Cooperative Source Seeking in Scalar Field: A Virtual Structure-Based Spatial-Temporal Method

Cheng Xu[1,2,3], Yulin Chen[1,2,3], Shihong Duan[1,2,3(✉)], Hang Wu[1,2,3], and Yue Qi[1,2,3]

[1] School of Computer and Communication Engineering, University of Science and Technology Beijing, Beijing, China
duansh@ustb.edu.cn
[2] Shunde Graduate School, University of Science and Technology Beijing, Beijing, China
[3] Beijing Key Laboratory of Knowledge Engineering for Materials Science, Beijing, China

Abstract. Source seeking problem has been faced in many fields, especially in search and rescue applications such as first-response rescue, gas leak search, etc. We proposed a virtual structure based spatial-temporal method to realize cooperative source seeking using multi-agents. Spatially, a circular formation is considered to gather collaborative information and estimate the gradient direction of the formation center. In terms of temporal information, we make use of the formation positions in time sequence to construct a virtual structure sequence. Then, we fuse the sequential gradient as a whole. A control strategy with minimum movement cost is proposed. This strategy rotates the target formation by a certain angle to make the robot team achieve the minimum moving distance value when the circular team moves to the next position. Experimental results show that, compared with state-of-the-art, the proposed method can quickly find the source in as few distances as possible, so that the formation can minimize the movement distance during the moving process, and increase the efficiency of source seeking. Numerical simulations confirm the efficiency of the scheme put forth. Compared with state-of-the-art source seeking methods, the iterative steps of our proposed method is reduced by 20%, indicating that the method can find the signal source with higher efficiency and lower energy consumption, as well as better robustness.

Keywords: Cooperative computing · Gradient estimation · Source seeking · Circular formation · Spatial-temporal information

1 Introduction

In the last decade, *cooperative source seeking* based on multi-agents (or multi-robots) has been drawing more and more attention and widely used in many

H. Gao et al. (Eds.): CollaborateCom 2020, LNICST 350, pp. 320–336, 2021.
https://doi.org/10.1007/978-3-030-67540-0_19

fields, such as oil exploration [1–3], odor source search [4], environmental monitoring [5,6], pollution detection [7], and first-response search and rescue (SAR) tasks [8–10], etc. The target signal source can generally be an electromagnetic signal, an acoustic signal, or a chemical or biological signal. For example, in the event of a gas leak, rescuers need to be dispatched to find the source of the leak. At the same time, the lives of the participating rescuers must be protected as much as possible to avoid causing serious safety accidents. For the sake of safety and efficiency, robots (also known as agents) replace search and rescue personnel to enter dangerous areas and perform SAR operations. For example, when looking for a missing tour pal with a positioning signal device in the wild, in the face of complex terrain conditions, drones or unmanned vehicles can be used to perform search and rescue missions. To sum up, a source search algorithm with higher accuracy and consistency is of practical significance in many aspects, such as industrial production, military security, and civil security.

Many existing signal source seeking algorithms have evolved from the inspiration of biological behavior. Based on observations of spiny lobster behaviors, Consi et al. [11] proposed an agent that simulates the behavior of lobsters. In [12], inspired by silkworm moths, Kuwana et al. proposed a sourcing method that imitated the behaviors of silkworm moths seeking odor sources. Russell et al. [13] also verified the applicability of the above-mentioned algorithm to chemical source tracing in airflow environments.

However, the aforementioned biometric methods are based on the inspiration of individual biological behavior and still face obvious limitations. They rely on the information generally collected by only one single agent, such as in [11] and [12]. The agent uses its own sensor to perform the sourcing task, which lacks information interaction with other agents, making it difficult for researchers to apply it to scenarios where multiple agents work together. Although the efficiency of source seeking methods has been improved, the limitations of single agents will lead to the insufficient information collection and low robustness [11,12].

Many organisms in the natural world forage and reproduce through group behaviors [14–16]. Clustered organisms can efficiently find food and avoid natural enemies through an individual division of labor and information exchange. It has the characteristics of high efficiency and strong adaptability. Through the observation of biological populations, researchers proposed multiple swarm optimization algorithms, such as ant colony [14], bee colony [15], and wolf colony [16]. In 1992, Marco Dorigo [14] proposed an ant colony optimization algorithm by simulating the principles of ant social division of labor and cooperative foraging. Based on the inspiration of bee colonies to find nectar sources, Karaboga proposed the Artificial Bee Colony (ABC) [15] in 2005, which has the advantages of high accuracy and fewer control parameters. The algorithm was successfully applied to many fields such as artificial neural network training and combination optimization [17]. In [16], Wu et al. were inspired by the cooperative hunting behavior of wolves and proposed the Wolf Pack Algorithm (WPA). The swarming behavior of fish schools has also attracted researchers' attention. In [22,23], Wu et al. and Said Al-Abri et al. were inspired by the behavior of fish swarm clusters to study a cooperative collaborative mobile strategy called *acceleration-deceleration*. It can simulate the swarm behavior of fish schools avoiding light, and move the agent team to the position of the signal source. Kennedy and Eberhart et al. [18] firstly proposed Particle Swarm Optimization (PSO), which was originally proposed to simulate the motion of bird swarms. Jatmiko et al. [19,20] applied the particle swarm optimization algorithm to the field of odor source localization, which uses multiple agents to find stationary odor sources. Li et al. [21] proposed an improved probabilistic particle swarm optimization algorithm for source seeking in a ventilated environment.

As intelligent robots and sensors work very differently from real creatures, biological behavior-inspired methods also have limitations. Besides, uncoordinated agents can cause resource competition and conflicts, which affects the overall performance of multiple agents.

Gradient-based methods are also widely considered in source seeking applications, which can be mainly divided into single-agent methods and multi-agents cooperative ones. As for a single agent, in [24–26], researchers used random gradient estimation to make the agent randomly move in the signal field, measure the spatial information of the signal field, and calculate the gradient direction of the signal field. Krstic et al. [24,25] applied extreme value search control to make a single agent move to the local signal maximum in a noise-free signal field. Anatasov et al. [26] made a single agent calculate the gradient by random movement, and drove the agent to signal source with gradient information. However, the above-mentioned single-target-based gradient source seeking method still faces the following main problems:

1) Random gradient estimation can avoid local extremes to a certain extent, but the agent needs to constantly move back and forth to measure the signal strength, so as to calculate the gradient. This may lead to the inefficiencies of searching process.
2) The sudden failure of the single robot may lead to failure of the whole source seeking task, which means a low fault tolerance and robustness.

Due to above-mentioned drawbacks, more and more researchers are focusing on multi-agent cooperative source seeking. Petter Ögren et al. [27] used a coordinated movement strategy to make the agent team form a sensor network, and drive the team to find a signal source by the least square method. Zhu et al. [28] utilized the $leader - follower$ strategy combined with least squares to calculate the gradient direction. In [29], Li et al. used the method of least squares estimator to enable the agent team to collaboratively calculate the gradient. The method proposed by Ruggero Fabbiano et al. in [30] enables the agents to sustain a circular formation by maintaining the same relative angle, and drive the agent team to the signal source with gradient descent algorithm. In [31,32], Lara-Brinon et al. proposed a circular formation method, which enables the team to maintain a uniformly distributed circular formation. With use of this means, agents calculate the gradient and drive themselves to the signal source.

However, the above-mentioned cooperative source seeking methods take the agent-formation into consideration, but only make use of the spatial information of the scalar signal field. They do not effectively use the information in the time sequence alongside the gradient direction. Signal strength measurement and intelligent formation control will inevitably have errors. The source seeking results obtained by gradient estimation using error signals and position information are often of poor accuracy.

In this paper, we aim to propose a source seeking method that utilize both spatial and temporal information in the scalar signal field, which can improve the source seeking efficiency. The rest of this paper is organized as follows. Section 2 puts forward the specific definition of a cooperative source seeking problem. Section 3 focuses on the details of a proposed virtual structure-based method. Section 4 demonstrates the experimental verification and analysis. Conclusions are drawn in Sect. 5.

2 Problem Formulation

The problem of source seeking mainly refers to the searching of signal source the unknown scalar fields. In this section, we proposed a virtual structure-based method with the fusion of spatial and temporal information collected by agents in formation. The agents advance along the gradient descent direction of the signal field and finally reach the position of the signal source.

We assume that the agent team moves in a two-dimensional space. For each agent i in the team, its dynamic equations could be denoted as follows:

$$\dot{p}_i = v_i \tag{1}$$

$$\dot{v}_i = u_i \tag{2}$$

where p_i is the position vector in the two-dimensional plane, v_i is the velocity vector and u_i is the acceleration input.

The nodes are collected into a vertex set $\mathcal{V}$, while links between nodes are collected in an edge set $\mathcal{E}$. Communication and measurements between nodes are bidirectional, so that $(\mathcal{V}, \mathcal{E})$ forms an undirected graph $\mathcal{G}$. The agents can communicate with each other within the communication topology and generally include the following information: the coordinates, the speed, and the measured signal strength information, etc. For simplicity, we assume that the communication between robot pairs are bidirectional and fully connected, and any pair of agents can exchange information with each other. The performance of the proposed algorithm under limited communication conditions is not within the scope of this paper and left for further studies.

The scalar signal field distribution function is represented as $z(p) : \mathbb{R}^2 \to \mathbb{R}$, which does not change over time. Its independent variable is coordinates of the sampling points denoted as p, and the signal strength reaches a maximum value at the source location p_s.

3 Virtual Structure Based Method

In this section, we detailed the proposed virtual structure based source seeking method, which combines the spatial and temporal information of the signal strength in the signal field. We firstly give a brief introduction to circular formation based collective gradient estimation criterion, so as to utilize the spatial distribution of the signal field. Then, we fuse the temporal information of the signal field by sequential sampling. Finally, we recursively perform the above steps, calculating the direction of the gradient until we find the source.

3.1 Gradient Estimation with Spatial Information

In order to better exploit spatial information in the signal field for gradient estimation, we considered a circular formation for multi-agents. A vector is formed from each agent to the center of the formation. The uniform and symmetric distribution of the circular formation makes the sum of these vectors a zero, which means we could make use of the individual measurement of each agent to accomplish spatial information fusion to estimate the gradient as a whole.

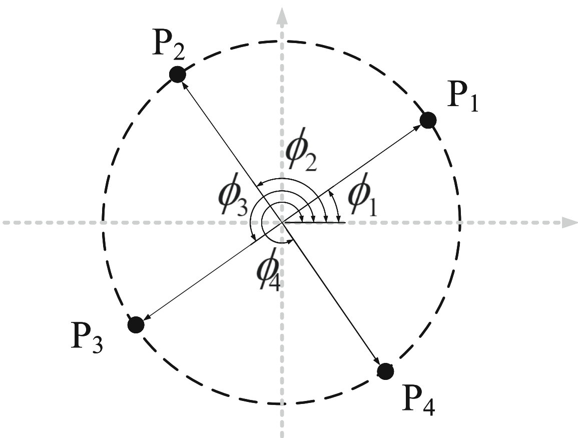

Fig. 1. The robots are evenly distributed on circular formation

We consider a robot team consisting of N agents, and they are organized in a circular formation. These N robots are evenly distributed on a circle with radius R, each of whose coordinates is represented as $p_i = (x_i, y_i)$, where x and y are respectively the abscissa and ordinate of robot i. A typical example is shown as Fig. 1, in which condition N is set as 4. The agents in the team are uniformly distributed on a circle with a radius R and a center position of $c(x_c, y_c)$. Therefore, the ith robot among the evenly distributed agents could be observed as a position p_i in the circular formation, which could be formulated as follows:

$$p_i = c + RD(\phi_i) \tag{3}$$

where R is the radius of the circular formation, $D(\phi_i)$ is the direction function of the ith agent in the circular formation, and $\phi_i = \phi_0 + \frac{2\pi i}{N}$ represents the azimuth angle of the ith agent, which could be referred to Fig. 1. It could be easily obtained that $D(\phi_i) = (\cos\phi_i, \sin\phi_i)$.

All agents in this circle formation are full-connected. The swarm agent team can approximate the signal field gradient through the weighted average of the signal strength collected by each individual agent [31], so as to achieve the purpose of spatial information fusion. Consider each agent measures the signal strength $z(p_i)$ at its current position $p_i(x_i, y_i)$ in the working space W. The gradient direction of the circular formation center c is denoted as $\widehat{\nabla} z(c)$, which could be calculated by combining the measured values of multiple agents around the circular formation center c with a radius R, namely

$$\frac{2}{NR^2}\sum_{i=1}^{N} z(p_i)(p_i - c) = \widehat{\nabla} z(c) + o(R) \tag{4}$$

where N is the number of agents in formation, and the approximation error term $o(R)$ is bounded by

$$\|o(R)\| \le \lambda_{max}(H_z)R \tag{5}$$

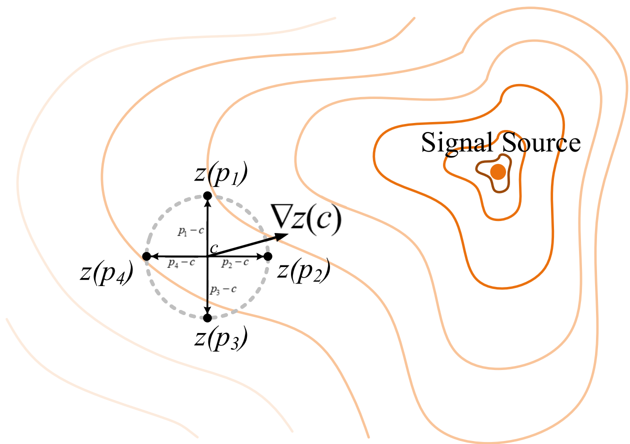

Fig. 2. Gradient estimation of circular formations

Proof: We assume that the agent team is evenly distributed in the circular formation, then it could be easily concluded that $\sum_{i=1}^{N}(p_i - c) = 0$. Using the first-order Taylor expansion of each measurement $o_i(R)$ about the point

c and recalling that $||p_i - c|| = R$, then the following equation holds for all $i = 1, 2, ..., N$:

$$z(p_i) - z(c) = \nabla z(c)^{\mathrm{T}}(p_i - c) + o_i(R) \tag{6}$$

where $o_i(R)$ denotes the remainder of the Taylor expansion. Multiplying Eq. 6 by $\frac{2}{D^2 N}(p_i - c)$ and averaging the sum, we get

$$\frac{2}{ND^2}\sum_{i=1}^{N} z(p_i)(p_i - c) + \frac{2}{ND^2}\sum_{i=1}^{N} z(c)(p_i - c) =$$
$$\frac{2}{ND^2}\sum_{i=1}^{N} \nabla z(c)^{\mathrm{T}}(p_i - c)(p_i - c) + \frac{2}{ND^2}\sum_{i=1}^{N} o_i(R)(p_i - c).$$

Since the agents are uniformly distributed along a fixed circle, then we have $\sum_{i=1}^{N}(p_i - c) = 0$ and thus

$$\frac{2}{ND^2}\sum_{i=1}^{N} z(p_i)(p_i - c) =$$
$$\frac{2}{ND^2}\sum_{i=1}^{N} [(p_i - c)(p_i - c)]\nabla z(c)^{\mathrm{T}} + o(R),$$

where $o(R) = \frac{2}{ND^2}\sum_{i=1}^{N} o_i(R)(p_i - c)$. We analyze the second term of the previous equation using Eq. 4 to express the position of the agents p_i to obtain

$$\sum_{i=1}^{N}(p_i - c)(p_i - c)^{\mathrm{T}} = \sum_{i=1}^{N} D(\phi_i)D(\phi_i)^{\mathrm{T}}$$
$$= R^2 D(\phi_0)(\sum_{i=1}^{N} D(2\pi i/N)D(2\pi i/N)^{\mathrm{T}})D(\phi_0)^{\mathrm{T}}$$
$$= R^2 D(\phi_i)\sum_{i=1}^{N}\begin{bmatrix}\cos^2(2\pi i) & 0.5\sin(4\pi i)\\ 0.5\sin(4\pi i) & \sin^2(2\pi i)\end{bmatrix} D(\phi_i)^{\mathrm{T}}$$
$$= R^2 D(\phi_0)(\frac{N}{2}I_2)D(\phi_0)^{\mathrm{T}} = \frac{NR^2}{2}I_2.$$

Since $\cos^2\phi = 0.5(1 + \cos(2\phi)), \sin^2\phi = 0.5(1 - \cos(2\phi))$, and $\sum_{i=1}^{N}\cos(2\frac{2\pi i}{N}) = \sum_{i=1}^{N}\sin(2\frac{2\pi i}{N}) = 0$ for $N > 2$, where $I_2 \in \mathbb{R}^{2\times 2}$represents the identity matrix. Thus, the equality of (4) is satisfied. Thanks to the Taylor's Theorem cite each remainder $o_i(R)$ satisfies the inequality

$$|o_i(R)| \leq \frac{1}{2}\lambda_{max}(H_z)\left\|p_i - c\right\|^2, \forall i.$$

Therefore, the function $o_i(R)$ can be bounded as

$$\|o_i(R)\| \leq \frac{2}{R^2 N} |o_i(R)| \|p_i - c\| \leq \lambda_{max}(H_z) R.$$

■

As shown in Fig. 2, the gradient direction calculated by Eq. 4, incorporates the signal strength information of N agents at different spatial positions, so that the agent team can effectively estimate the gradient direction.

3.2 Gradient Estimation with Temporal Information

The above-mentioned gradient estimation method with circular formation, makes full use of the spatial information in the cooperative network. To some extent, it can avoid the noise of individual source detection, and improve the source seeking accuracy and robustness. However, existing studies, including the spatial fusion method in the above section, do not take the time-series information into consideration. It is with this in mind that in this section, we proposed a virtual structure-based method to reduce the influence of noise on the gradient estimation by introducing time-sequential information to the circular formation. Numerical simulations confirm the efficiency of the scheme put forth.

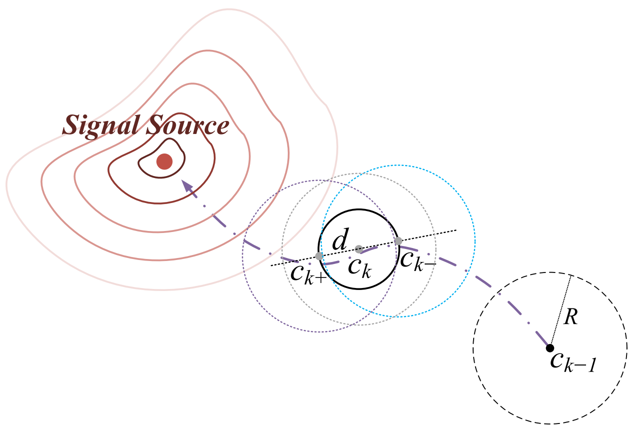

Fig. 3. Illustration of two-point gradient estimation

To integrate the time-sequential, we simulate two virtual positions (denoted as c_{k+} and c_{k-} respectively) where the agents in formation may appear. Then, we fuse these two estimations to obtain more accurate outputs. The detailed calculation process is listed as follows.

Firstly, when the agent team calculates the next formation center in time slot k (i.e., the target position c_k), we randomly select a point c_{k-} on the formation circle, centering at the target position c_k as the center, with a diameter of d, as shown in Fig. 3.

Then, as shown in Fig. 4 alongside the diameter we obtain another intersect c_{k+}. The circular formation passed these two points in succession, and the gradient directions of the circular team at these two points are respectively calculated. The weighted average of these two is used as the next gradient estimation, namely

$$\widehat{g}_k = \frac{1}{2}\left(\nabla z(c_{k-}) + \nabla z(c_{k+})\right) \tag{7}$$

Finally, the center position c_{k+1} of the circular agent team in $(k+1)$ *th* round is given by

$$c_{k+1} = c_k - a_k \widehat{g}_k \tag{8}$$

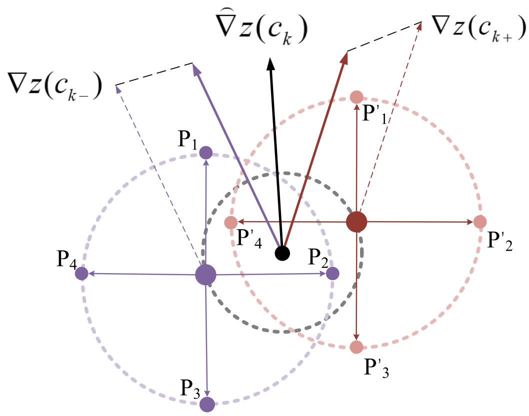

Fig. 4. Illustration of virtual structure based method.

where a_k is the step coefficient. The agent team continuously generates three series of points $\{c_{k-}, c_k, c_{k+}\}$ through above steps, and gradually moves to the vicinity of the signal source.

As for the step coefficient, it is generally denoted as

$$a_k = \frac{a}{(k+1+s)^\alpha}, \quad \mathrm{k} = 0, 1, ... \tag{9}$$

where a is a positive constant. s is a stability factor, which makes the algorithm have a large step size in the early iteration without causing instability, and should be set to 5% or 10% of the expected iterations of the algorithm. α controls the attenuation rate of the gain, and should be set to $\alpha = 0.602$, as suggested in [33].

However, we need to modify a_k. Because if the numerator a is constant, the gain coefficient a_k decreases monotonically, which is not desirable. As the step length of the agent will gradually decrease, it may be trapped in where the gradient estimation value is small. In our proposed method, we use a variable instead, which is inversely proportional to the magnitude of the gradient estimate. When the gradient estimation value is large, the agent travels in smaller steps; if the magnitude of the gradient estimation decreases, the gain coefficient increases by increasing the step size. Even if the signal field is very flat, the agent can perceive the gradient by increasing the step size. According to the above principles, the expression of a_k is formulated as

$$a_k = \frac{\mathrm{r} \times (1+s)^{\alpha}}{\frac{1}{\omega}\sum_{j=k-\omega+1}^{k}\frac{1}{n}\|\widehat{g}_j\|} \tag{10}$$

where r is the greedy coefficient, and the larger the value of r, the larger the step size coefficient of each step.

3.3 Control Strategy with Minimal Moving Cost

An algorithm based on gradient descent can drive the agent team to the target signal source position. However, without a good control strategy, the gradient information cannot be well used by the agents. In order to more efficiently complete the source seeking task, we propose a minimal moving distance control strategy.

During the movement of the agent team, the control strategy in this paper does not require the agent to maintain a fixed circular formation all the time. It aims to minimize the overall movement distance from the start point to the source. To achieve the control goal, after the next formation center coordinates are calculated, the circular agent team can rotate an arbitrary angle around the formation center, and then we use optimization methods to find the angle that minimizes the moving distance.

We denote the distance that the ith agent need move from current position p_i to the next as $L(i) = \|p_i - {p'}_i\|$, where p'_i is the next step position of agent i. Our purpose is to minimize the overall moving cost of the agent team, so for the robot movement strategy we display as

$$L(\mathcal{P}) = \underset{{p'}_i \in \mathcal{P}}{arg\,min} \sum_{i}^{N} \|p_i - {p'}_i\| \tag{11}$$

where $\mathcal{P} = \{p'_1, ..., p'_N\}$.

To sum up, based on our proposed spatial-temporal source seeking method, together with the control strategy, the general seeking process could be formulate as displayed in Algorithm 1.

4 Experimental Verification and Analysis

In order to verify the effectiveness, we compared our method proposed with two state-of-the-art methods [15,21] in the numerical experiments. The method in

Algorithm 1. Virtual Structure based Method

Input: Initialize the coordinates of formation center c_0 and signal source p_s, initialize iteration indicator $k = 1$.

Output: Estimated position c

Calculate the next position c_1 at initial position c_0 using (9), (11) and (10), then sample c_{1-} and c_{1+} on a circle of radius R along an arbitrary diameter;
Repeat
Move the agents to the position c_{k-} and c_{k+} successively considering control strategy with minimal moving cost by optimizing (11).
Calculate the gradients $\nabla z(c_{k-})$ and $\nabla z(c_{k+})$;
Calculate calculate the next target position c_{k+1} using (9), (11) and (10);
Sample $c_{(k+1)-}$ on the circle centered at c_k;
The other point on the diameter of the circle is taken as $c_{(k+1)+}$, which is the opposite of $c_{(k+1)-}$ on the circle centered at c_k.
if $\|c_{(k+1)-} - c_{k+}\| > \|c_{(k+1)+} - c_{k+}\|$ **then**
  $c_{(k+1)+} \leftarrow c'_{(k+1)}$, $c_{(k+1)-} \leftarrow c''_{(k+1)}$;
end if
Untill $||c_k - p_s|| < 1$
Let $c \leftarrow c_k$;
return c

[26] has only one robot for the source seeking task, while the method in [31] presented a circular team of four agents for source seeking.

A scalar signal source simulates a working space W with the coordinates of signal source $p_s = (5m, 25m)$. The collective information, as well as signal strength in this experiment, received by each agent in the working space is given by Eq. 12. The overall signal source model used in the experiment is denoted as follows [26]:

$$z(p) = -20.05 - 20\log_{10} \|p - p_s\| - N \tag{12}$$

where p is the current position of the robot, p_s is the position of the signal source, and N is the signal noise [15]. Thereinto, $N = \sqrt{\alpha^2 + \beta^2}$, and α and β both have normal distributions respectively, $\alpha \sim \mathcal{N}(\nu \cos\theta, \sigma^2)$, $\beta \sim \mathcal{N}(\nu \sin\theta, \sigma^2)$, σ is the standard deviation, and ν and θ are related-factors with the mean of the distribution [26]. The initial positions of the center of the robot and the robot team are $p_{init} = (25m, 5m)$. This article conducts experimental verification and analysis under the above experimental conditions. In the experimental verification and analysis part, we compare the method proposed in this article with the methods in [26] and [31]. Numerical simulations confirm the efficiency of the scheme put forth.

4.1 Typical Experimental Results

As it could be seen from Fig. 5, the two comparative methods are more easily affected by signal noises, as their trajectories (green and magenta ones) fluctuated more heavily and are more tortuous. In contrast, the trajectory curve

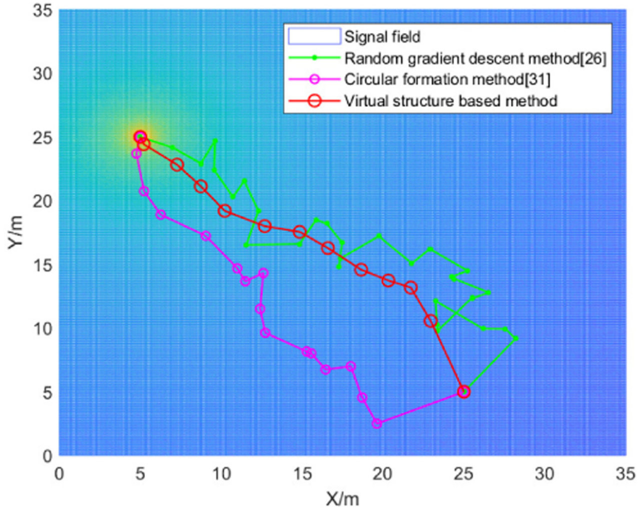

Fig. 5. Comparison of trajectories of three methods. The red trajectory is the trajectory of the robot formation center of the virtual structure based method proposed in this paper. The magenta trajectory is the trajectory of the center of the robot formation of the circular formation, and the green trajectory is the trajectory of the single robot center in the random method.

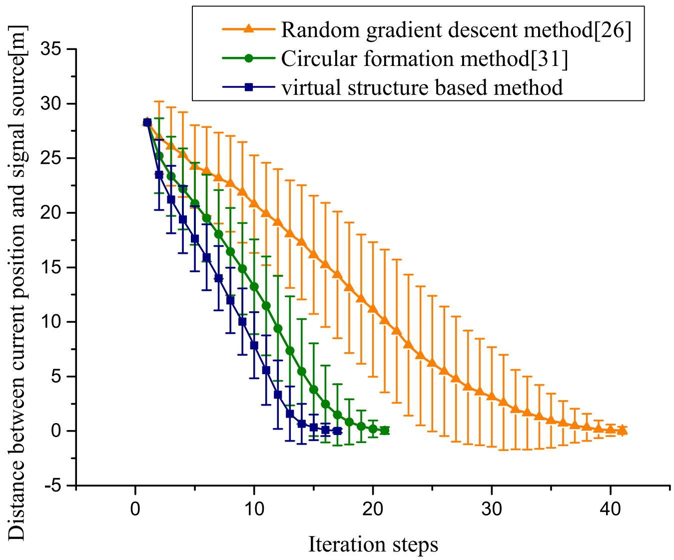

Fig. 6. The distance from the center of the robot to the source varies with the number of iteration steps.

obtained by the method described in this paper (red one) is smoother and simpler, which implicit that our approach is more efficient and robust to various changing signals. The method proposed in this paper reduces the interference of noise on the gradient calculation, and makes the trajectory more consistent with the direction of the gradient increase in the signal field. This shows that our proposed virtual structure based method has better stability and anti-noise characteristics.

Furthermore, the two comparative methods, as well as our own proposed one, are performed 100 times in the same experimental scenario. The signal field used is given by Eq. 12. For the generated noise term R, we make its parameters to be $\nu = 2$, $\theta = \frac{\pi}{3}$, $\sigma = 1$. These parameters are used to generate the experimental signal field. The averages of 100 experimental results are taken as the final experimental output. The variance of calculated total movement distances are drawn as error bars in Fig. 6, which demonstrates the detailed the trend of distance variance along with the iteration steps.

When the distance between the source and the agents center approaches to 0, we take it as the agents in formation successfully find the source. The speed of finding signal sources is one of the important indicators to measure the efficiency of the algorithm. The fewer iteration steps that the agents take to reach the signal source, the faster the distance converges to zero, as well as the higher the algorithm's efficiency.

Figure 6 shows the distance between the current agent formation center and the signal source, along with the number of iteration steps. It can be seen that our proposed virtual structure-based method convergences faster than comparative methods. The circular formation method that is superior to the [31] is also far superior to the random approximation method of [26], indicating that the cooperative estimation among agents outperforms independent work, and could solve the source seeking problem in a more efficient way.

4.2 Cumulative Distances with Noise Variance

The cumulative distances represent the total movement distances made by the agents, traveling from the starting position to the signal source. The lower the cumulative distances, the shorter the distances the robots move to find the signal source, which indicates that the source seeking method is more efficient.

The control variable of this experiment is the noise variance of the signal field, that is, the noise term N, in Eq. 12. We vary the noise variance of the signal field, and then compare the differences of the cumulative distances of the agents calculated by the three different source seeking methods. Let $\sigma = \{1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5\}$. In different signal fields generated by different noise parameters σ, we performed three algorithms 100 times in various signal fields, and then recorded the averages and variances of each experiment.

Figure 7 shows the robot's cumulative distances of all three methods as the noise variance of σ changes. The error bars in the figure are given by the variance of 100 experimental results. It can be seen from Fig. 7 that our proposed virtual structure-based method has significantly lower cumulative distances than those in [26] and [31] under the same noise condition. The virtual structure-based method enables the agents to find the signal source with a shorter movement distance, and is of much higher search efficiency.

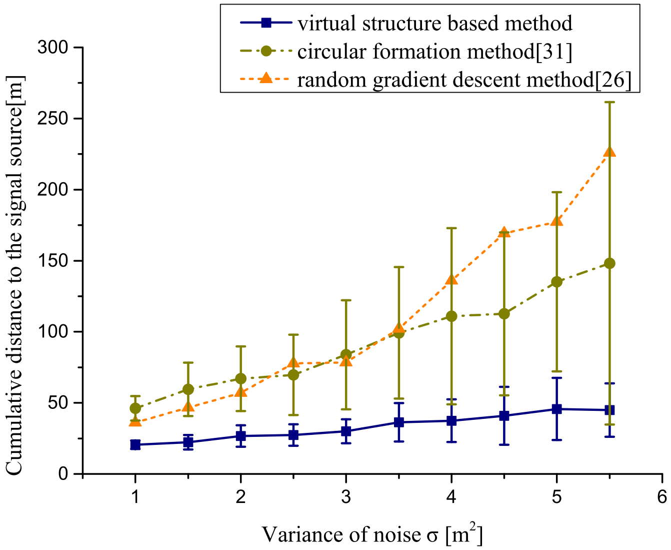

Fig. 7. Cumulative distance changes with noise variance σ.

On the other hand, as the noise variance σ increases, the curves of the three methods all have an upward trend. The growth rate of the proposed virtual structure-based method is significantly lower than that of the others, and the gap with the other two methods is getting wider and wider. It indicates that the proposed method is significantly more robust to noise interference than the methods in [26] and [31]. In addition, the amplitude of the error bars of virtual structure-based method is also smaller, which means more stable and robust to noise characteristics.

5 Conclusions

In this paper, we proposed a virtual structure-based method for multi-robot cooperative source seeking, in order to fuse both spatial and temporal information in the scalar field. Compared with state-of-the-art multi-robot collaborative methods, our proposed method enables to find signal sources with smaller iteration steps and cumulative distances, which can effectively reduce task overhead and improve efficiency. Our future work will focus on reducing the formation error of the robot team, which may provide a more accurate gradient estimation with circular or arbitrary formation.

Acknowledgments. This work is supported in part by National Postdoctoral Program for Innovative Talents under Grant BX20190033, in part by Guangdong Basic and Applied Basic Research Foundation under Grant 2019A1515110325, in part by Project funded by China Postdoctoral Science Foundation under Grant 2020M670135, in part by Postdoctor Research Foundation of Shunde Graduate School of University of Science and Technology Beijing under Grant 2020BH001, and in part by the Fundamental Research Funds for the Central Universities under Grant 06500127 and FRF-GF-19-018B.

References

1. Waleed, D., et al.: An in-pipe leak detection robot with a neural-network-based leak verification system. IEEE Sens. J. **19**(3), 1153–1165 (2019)
2. Berjaoui, S., Alkhatib, R., Elshiekh, A., Morad, M., Diab, M.O.: Free flowing robot for automatic pipeline leak detection using piezoelectric film sensors. In: International Mediterranean Gas and Oil Conference (MedGO). Mechref 2015, pp. 1–3 (2015)
3. Watiasih, R., Rivai, M., Penangsang, O., Budiman, F., Tukadi, Izza, Y.: Online gas mapping in outdoor environment using solar-powered mobile robot. In: 2018 International Conference on Computer Engineering, Network and Intelligent Multimedia (CENIM), Surabaya, Indonesia, pp. 245–250 (2018)
4. Che, H., Shi, C., Xu, X., Li, J., Wu, B.: Research on improved ACO algorithm-based multi-robot odor source localization. In: 2018 2nd International Conference on Robotics and Automation Sciences (ICRAS), Wuhan, pp. 1–5 (2018)
5. Cao, X., Jin, Z., Wang, C., Dong, M.: Kinematics simulation of environmental parameter monitor robot used in coalmine underground. In: 2016 13th International Conference on Ubiquitous Robots and Ambient Intelligence (URAI), Xi'an, pp. 576–581 (2016)
6. Shin, H., Kim, C., Seo, Y., Eom, H., Choi, Y., Kim, M.: Aerial working environment monitoring robot in high radiation area. In: 2014 14th International Conference on Control, Automation and Systems (ICCAS 2014), Seoul, pp. 474–478 (2014)
7. Shin, D., Na, S.Y., Kim, J.Y., Baek, S.: Fish robots for water pollution monitoring using ubiquitous sensor networks with sonar localization. In: 2007 International Conference on Convergence Information Technology (ICCIT 2007), Gyeongju, pp. 1298–1303 (2007)
8. Kanwar, M., Agilandeeswari, L.: IOT based fire fighting robot. In: 2018 7th International Conference on Reliability, Infocom Technologies and Optimization (Trends and Future Directions) (ICRITO), Noida, India, pp. 718–723 (2018)
9. Chen, X., Zhang, H., Lu, H., Xiao, J., Qiu, Q., Li, Y.: Robust SLAM system based on monocular vision and LiDAR for robotic urban search and rescue. In: IEEE International Symposium on Safety, Security and Rescue Robotics (SSRR), Shanghai 2017, pp. 41–47 (2017)
10. Denker, A., İşeri, M.C.: Design and implementation of a semi-autonomous mobile search and rescue robot: SALVOR. In: International Artificial Intelligence and Data Processing Symposium (IDAP), Malatya, pp. 1–6 (2017)
11. Consi, T., Atema, J., Goudey, C., Cho, J., Chryssostomidis, C.: AUV guidance with chemical signals. In: Proceedings of the 1994 Symposium on Autonomous Underwater Vehicle Technology, AUV 1994, pp. 450–455. IEEE (1994)

12. Kuwana, Y., Nagasawa, S., Shimoyama, I., Kanzaki, R.: Synthesis of the pheromone-oriented behaviour of silkworm moths by a mobile robot with moth antennae as pheromone sensors1. Biosens. Bioelectron. **14**(2), 195–202 (1999)
13. Russell, R.A., Bab-Hadiashar, A., Shepherd, R.L., Wallace, G.G.: A comparison of reactive robot chemotaxis algorithms. Robot. Auton. Syst. **45**(2), 83–97 (2003)
14. Dorigo, M., Maniezzo, V., Colorni, A.: Ant system: optimization by a colony of cooperating agents. IEEE Trans. Syst. Man Cybern. Part B (Cybern.) **26**(1), 29–41 (1996)
15. Karaboga, D.: An idea based on honeybee swarm for numerical optimization. Technical Report TR06. Erciyes University, Engineering Faculty, Computer Engineering Department (2005)
16. Wu, H.S., Zhang, F., Wu, L.: New swarm intelligence algorithm-wolf pack algorithm. Syst. Eng. Electron. **35**(11), 2430–2438 (2013)
17. Neto, M.T.R.S., Mollinetti, M.A.F., Pereira, R.L.: Evolutionary artificial bee colony for neural networks training. In: 2017 13th International Conference on Natural Computation, Fuzzy Systems and Knowledge Discovery (ICNC-FSKD), Guilin, pp. 44–49 (2017)
18. Kennedy, J., Eberhart, R.: Particle swarm optimization. In: Proceedings of ICNN 1995 - International Conference on Neural Networks, Perth, WA, Australia, vol. 4, pp. 1942–1948 (1995)
19. Jatmiko, W., Sekiyama, K., Fukuda, T.: Apso-based mobile robot for odor source localization in dynamic advection-diffusion with obstacles environment: theory, simulation and measurement. IEEE Comput. Intell. Mag. **2**(2), 37–51 (2007)
20. Jatmiko, W., et al.: Robots implementation for odor source localization using PSO algorithm. WSEAS Trans. Circuits Syst. **10**(4), 115–125 (2011)
21. Li, F., Meng, Q.-H., Bai, S., Li, J.-G., Popescu, D.: Probability-PSO algorithm for multi-robot based odor source localization in ventilated indoor environments. In: Xiong, C., Huang, Y., Xiong, Y., Liu, H. (eds.) ICIRA 2008. LNCS (LNAI), vol. 5314, pp. 1206–1215. Springer, Heidelberg (2008). https://doi.org/10.1007/978-3-540-88513-9_128
22. Wu, W., Zhang, F.: A speeding-up and slowing-down strategy for distributed source seeking with robustness analysis. IEEE Trans. Control Netw. Syst. **3**(3), 231–240 (2016)
23. Al-Abri, S., Wu, W., Zhang, F.: A gradient-free three-dimensional source seeking strategy with robustness analysis. IEEE Trans. Autom. Control **64**(8), 3439–3446 (2019)
24. Liu, S.-J., Krstic, M.: Stochastic source seeking for nonholonomic unicycle. Automatica **46**(9), 1443–1453 (2010)
25. Cochran, J., Krstic, M.: Source seeking with a nonholonomic unicycle without position measurements and with tuning of angular velocity part I: stability analysis. In: 2007 46th IEEE Conference on Decision and Control, New Orleans, LA, pp. 6009–6016 (2007)
26. Atanasov, N., Le Ny, J., Michael, N., Pappas, G.J.: Stochastic source seeking in complex environments. In: 2012 IEEE International Conference on Robotics and Automation, Saint Paul, MN, pp. 3013–3018 (2012)
27. Ogren, P., Fiorelli, E., Leonard, N.E.: Cooperative control of mobile sensor networks: adaptive gradient climbing in a distributed environment. IEEE Trans. Autom. Control **49**(8), 1292–1302 (2004)
28. Zhu, S., Wang, D., Low, C.B.: Cooperative control of multiple UAVs for moving source seeking. In: 2013 International Conference on Unmanned Aircraft Systems (ICUAS), Atlanta, GA, pp. 193–202 (2013)

29. Li, S., Kong, R., Guo, Y.: Cooperative distributed source seeking by multiple robots: algorithms and experiments. IEEE/ASME Trans. Mechatron. **19**(6), 1810–1820 (2014)
30. Fabbiano, R., Garin, F., Canudas-de-Wit, C.: Distributed source seeking without global position information. IEEE Trans. Control Netw. Syst. **5**(1), 228–238 (2018)
31. Briñón-Arranz, L., Renzaglia, A., Schenato, L.: Multi-robot symmetric formations for gradient and hessian estimation with application to source seeking. IEEE Trans. Robot. **35**, 782–789 (2019)
32. Briñón-Arranz, L., Seuret, A., Pascoal, A.: Circular formation control for cooperative target tracking with limited information. J. Franklin Inst. **356**, 1771–1788 (2019). https://doi.org/10.1016/j.jfranklin.2018.12.011
33. Spall, J.: Intro to Stochastic Search and Optimization. Wiley, Hoboken (2003)