positioning autonomy and ensure the positioning safety of the UE, we propose an autonomous positioning algorithm for the UE in cellular networks in this paper. UE first uses TDOA measured values and multiple UE positioning results issued by the network side to inversely calculate the position of the BSs participating in the positioning, meanwhile the coordinates of these BSs will be obtained. Also we know that the TDOA measurements are obtained by the UE itself by calculating the time difference between receiving downlink signals of different BSs. Then, the UE can use the calculated BS position coordinates and TDOA measured values to calculate its own position without the BS's help.

The remainder of this paper is organized as follows. On account of the most TDOA positioning technology used currently, a mathematical model of positioning algorithm is given in Sect. 2, and Sect. 3 elaborates the analysis and algorithm, including the application of genetic algorithm in inverse calculation of the BS position. Section 4 shows the simulation results and proves the rationality of analysis. Finally Sect. 5 concludes this paper.

Notations: Uppercase letter X and Y denote the horizontal and vertical coordinates of the BS respectively, and lowercase letter x and y denote the horizontal and vertical coordinates of the UE, respectively. The transpose and inverse of a matrix are expressed as $(\cdot)^T$, $(\cdot)^{-1}$, respectively.

2 System Model

TDOA positioning technique, also called hyperbolic positioning, calculates the position coordinates of target UE by measuring the time difference of the radio wave from the UE to two different base stations [8]. There are two methods for obtaining TDOA measured values. One is based on the difference between TOA measurements of the two base stations. This method requires strict clock synchronization between the UE and the BS. The other is to use related technologies to cross-correlate the signal received by one BS with the signal received by another BS to obtain the TDOA measurements. This method can estimate TDOA measurements when the BS and the UE clocks are not synchronized.

Once a certain TDOA value is obtained, the distance from the UE to the two base stations can be calculated. Several TDOA measured values can form a system of curvature equations about the position of the UE. Then the estimated position of the UE will be captured by solving this system of the curvature equations.

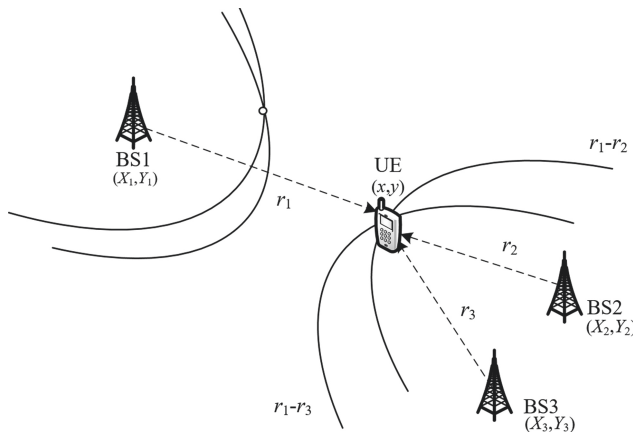


Fig. 1. TDOA-based position method.

According to the geometric principle, the UE must be on the hyperbola with BS_1 and BS_2 as the focal point and the distance difference $r_{21} = r_2 - r_1$ as the focal length. Similarly, if the distance difference r_{31} between BS_3 and BS_1 and the UE is obtained, another hyperbola can be constructed. One of the intersection points of these two hyperbolic curves is the position of the UE as shown in Fig. 1. (x, y) is the position coordinate of the UE to be estimated, (X_i, Y_i) is the known position of the i -th BS, and the distance between the UE and the BS_i is

$$r_i^2 = (X_i - x)^2 + (Y_i - y)^2 = K_i - 2X_i x - 2Y_i y + x^2 + y^2 \quad (1)$$

where $K_i = X_i^2 + Y_i^2$.

Let r_{i1} indicates range difference between the distance from the UE to the BS_i and the distance from the UE to the BS_1 (service base station).

$$r_{i1} = c\tau_{i1} = r_i - r_1 = \sqrt{(X_i - x)^2 + (Y_i - y)^2} - \sqrt{(X_1 - x)^2 + (Y_1 - y)^2} \quad (2)$$

In the formula, $c = 3 \times 10^8$ m/s is the propagation speed of the electric wave in the air and τ_{i1} is the measured value of TDOA. These nonlinear system of equations need to be linearized, and the final linear equations can be obtained as follow

$$r_{i1}^2 + 2r_{i1}r_1 = K_i - 2X_{i1}x - 2Y_{i1}y - K_1 \quad (3)$$

where $X_{i1} = X_i - X_1$ and $Y_{i1} = Y_i - Y_1$. Among these equations, only x , y and r_1 are unknown values. Therefore, formula (3) can be regarded as a system of linear equations about the UE position coordinates (x, y) and r_1 , and the coordinate to uniquely determine the position of the UE can be obtained by solving the system of equations.

In traditional positioning technologies, most are network-based positioning schemes which detect and receive the signals transmitted by the UE through multiple base stations with known positions and process the TDOA information carried in the received signals. These methods only require UE to send TDOA. However, if the network side doesn't report positioning information to the UE, UE will never know its positioning result. Therefore, an autonomous positioning algorithm for the UE in cellular networks is needed.

3 Application of Genetic Algorithm in Inverse Calculation of Base Station Location

In this paper, we propose an autonomous positioning algorithm for the UE in cellular networks. First the UE uses TDOA measured values and the multiple UE positioning results sent by the network side to inversely calculate the position of the BS participating in the positioning, thereby obtaining the coordinates of the BS. After that, the UE can use the calculated BS position coordinates and TDOA measured values obtained by the UE by calculating the time difference between receiving downlink signals of different BSs to realize its own positioning. In this chapter, we first propose the system model of the inverse base station and then give the solution process of it.

3.1 Inverse Base Station Positioning Model

On the two-dimensional plane, assuming that there are M base stations participate in positioning, the UE obtains N measurement positions. (X_i, Y_i) , $i = 1, 2, \dots, M$ is the position coordinate of the i -th base station BS_i to be solved, and (x_j, y_j) , $j = 1, 2, \dots, N$ is the measurement positioning coordinate of the UE. The distance from the j -th UE positioning (x_j, y_j) to the i -th base station (X_i, Y_i) is R_i^j . Taking the base station with serial number 1 as the serving base station, R_{i1}^j represents the distance difference between the j -th UE positioning to $BS_i (i \neq 1)$ and BS_1 (serving base station), as

$$R_{i1}^j = R_i^j - R_1^j + cn_{i1}^j \quad (4)$$

and also can be expressed as

$$R_{i1}^j = \sqrt{(X_i - x_j)^2 + (Y_i - y_j)^2} - \sqrt{(X_1 - x_j)^2 + (Y_1 - y_j)^2} + cn_{i1}^j \quad (5)$$

where $i = 1, 2, \dots, M$, $j = 1, 2, \dots, N$. In the case of considering error, n_{i1}^j is the error introduced in TDOA measurement.

Taking the inverse calculation of the coordinate of the i -th ($i = 2, 3, \dots, M$) BS as an example, inversely calculating the position of the base station (X_i, Y_i) and (X_1, Y_1) is to solve the nonlinear equations composed of N equations as (6). $\Delta \mathbf{R}$, \mathbf{R}_1 , \mathbf{R}_i , \mathbf{n} are N -dimensional vectors formed by N UE measuring and positioning, and can be expressed as

$$\begin{aligned} \Delta \mathbf{R} &= [R_{i1}^1, R_{i1}^2, \dots, R_{i1}^N]^T_{N \times 1} \\ \mathbf{R}_1 &= [R_1^1, R_1^2, \dots, R_1^N]^T_{N \times 1} \\ \mathbf{R}_i &= [R_i^1, R_i^2, \dots, R_i^N]^T_{N \times 1} \\ \mathbf{n} &= [n_{i1}^1, n_{i1}^2, \dots, n_{i1}^N]^T_{N \times 1} \end{aligned} \quad (6)$$

substitute into (5) to get

$$\Delta \mathbf{R} = \mathbf{R}_i - \mathbf{R}_1 + \mathbf{cn} = \begin{bmatrix} \sqrt{(X_i - x_1)^2 + (Y_i - y_1)^2} \\ \sqrt{(X_i - x_2)^2 + (Y_i - y_2)^2} \\ \vdots \\ \sqrt{(X_i - x_N)^2 + (Y_i - y_N)^2} \\ -\sqrt{(X_1 - x_1)^2 + (Y_1 - y_1)^2} \\ -\sqrt{(X_1 - x_2)^2 + (Y_1 - y_2)^2} \\ \vdots \\ -\sqrt{(X_1 - x_N)^2 + (Y_1 - y_N)^2} \end{bmatrix} + \mathbf{cn} \quad (7)$$

This paper considers the case of $N \geq 3$, using maximum likelihood to estimate the BS coordinates (X_i, Y_i) and (X_1, Y_1) . Since the elements in $\mathbf{R}_i - \mathbf{R}_1$ are

known TDOA measurements and n is a normal distribution function with mean 0 and variance σ^2 , all elements in $\Delta\mathbf{R}$ follow a normal distribution with mean R_{i1}^j and variance σ^2 . Each measurement value is independent, the corresponding likelihood function is

$$\begin{aligned} & \prod_{j=1}^N \left[\frac{1}{\sqrt{2\pi}\sigma} \exp \left\{ -\frac{(\Delta R_i^j - R_i^j + R_1^j)^2}{2\sigma^2} \right\} \right] \\ & = \left[\frac{1}{\sqrt{2\pi}\sigma} \right]^N \exp \left(-\frac{(\Delta\mathbf{R} - \mathbf{R}_i + \mathbf{R}_1)^T (\Delta\mathbf{R} - \mathbf{R}_i + \mathbf{R}_1)}{2\sigma^2} \right) \end{aligned} \quad (8)$$

Finding the coordinate value that maximizes the maximum likelihood probability is equivalent to solving the following formula

$$(X_i, Y_i, X_1, Y_1) = \arg \min \left[(\Delta\mathbf{R} - \mathbf{R}_i + \mathbf{R}_1)^T (\Delta\mathbf{R} - \mathbf{R}_i + \mathbf{R}_1) \right] \quad (9)$$

Formula (9) is a non-linear function. If the analytical method is used, the process will be very complicated, and bring a huge amount of calculation. In addition, for the inverse calculation of the BS position, since the location difference between the two positions of the mobile terminal and the same base station cannot be obtained, the linear algorithm of the TDOA principle cannot be used to calculate the position of the base station. Therefore for formula (9), the genetic algorithm is used to solve this problem, and the optimal solution is searched in the entire potential solution space to calculate the position of the base station.

3.2 Process of Genetic Algorithm in Inverse Calculation of Base Station Location

Genetic Algorithm (GA) is a kind of random optimization algorithm obtained from the idea of biological inheritance and evolution [9]. GA expresses the problem-solving process as the evolution of chromosomes. Each chromosome represents an individual in the group and is also a solution to the problem [10]. According to the fitness of chromosomes and the corresponding evolutionary rules, through the selection, crossover, and mutation between each generation of chromosomes, it eventually converges to the state of the most suitable environment, and the optimal solution to the problem is obtained [11]. The steps to realize the back-calculation of the location of the base stations using genetic algorithm are as follows.

Initialize Population and Chromosome Coding. First, we need to determine the search space. Obtain the ID of the cell where the UE is located. Due to the uniqueness of the ID, the range of the cell can be seen as the search space. Assume that the upper limit of cell coordinates is x_{max} , y_{max} , the lower limit is x_{min} , y_{min} , Then the BS coordinates (X_i, Y_i) , where $i > 1$, and the serving BS coordinate (X_1, Y_1) satisfy the following conditions

$$\begin{cases} x_{\min} \leq X_i \leq x_{\max} \\ y_{\min} \leq Y_i \leq y_{\max} \end{cases} \begin{cases} x_{\min} \leq X_1 \leq x_{\max} \\ y_{\min} \leq Y_1 \leq y_{\max} \end{cases} \quad (10)$$

Then use binary coding for individual coding. This coding method is simple and easy to operate, which is conducive to the realization of crossover and mutation. This algorithm only considers positioning on a two-dimensional plane. Taking (X_i, Y_i) as an example, first encode the base station coordinates $(X_i$ and $Y_i)$ separately, and the encoded bit strings are A and B . Cascade A and B to get the bit string C , then C represents an individual's chromosome. With this coding method, the search space of the genetic algorithm is mapped to the space coordinates within the cell range.

At last, the population and parameters need to be Initialized. Set the population size P and randomly generate a $P \times 4$ dimensional matrix as the initial population within the cell range. Set evolution termination algebra G , crossover rate P_c and mutation rate P_m .

Fitness Function. When Eq.(8) obtains the minimum value, the estimated coordinates of the BS are the best. So the fitness function is taken as

$$f = \frac{1}{(\Delta \mathbf{R} - \mathbf{R}_i + \mathbf{R}_1)^T (\Delta \mathbf{R} - \mathbf{R}_i + \mathbf{R}_1)} \quad (11)$$

The larger the individual fitness value, the closer it is to the optimal solution.

Adaptive Operator. The crossover rate P_c and mutation rate P_m have a great influence on the optimization process of the genetic algorithm, and an appropriate value should be selected to prevent the occurrence of premature convergence.

Genetic Algorithm Process

Select Operation. Use the roulette strategy to select individuals and ensure that highly adaptive individuals can enter the next generation population with a higher probability.

The roulette strategy is to calculate the probability that each individual can be inherited according to the fitness of the individual. According to this probability, individuals in the contemporary population are randomly selected to form the offspring population. The fitness function given in Eq. (10) is judged as a maximization problem, and the steps to solve the maximization problem with this strategy are given below.

Calculate the sum of the fitness values of all individuals in the contemporary population

$$F = \sum_{j=1}^P f_j \quad (12)$$

where P is the population size, f_j is the fitness value of the j -th individual. The fitness value of each individual is divided by the total F to obtain the probability that the individual is selected

$$p_j = \frac{f_j}{F} \quad (13)$$

Calculate individual cumulative probability and construct roulette wheel. Then generate a random number in the range of $[0, 1]$ when selecting, if the random number is less than or equal to the cumulative probability of the individual (cumulative probability is the individual probability of all individuals in front of the individual in the individual list sum) and greater than the cumulative probability of individual 1, select the individual to enter the progeny population.

Cross Operation. Uniform arithmetic crossover is used in this algorithm. That is, the genes at each locus of two paired individuals are exchanged with the same crossing probability, thereby forming two new individuals.

Mutation Operation. This algorithm uses non-uniform mutation. Randomly disturb the original gene value, and use the disturbed result as the new gene value after mutation. For example, $x = (x_1, x_2, \dots, x_n)$ is the individual to be mutated ($n = 4$ for the algorithm, the mutated individual is X_i, Y_i, X_1, Y_1). For the mutated individual x_k , the new offspring x'_k is

$$x'_k = \begin{cases} x_k + r \cdot (x_{max} - x_k) \cdot \left(1 - \frac{g}{G}\right)^b, & a = 0 \\ x_k + r \cdot (x_k - x_{min}) \cdot \left(1 - \frac{g}{G}\right)^b, & a = 1 \end{cases} \quad (14)$$

where r is a random number on $[0, 1]$, g is the current evolutionary algebra, G is the maximum evolutionary algebra, b is a parameter to determine the non-uniformity, usually 2–5, and a is 0, 1 random number.

According to the above theoretical analysis, the process of genetic algorithm optimization is the change process of objective function and fitness function. During the operation of the genetic algorithm, individuals with low fitness in the population are continuously eliminated and the number of eligible individuals will increase which are closer to the optimal solution of the problem. Then the optimal result of the problem, that is, the position coordinates of the BS can be obtained.

4 Algorithm Simulation and Result Analysis

4.1 Simulation Conditions

This section tests the performance of the previously proposed algorithm in a Gaussian noise environment, and simulates the algorithm with different measurement errors, cell radii, and the mobile terminal acquiring different numbers of position coordinates. The main parameter settings in the paper are as follows: a cellular structure with 3 base stations, the serving base station is BS₁ (0,0), and the remaining BS coordinates are: BS₂ (0,1000) and BS₃ (500√3,500). The actual position of the UE takes a random number within the coverage of the three base stations. TDOA measurement error is Gaussian distributed, whose mean value is 0 and the setting standard deviation (converted into distance) are 2 m, 4 m, 6 m, 8 m, and 10 m, respectively.

Genetic algorithm simulation parameters: population size $P = 200$, maximum termination evolution generation $G = 100$, cross rate $P_c = 0.3$, variation rate $P_m = 0.05$, parameter to determine the non-uniform variability $b = 4$.

4.2 Simulation Results and Analysis

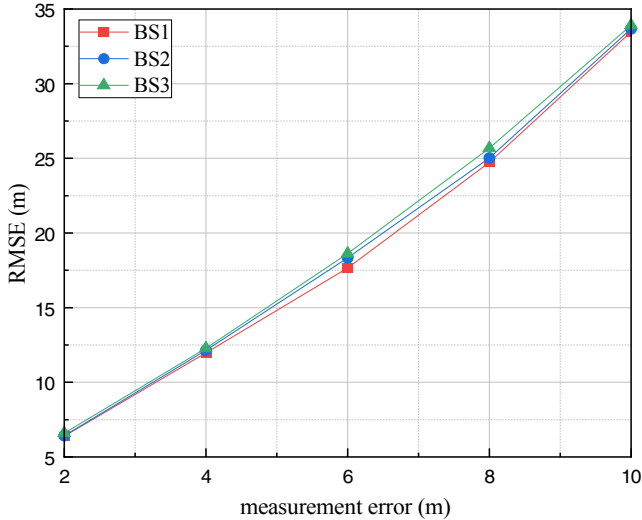


Fig. 2. Inverse calculation results with 3 positioning coordinates

As shown in Fig. 2, under Gaussian mode, the cell radius is 500m, the measurement error is taken between 2m to 10m and the UE uses three positioning positions to inversely calculate the coordinates of the BS. The simulation results show that the positioning error of base station inverse algorithm increases with the increase of measurement error. And since the UE positions are randomly distributed relative to the three base stations, the positioning performance of the three base stations is very close. From the positioning effect, the error values of the three base stations are within the acceptable range and demonstrates excellent positioning performance, so that the feasibility of the inverse algorithm is verified.

As shown in Fig. 3, under Gaussian mode, the cell radius is 500m, the measurement error is taken between 2m to 10m and the UE uses four and five positioning positions to inversely calculate the coordinates of the BS, respectively. And Fig. 4 shows the results using different numbers of UE positions. Compared with Fig. 2, the UE uses more positioning positions to calculate base station coordinates. The simulation results show that the positioning accuracy of the three base stations has been improved. From the perspective of positioning

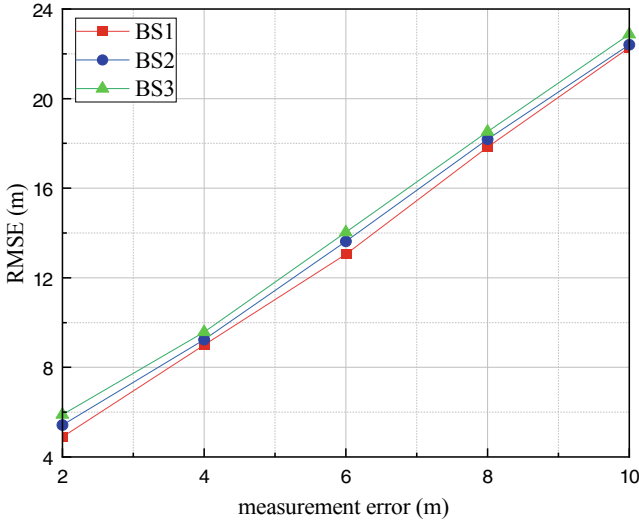


Fig. 3. Inverse calculation results with 4 positioning coordinates

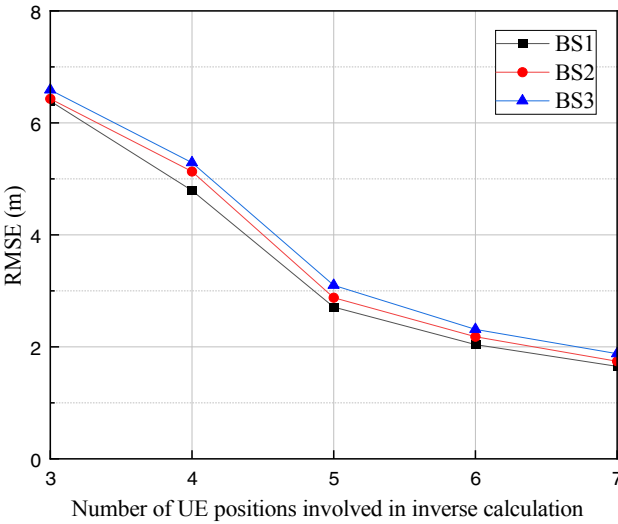


Fig. 4. Inverse calculation results with different number of positioning coordinates

effect, when the UE obtains more position coordinates for inverse calculation, the positioning performance of the algorithm can be improved.

As shown in Fig. 5, under Gaussian mode, the measurement error is taken to be 10 m, the cell radius ranges between 500 m and 2500 m and the UE uses four positioning positions to inversely calculate the coordinates of the BS. The simulation results show that the positioning errors of the three base stations all

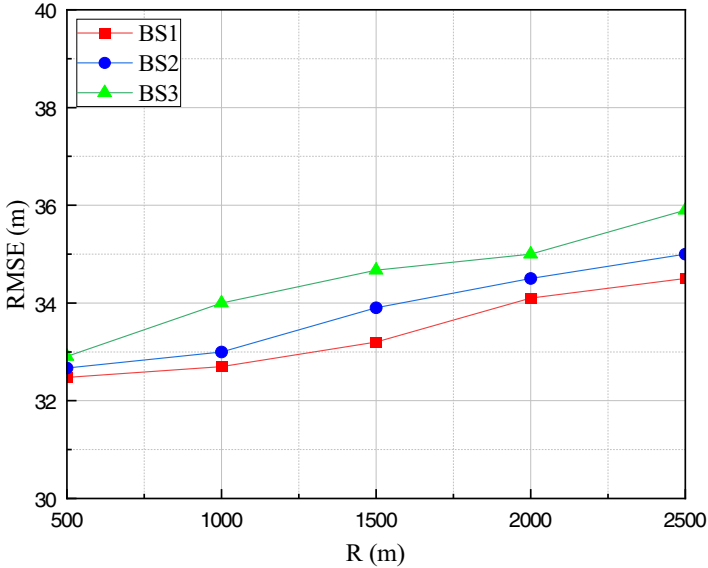


Fig. 5. Relation between standard error and cell radius

decrease as the radius decreases, the errors are all within the acceptable range and the error curve of our algorithm is in a stationary state which demonstrates excellent stability of the algorithm.

5 Conclusion

In order to improve the autonomy and security of the UE, in this paper, we propose an autonomous positioning algorithm for the UE in cellular networks. UE first uses TDOA measured values and multiple UE positioning results issued by the network side to inversely calculate the position of the BSs participating in the positioning, meanwhile the coordinates of these BSs will be obtained. After that, the UE can use the calculated BS position coordinates and TDOA measured values to calculate its own position independently. The simulation results show that: 1. The algorithm is feasible, and the error of the calculated base station position coordinates is within the acceptable range. 2. As the position coordinates of the participating algorithms obtained by UE increase, more accurate base station position coordinates can be obtained. 3. As the cell radius increases, the error curve of our algorithm is in a stationary state which demonstrates excellent stability of the algorithm. It can be seen from the analysis and simulation results that our algorithm is effective and also demonstrates stability and higher positioning accuracy.

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