Outage Probability for Device to Device and Cellular Heterogeneous Networks over Nakagami-m Channels

Yu Zhou[1](✉), Xiao Chen[2], Xuesong Shao[1], Fan Gao[1], Qixin Cai[1], and Yue Li[1]

[1] Marketing Service Center, State Grid Jiangsu Electric Power Co. Ltd., Nanjing, China
[2] State Grid Jiangsu Electric Power Co. Ltd., Nanjing, China

Abstract. Device-to-Device (D2D) communication has become an important component in future communications, because it has the advantages of offloading traffic from the base stations and reducing the distance between the transmitter and receiver. In this paper, we derive a closed-form expression for the outage probability in uplink D2D communication and cellular heterogeneous networks over Nakagami-m fading, where the fading parameter m can vary among the devices with any positive value. The analytical results are presented and verified the simulation results.

Keywords: Device-to-Device communication · Nakagami-m fading · Outage probability

1 Introduction

Nowadays, the explosive growth of communication devices led to an increasing demand for higher capacity, data rates and radio spectrum resources. Device-to-Device (D2D) communication has been seen as a key component in the fifth generation (5G) wireless network, since it allows two adjacent D2D-enabled users, denoted as a D2D pair, to directly establish communication link without base stations assistance or control. Therefore, underlay D2D in conventional cellular networks has the potential to reduce the transmission distance, improve spectrum frequency and offload the traffic from base stations. It can be seen as a two tier heterogeneous networks (HetNets) where one tier is composed of the users that communicate over the D2D link and the other tier is the users transmitting signals through the base station. In D2D cellular heterogeneous networks, time/frequency resources can be portioned or shared by the D2D tier and cellular tier [1]. To avoid the inter-tier interference, the cellular and D2D users use the orthogonal resources by dividing the uplink frequency band into two non-overlapping portions [2,3]. However, sophisticated resource allocation schemes are required for system implement [4]. In [5] and [6], D2D pairs are allowed

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to reuse the uplink resource of the cellular users. Compared with the orthogonal spectrum scheme, the shared frequency network is efficient in the network resources and more flexible to perform.

However, the shared D2D-cellular HetNets improve the spectrum utilization with the cost of introducing the interference from both the D2D tier and cellular tier. In recent years, the performance analysis of the D2D communication in cellular networks has been extensively investigated. The outage probability is one of the key performance metrics to model the D2D communication underlaying the cellular network. The performance analysis of D2D-cellular HetNets under Rayleigh fading channel has been well studied in the literature. Reference [7] provided analytical coverage probability expression for such D2D and cellular users under Rayleigh fading channels. Outage probability for the uplink D2D cellular network with the multiple antennas has been analyzed by considering the Rayleigh faded channel [8]. It demonstrated that the performance was improved by equipping the D2D users with multiple antennas. Recently, Nakagami-m fading model has been proposed for modeling differing line-of-sight conditions and short-range communication i.e., D2D communication links. Nakagami-m channel is a generalization fading model which incorporates other fading scenarios by setting different fading parameters, i.e., Rayleigh for $m = 1$, the Rician shadow fading distribution for $m = (K + 1)^2/(2K + 1)$ and no fading for $m = \infty$ as special cases. Reference [9] derived the outage probability in an interference-limited system and assumed the Nakagami-m channels for desired signals and Rayleigh channels for interference signals. Reference [10] and [11] analyzed coverage performance of heterogeneous networks over the Nakagami-m channels by assuming the fading parameter m was a positive integer. However, to the best of the authors knowledge, none of these previous studies accommodate thermal noise and the outage performance of D2D-cellular HetNets over Nakagami-m fading channel with arbitrary fading parameters has not been reported in the literature.

In this paper, we derive an analytical expression for the outage probability of the D2D and cellular heterogeneous networks, where the uplink frequency resources are shared between the D2D users and cellular users. And we assume that the typical D2D transmitter is equipped with multiple antennas. The D2D transmitters and cellular users are equipped with single omnidirectional antennas. Both the desired signals and co-channel interference experience the Nakagami-m fading with different fading parameters. Then the closed-form expression for the outage probability is derived for D2D links over Nakagami-m channels, which is valid both for integer and non-integer values of m. By using the outage probability as the performance metric, we investigate impact of system parameters on the D2D communication.

The paper is organized as follows: the system model and Nakagami-m channel model are proposed in Sect. 2. The outage probability over D2D links has been derived in Sect. 3. The Monte-Carlo simulations and analytical results are compared in Sect. 4. Finally, some conclusions are given in Sect. 5.

2 System Model and Channel Model

2.1 System Model

In this section, we consider a two-tier uplink heterogeneous networks where the D2D users and cellular users are placed in the area as shown in Fig. 1. By reusing the uplink frequency band of cellular networks, D2D pair can directly communicate without the base stations assistance. The received signal at any receiving node is assumed to be corrupted by desired signal, additive white Gaussian noise (AWGN) and interference from both D2D transmitters and cellular users. Here, we consider a multi-antennas D2D communication system, where the typical D2D transmitter is equipped with N_R antennas. Other D2D devices and the cellular users are equipped with a single antenna. The channel fading of desired link and interference link are considered to follow the non-identically independent Nakagami-m fading distribution. Through this paper, we consider a typical D2D receiver at the center. Denoting d_0 and $\boldsymbol{h_0}$ are the distance and the channel fading between the D2D transmit and receive nodes in a typical D2D pair. The received signal at the D2D receive node is given as

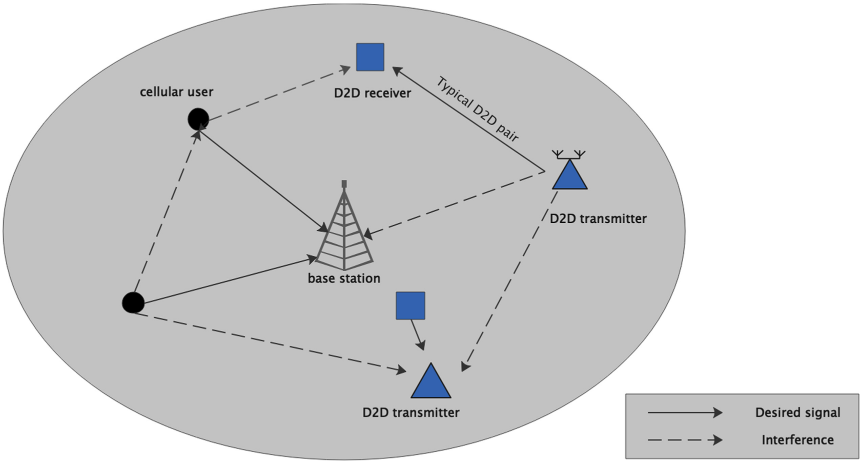

Fig. 1. System model of the D2D-cellular HetNets

$$y_0 = P_0 d_0^{-\alpha/2} \boldsymbol{h}_0^H \boldsymbol{x}_0 + \sum_{i=1}^{I} P_0 d_i^{-\alpha/2} \boldsymbol{h}_{D_i}^H \boldsymbol{x}_i + \sum_{j=1}^{J} P_1 d_j^{-\alpha/2} \boldsymbol{h}_{U_j}^H \boldsymbol{x}_j + \boldsymbol{n_0} \qquad (1)$$

where P_0 and I are the transmit power and number of the D2D transmitters, P_1 and J are the transmit power and number of cellular users, d_i and d_j are the distance from the D2D transmitters and cellular users to the typical D2D receiver, α is the path-loss parameter, $\boldsymbol{x_i}$ is the transmitted signal with

$\mathbb{E}[||x_i||^2] = 1$. $\boldsymbol{h_0} = \{h_{0,1}, h_{0,2}, \cdots, h_{0,N_R}\}$ is a $1 \times N_R$ vector and the envelope for each subchannel $||h_{0,n}||$ follows the i.i.d Nakagami-m distribution, denoted as $||h_{0,n}|| \sim Nakagami(m, \Omega)$. In addition, $\boldsymbol{h}_{D_i}$ and $\boldsymbol{h}_{U_j}$ are the $1 \times N_R$ channel vectors defining the channel fading between ith D2D transmitter and jth cellular users to the D2D receiver, i.e. $\boldsymbol{h}_{D_i} = \{h_{D_{i,1}, D_{i,2}, \cdots, D_{i,N_R}}\}$ and $\boldsymbol{h}_{U_i} = \{h_{U_{j,1}, U_{j,2}, \cdots, U_{j,N_R}}\}$. Similarly, we assume the envelope for each subchannel follows the Nakagami-m distribution, denoted as $||h_{D_{i,n}}|| \sim Nakagami(m_{D_i}, \Omega_{D_i})$ and $||h_{U_{j,n}}|| \sim Nakagami(m_{U_j}, \Omega_{U_j})$. n_0 is an $1 \times N_R$ vector of an i.i.d. zero mean complex AWGN with a variance σ_n^2.

From (1), the signal-to-interference-and-noise-ratio (SINR) at the typical D2D receiver is given as

$$\mathrm{SINR}_0 = \frac{P_0 d_0^{-\alpha} |\boldsymbol{h}_0|^2}{\sum_{i=1}^{I} P_0 d_i^{-\alpha} |\boldsymbol{h}_{D_i}|^2 + \sum_{j=1}^{J} P_1 d_j^{-\alpha} |\boldsymbol{h}_{U_j}|^2 + \sigma_n^2} \tag{2}$$

2.2 Channel Model

Since the element of fading channel vector $h_{0,n}$ follows the i.i.d Nakagami-m distribution, the fading power of each subchannel (equivalently squared Nakagami-m random variable) is considered to follow the Gamma distribution denoted as $||h_{0,n}||^2 \sim \mathcal{G}(m_0, \frac{\Omega_0}{m_0})$. The probability density function (PDF) of a Gamma random variable (RV) is expressed as [12]

$$f_{||h_{0,n}||^2}(x) = \frac{{m_0}^{m_0} x^{m_0 - 1}}{\Gamma(m_0) \Omega_0^m} \exp(-\frac{m_0}{\Omega_0} x) \tag{3}$$

Since the MRT used at the typical D2D receiver, the total fading power of N_R subchannels is given as [13]

$$\chi_0^2 = |\boldsymbol{h_0}|^2 = \sum_{n=1}^{N_R} ||h_{0,n}||^2, \tag{4}$$

where χ_0^2 is the summation of N_R subchannel gains. It is well known that the distribution of the sum of Gamma RVs can be approximated by single Gamma distribution [14]. By the moment matching method, the PDF of χ_0^2 is approximated by

$$f_{\chi_0^2}(x) \cong \frac{m_0^{m_0 N_R} x^{m_0 N_R - 1}}{\Gamma(m_0 N_R) \Omega_0^{m_0 N_R}} e^{-\frac{m_0}{\Omega_0} x}, \tag{5}$$

where $m_0 N_R$ is the shape parameter and Ω_0 / m_0 is the scale parameter of the Gamma RV χ_0^2. And the complementary cumulative distribution function (CCDF) of χ_0^2 is given by [15]

$$\begin{aligned} F_{\chi_0^2}(x) &= 1 - \frac{\gamma(m_0 N_R, m_0 x / \Omega_0)}{\Gamma(m_0 N_R)} \\ &= \sum_{k=0}^{\lfloor t \rfloor} \frac{U_k (m_0 x / \Omega_0)^k}{k!} \exp(-m_0 x / \Omega_0) \end{aligned} \tag{6}$$

where $t = m_0 N_R$, $\lfloor\cdot\rfloor$ is the floor function, $U_k = 1$ when $k = 0, 1, \cdots, \lfloor t \rfloor - 1$ and $U_k = t - \lfloor t \rfloor$ when $k = \lfloor t \rfloor$. Note that, (6) is valid to characterize the CCDF of Gamma distribution for both integer and non-integer shape parameter. And it makes the performance analysis tractable in multi-antennas D2D communication over Nakagami-m channels. Similarly, the fading gains from the ith interfering D2D transmitter and jth cellular users follow the Gamma distribution, denoted as $\chi^2_{D_i} = ||h_{D_i}||^2 \sim \mathcal{G}(m_{D_i}, \frac{\Omega_{D_i}}{m_{D_i}})$ and $\chi^2_{U_j} = ||h_{U_j}||^2 \sim \mathcal{G}(m_{U_j}, \frac{\Omega_{U_i}}{m_{U_j}})$, respectively [11].

3 Outage Probability

Outage probability is defined to be the SINR falls than a predefined threshold, i.e. $P[\text{SINR} < \gamma]$. For a typical D2D receiver, the outage probability is given as

$$\begin{aligned}
P_0 &= P[\text{SINR}_0 < \gamma] \\
&= P[\frac{P_0 d_0^{-\alpha} \chi_0^2}{\sum_{i=1}^{I} P_0 d_i^{-\alpha} \chi^2_{D_i} + \sum_{j=1}^{J} P_1 d_j^{-\alpha} \chi^2_{U_j} + \sigma_n^2} < \gamma] \\
&= 1 - P[\chi_0^2 > \frac{\gamma d_0^{\alpha}}{P_0}(I + \sigma_n^2)] \\
&\overset{(a)}{=} 1 - \mathbb{E}_I[\sum_{k=0}^{\lfloor t \rfloor} \frac{U_k s^k (I + \sigma_n^2)^k}{k!} \exp(-s(I + \sigma_n^2))] \\
&\overset{(b)}{=} 1 - \sum_{k=0}^{\lfloor t \rfloor} \frac{U_k s^k \exp(-s\sigma_n^2)}{k!} \sum_{n=0}^{k} \binom{n}{k} (\sigma_n^2)^{n-k} \mathbb{E}_I[I^n e^{-sI}], \qquad (7)
\end{aligned}$$

where $I = \sum_{i=1}^{I} P_0 d_i^{-\alpha} \chi^2_{D_i} + \sum_{j=1}^{J} P_1 d_j^{-\alpha} \chi^2_{U_j}$, $\binom{n}{k}$ is the binomial coefficient, $s = \gamma d_0^{\alpha} m_0 / P_0 \Omega_0$, (a) is derived from (6) and (b) is derived by the binomial expansion.

Now, we evaluate the PDF of the total interference I. Note that I is represented in terms of a series of weighted summations of Gamma RVs. As per the fact of weighted Gamma RV i.e. $X \sim \mathcal{G}(a, b)$ then $Y = cX \sim \mathcal{G}(a, cb)$. Since $\chi^2_{D_i}$ and $\chi^2_{U_j}$ are independent Gamma RVs, the distribution of the total interference received follows $I = \sum_{i=1}^{I} P_0 d_i^{-\alpha} \chi^2_{D_i} + \sum_{j=1}^{J} P_1 d_j^{-\alpha} \chi^2_{U_j} \sim \mathcal{G}(\beta, \lambda)$, where β and λ are the shape and scale parameters of the Gamma RV. Based on the second-order moment matching method, the parameters are given by [16]

$$\beta = \frac{(\sum_{i=1}^{I} P_0 m_{D_i} \Omega_{D_i} d_i^{-\alpha} + \sum_{j=1}^{J} P_1 m_{D_i} \Omega_{U_j} d_j^{-\alpha})^2}{\sum_{i=1}^{I} m_{D_i} (P_0 \Omega_{D_i} d_i^{-\alpha})^2 + \sum_{i=1}^{I} m_{U_j} (P_1 \Omega_{U_j} d_j^{-\alpha})^2} \qquad (8)$$

and

$$\lambda = \frac{\sum_{i=1}^{I} m_{D_i} (P_0 \Omega_{D_i} d_i^{-\alpha})^2 + \sum_{i=1}^{I} m_{U_j} (P_1 \Omega_{U_j} d_j^{-\alpha})^2}{\sum_{i=1}^{I} P_0 m_{D_i} \Omega_{D_i} d_i^{-\alpha} + \sum_{j=1}^{J} P_1 m_{U_j} \Omega_{U_j} d_j^{-\alpha}} \qquad (9)$$

With the shape and scale parameters in (8) and (9), we have

$$\begin{aligned}
&\mathbb{E}_I[I^n \exp(-sI)] \\
&= \int_0^\infty x^n \exp(-sx) \frac{x^{\beta-1}\exp(-\frac{x}{\lambda})}{\lambda^\beta \Gamma(\beta)} dx \\
&= \int_0^\infty \frac{x^{n+\beta-1}\exp(-(s+\frac{1}{\lambda})x)}{\lambda^\beta \Gamma(\beta)} dx \\
&= \frac{\lambda^n \Gamma(n+\beta)}{(1+s\lambda)^{n+\beta}\Gamma(\beta)},
\end{aligned} \tag{10}$$

Using the above expression, the outage probability defined in (7) is rewritten as

$$P_0 = 1 - \sum_{k=0}^{\lfloor t \rfloor} \sum_{n=0}^{k} \binom{n}{k} \frac{s^k \lambda^n U_k \exp(-s\sigma_n^2)\Gamma(n+\beta)}{k!\Gamma(\beta)(1+s\lambda)^{n+\beta}} \tag{11}$$

As in the case of integer parameter t, the outage probability is then simplified as

$$P_0 = 1 - \sum_{k=0}^{t-1} \frac{(s\lambda)^k \Gamma(k+\beta)}{k!\Gamma(\beta)(1+s\lambda)^{k+\beta}}. \tag{12}$$

4 Numerical Results and Analysis

In this section, we compares the analytical results of coverage probability for D2D link with the simulation results. The simulation results are derived by the Monte-Carlo simulation. Simulation parameter settings are given in Tabel 1.

Table 1. Simulation parameter settings

Parameter	Values
Cell radius R	500 m
Distance between the typical D2D pair d_0	30 m
Number of D2D transmitters I	4
Number of CUs J	4
Number of antennas at D2D transmitter N_R	3
Transmit power of cellular user P_1	28 dBm
Transmit power of D2D user P_0	25 dBm
Path loss exponent α	3.5
Thermal noise power σ_n^2	−174 dBm
Distance between interfering D2D users and typical D2D user	{40, 45, 50, 55} m
Distance between interfering cellular users and typical D2D user	{50, 80, 150, 200} m

Figure 2 shows the coverage probability of the D2D receiver with respect to SINR threshold in terms of different fading parameters. And we compare the analytical results with the Erlang approximation (denoted as EA) in [15]. From this illustration, we observe that the proposed expression in (11) closely match the corresponding simulation results compared to the Erlang approximation. Erlarng approximation is tight with the simulation results when the shaping parameter is an integer. We further observe that the outage probability of the D2D communication decreases as the fading parameter m_0 increases, due to the decrease in channel fading.

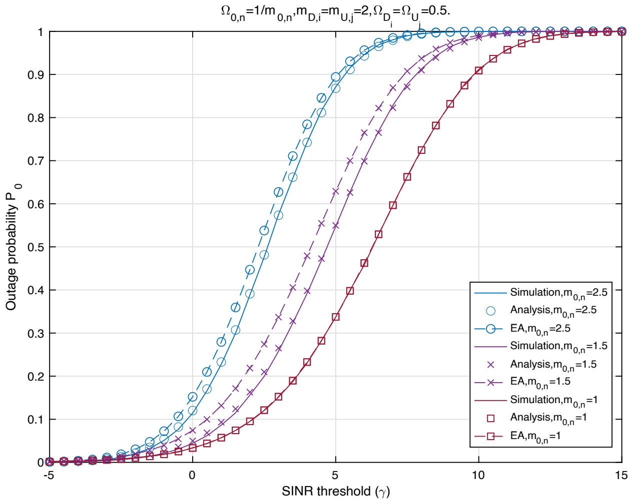

Fig. 2. Comparison the outage probability in terms of different fading parameters.

Figure 3 presents the outage probability with respect to the number of antennas at the receiver side N_R. Its clearly seen that the analytical results are very tight with the simulation results for different number of antennas, which implies that the approximation is reasonable. Furthermore, its clearly seen that, the outage probability decreases with the increasing of the number of antennas at the D2D receiver at low values of SINR. It demonstrates that the D2D with multiple antennas assure a great outage performance improvement.

Figure 4 presents the outage probability versus the average SINR for different threshold γ. One sees that average SINR in general degrades the outage performance due to increased desired signal. One also sees that the outage performance increase with the increase of SINR threshold.

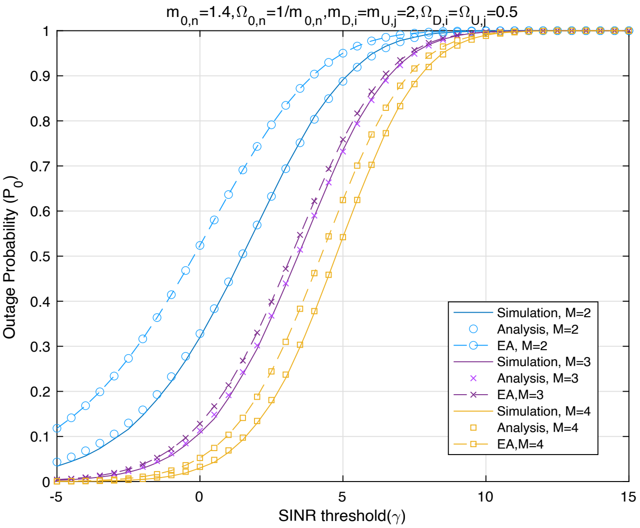

Fig. 3. Comparison the outage probability in terms of number of antennas at the D2D receiver.

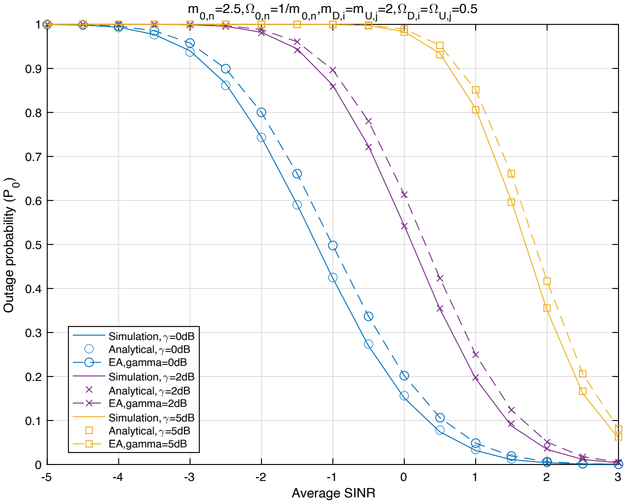

Fig. 4. Comparison the outage probability in terms of average SINR and threshold.

5 Conclusions

In this paper, outage probability for D2D links has been analyzed in uplink D2D communication and cellular HetNets. The closed-form expression for outage probability has been derived over the Nakagami-m fading. The expression is valid for both integer and non-integer values of fading parameters m and provides advantages in terms of less complexity in the numerical computations. Analytical results are very tight with the Monte-Carlo simulation. These results also show the impact of the number of D2D receiver antenna, Nakagami-m fading parameters and average SINR on outage performance.

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