



Time-Division Frequency Measurement and Localization Technology of Single-Satellite in Different Orbits Based on Optimal Observation Position

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Abstract. At present, the research on the single satellite frequency measurement and localization technology mainly focuses on the use of multiple satellite observation positions in a single flight orbit to forward the ground target source signals, but there are many limitations in locating the interference source with limited field of view. In this paper, based on the single-orbit single-satellite time-division frequency-measuring localization technology, a localization technology of single-satellite time-division Doppler frequency measurement in different orbits is proposed, which is based on the optimal observation satellite position. Firstly, the positioning model and the positioning equation of the single-satellite time-division Doppler frequency measurement and localization technology in different orbits are constructed. Secondly, a calculation formula for Geometric Dilution of Precision (GDOP) is derived and the optimal model of the observation satellite position is constructed. Finally, based on the positioning process, the minimum positioning error is targeted. Computer simulation results show that the availability and accuracy of the technique are improved significantly in the scene with limited field of view.

Keywords: Single-satellite passive localization · Doppler frequency · GDOP · Position optimization

1 Introduction

As a strong supplement to the ground communication system, satellite communication has been widely used because of its wide coverage and long communication distance. However, in the face of the influence of electromagnetic radiation in the environment, satellite communication will be interfered in the course of work, which makes the communication quality difficult to guarantee, and causes different degrees of influence and economic loss [1]. Therefore, how to accurately find the location of interference source has become an urgent problem in engineering [2, 3].

At present, satellite passive localization technology can rapidly locate the transmitted signals of sea, land and air targets because of its large localization area, good concealment and low interception rate, it plays an important role and has a broad application prospect in both civil and military fields [4, 5]. A method of instantaneous Doppler frequency difference passive location is proposed in reference [6]. This method requires the interference source and ground observation stations to see several satellites at the same time. However, the probability of instantaneous multi-satellite visibility is not studied in this paper. If the field of view is limited, the usability of this method is very low. The single-satellite multi-beam localization technology proposed in reference [7, 8] and the single-satellite direction finding and localization technology proposed in reference [9]. They require obtain the Direction of Arrival (DOA) or amplitude of the source signal, and the antenna structure is complex and high cost. The single-orbit single-satellite time-division frequency measurement and localization technology is proposed in reference [10], which can solve the problem of multi-satellite localization being unable to see multi-satellite instantaneously and low positioning availability. However, the positioning error of this method is large in the scene with limited field of view.

In order to solve the problem of low probability and large positioning error under the condition of limited field of view, this paper presents a single-satellite time-division frequency measurement and localization technique in different orbits based on the optimal observation position. By constructing the positioning model of single-satellite time-division frequency measurement and localization technology in different orbits, the positioning equation and the calculation formula of geometric dilution accuracy factor are derived, and the optimal positioning model and positioning steps are designed. Aiming at the minimum positioning error, the optimal observation position is obtained to locate the interference source in the scene with limited field of view.

The rest of this paper is organized as follows. In Sect. 2, the positioning model of the technology is presented. The positioning equation and the GDOP formula are derived, and the optimal model of the observation position is established. In Sect. 3, the positioning step of the technology is designed. The simulation results are shown in Sect. 4. Finally, the conclusions are given in Sect. 5.

2 Introduction to Localization Technology

2.1 Positioning Model and Positioning Equation

The single-satellite time-division Doppler frequency measurement and localization technology in different orbits is based on the single-orbit single-satellite time-division frequency measurement and localization technology. The difference between the transmitted signals received by the same ground station with different Doppler frequency is called Frequency Difference Of Arrival (FDOA). The position of the target is calculated by using the relation between the frequency difference of Doppler's arrival.

The medium-low orbit satellite is far from the earth, so the satellite and the radiation source can be regarded as the particle motion analysis approximately. Figure 1 is a positioning model for single-satellite time-division Doppler frequency measurement and localization technology in different orbits. Suppose that the position of an active

geostationary interference source in Earth Centered Earth Fixed (ECEF) is $\mathbf{s} = [x \ y \ z]^T$. The satellites with different orbits fly over the region of visibility at different time intervals, and the observation position at t_j is $\mathbf{s}_{ij} = [x_{ij} \ y_{ij} \ z_{ij}]^T$. The speed of this satellite is $\mathbf{v}_{ij} = [v_{x_{ij}} \ v_{y_{ij}} \ v_{z_{ij}}]^T$, where i is the number of the observing satellite's orbit l_i , where $i \in N^+$, j represents the time interval between the observation time t_j of the observing satellite in orbit l_i and the initial time t_0 of the satellite's entry into the visible region, where $j \in N$.

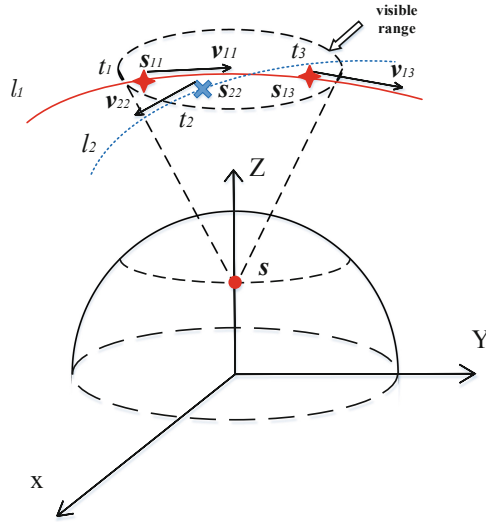


Fig. 1. Positioning model

Assuming that the Doppler frequency measured by observation satellite at different times in different orbits is f_{ij} , and the frequency of the radiation source is known as f_c , the measurement equation is

$$f_{ij} = f_c + \frac{f_c}{c} \cdot \dot{r}_{ij} \tag{1}$$

In the formula, $\dot{r}_{ij} = [(x_{ij} - x)v_{x_{ij}} + (y_{ij} - y)v_{y_{ij}} + (z_{ij} - z)v_{z_{ij}}]/r_{ij}$ represents the radial velocity of the satellite and the target; $r_{ij} = \sqrt{(x - x_{ij})^2 + (y - y_{ij})^2 + (z - z_{ij})^2}$ represents the relative position of a satellite to a target, where $i \in N^+$, $j \in N$, c represents the speed at which electromagnetic waves travel.

Taking into account the constraints of the Earth's surface, it is only necessary to obtain three different satellite observation positions are $\mathbf{s}_{\alpha m}$, $\mathbf{s}_{\beta n}$, $\mathbf{s}_{\gamma k}$, and corresponding velocities are $\mathbf{v}_{\alpha m}$, $\mathbf{v}_{\beta n}$, $\mathbf{v}_{\gamma k}$, as well as Doppler frequencies $f_{\alpha m}$, $f_{\beta n}$, $f_{\gamma k}$, which are observed at the corresponding time, where $\alpha, \beta, \gamma \in N^+$, $m, n, k \in N$. The geodetic

coordinates of the interference source can be obtained.

$$\begin{cases} f_{\alpha m} - f_{\beta n} = \frac{f_c}{c} \cdot (\dot{v}_{\alpha m} - \dot{v}_{\beta n}) \\ f_{\alpha n} - f_{\gamma k} = \frac{f_c}{c} \cdot (\dot{v}_{\alpha m} - \dot{v}_{\gamma k}) \\ x^2 + y^2 + \frac{z^2}{(1 - e^2)} = a^2 \end{cases} \quad (2)$$

where a is the radius of the earth and e is the first eccentricity of the earth.

2.2 Optimal Model of Observation Satellite Position

Due to the gradual improvement of the satellite system, the number of available satellites is increasing, and the selection of observation positions is becoming more and more complicated. Random selection of satellite observation position will make the positioning effect is not ideal. It is necessary to minimize the location error of the target source by optimizing the selection of observation satellites and their observation positions in different orbits. The optimization objective function can be expressed as

$$P([l_i, s_{ij}]) = \underset{(l_i, s_{ij})}{\text{Min}} \|\text{GDOP}(x, y, z)\| \quad (3)$$

In the formula, l_i represents the different satellite orbits, i represents the number of orbit, where $i \in N^+$, s_{ij} is the satellite observation position at t_j time on the corresponding orbit l_i . $\text{GDOP}(x, y, z)$ is the geometric dilution accuracy factor at the target source \mathbf{s} , where $\mathbf{s} = [x \ y \ z]^T$, it can be expressed as

$$\text{GDOP}(x, y, z) = \sqrt{\text{tr}(\mathbf{P}_{dX})} \quad (4)$$

where \mathbf{P}_{dX} is the positioning error covariance matrix, which can be derived from the total differential set of positioning Eq. 5 below.

The positions of the observing satellite are $\mathbf{s}_{\alpha m}$, $\mathbf{s}_{\beta n}$, $\mathbf{s}_{\gamma k}$, the velocities are $\mathbf{v}_{\alpha m}$, $\mathbf{v}_{\beta n}$, $\mathbf{v}_{\gamma k}$, where $\alpha, \beta, \gamma \in N^+$, $m, n, k \in N$, the position of the interference source is $\mathbf{s} = [x \ y \ z]^T$, the joint surface equation can be written as

$$\begin{cases} f_{\alpha m} = f_c + \frac{f_c}{c} \dot{v}_{\alpha m} = g_1(x, y, z) \\ f_{\beta n} = f_c + \frac{f_c}{c} \dot{v}_{\beta n} = g_2(x, y, z) \\ f_{\gamma k} = f_c + \frac{f_c}{c} \dot{v}_{\gamma k} = g_3(x, y, z) \\ H = [(1 - e^2)(x^2 + y^2) + z^2]^{1/2} - a = g_4(x, y, z) \end{cases} \quad (5)$$

After the total differential, the positioning error is covariance matrix $\mathbf{P}_{dX} = \text{E}[dXdX^T]$, where $dX = [dx \ dy \ dz]^T$. The frequency difference and elevation error is covariance matrix $\mathbf{R}_{fH} = \text{E}[dUdU^T]$, where $dU = [df_{\alpha m} \ df_{\beta n} \ df_{\gamma k} \ dH]^T$. The satellite positioning error is covariance matrix $\mathbf{R}_{X_{ij}} = \text{E}[dX_{ij}dX_{ij}^T]$, where $dX_{ij} =$

$[dx_{ij} dy_{ij} dz_{ij}]^T$. The satellite velocity error is covariance matrix $R_{V_{ij}} = E[dV_{ij}dV_{ij}^T]$, where $dV_{ij} = [dv_{x_{ij}} dv_{y_{ij}} dv_{z_{ij}}]^T, i = \alpha, j = m; i = \beta, j = n; i = \gamma, j = k$.

The positioning error covariance matrix can be expressed as

$$P_{dX} = E[dXdX^T] = C^{-1} \left(R_{fH} + \sum_{\substack{i = \alpha, j = m; \\ i = \beta, j = n; \\ i = \gamma, j = k;}} C_{X_{ij}} R_{X_{ij}} C_{X_{ij}}^T + \sum_{\substack{i = \alpha, j = m; \\ i = \beta, j = n; \\ i = \gamma, j = k;}} C_{V_{ij}} R_{V_{ij}} C_{V_{ij}}^T \right) C^{-T} \quad (6)$$

where $C = \frac{\partial \mathbf{g}(\mathbf{s})}{\partial \mathbf{s}}$, $C_{X_{\alpha m}} = \frac{\partial \mathbf{g}(\mathbf{s})}{\partial s_{\alpha m}}$, $C_{X_{\beta n}} = \frac{\partial \mathbf{g}(\mathbf{s})}{\partial s_{\beta n}}$, $C_{X_{\gamma k}} = \frac{\partial \mathbf{g}(\mathbf{s})}{\partial s_{\gamma k}}$, $C_{V_{\alpha m}} = \frac{\partial \mathbf{g}(\mathbf{s})}{\partial v_{\alpha m}}$, $C_{V_{\beta n}} = \frac{\partial \mathbf{g}(\mathbf{s})}{\partial v_{\beta n}}$, $C_{V_{\gamma k}} = \frac{\partial \mathbf{g}(\mathbf{s})}{\partial v_{\gamma k}}$, $\mathbf{g}(\mathbf{s}) = [g_1 g_2 g_3 g_4]^T$.

3 Positioning Process Steps

Through the discussion in Sect. 2, the principle of the technology in this paper has been clarified. When the position of the target source and the lowest visible elevation angle are known, in order to obtain the minimum positioning error corresponding to the optimal observed position, the specific positioning process is as follows:

- Step1: The number of satellites visible to the interference source is N . Obtain the corresponding orbit l_i , the initial time t_0 , the flight duration M and the ephemeris data of each satellite.
- Step2: Acquire the selection scheme of observation satellite position. Select two of the N satellites in different orbits as observation satellites. There are C_N^2 selection schemes. The observation interval of each observation satellite is 1 s. If the satellite that needs to acquire two observation positions is the observation main satellite, there are C_{M+1}^2 options for selecting two different observation positions on the observation main satellite and C_{M+1}^1 options for acquiring one observation position on the other observation satellite, a total of $C_N^2 \cdot C_{M+1}^2 \cdot C_{M+1}^1$ observation satellites position selection scheme.
- Step3: Acquire the position and instantaneous velocity of the observation satellite corresponding to each selection scheme, and calculate the observation frequency of each scheme by Eq. 1.
- Step4: Calculate the GDOP of each selection scheme by Eq. 4, Eq. 5 and Eq. 6. Select the minimum value of GDOP at the position of the target source according to the optimal model of the observation position selection. Obtain the optimal observation position and the minimum positioning error of the technology in this paper.

Through the above four steps, the minimum positioning error and the optimal satellite observation position can be obtained by the technology in this paper under the condition of the target source field of view limited. The simulation results and analysis of target source localization using the above method and process will be described in the next section.

4 Simulation Results and Analysis

4.1 Availability Analysis

The minimum elevation angle of the user is 71.57 if the depth of the deep well is 3 m and the diameter of the deep well is 1 m. Taking the four navigation satellite systems as an example, the number of visible satellites under the condition of the target source field of view limited is analyzed using Satellite Tool Kit (STK) software. The global coverage of the four satellite navigation systems by three satellites is shown in Fig. 2, and the global coverage of the four satellite navigation systems by single satellite is shown in Fig. 3. Different colors represent the percentage of visible time of day. Figure 2 shows that in a positioning scenario with a minimum elevation of 71.57, the percentage of a day in which three satellites share a common view instantaneously is less than 40% in most parts of the world. Therefore, the availability of instantaneous three satellites localization method proposed in reference [6] is very low. Figure 3 shows that in the positioning scenario with a minimum elevation of 71.57, the percentage of visible time of single satellite in a day is significantly increased, reaching 100% in some areas. The availability of single-satellite time-division frequency measurement technology is significantly improved compared with instantaneous multi-satellite positioning.

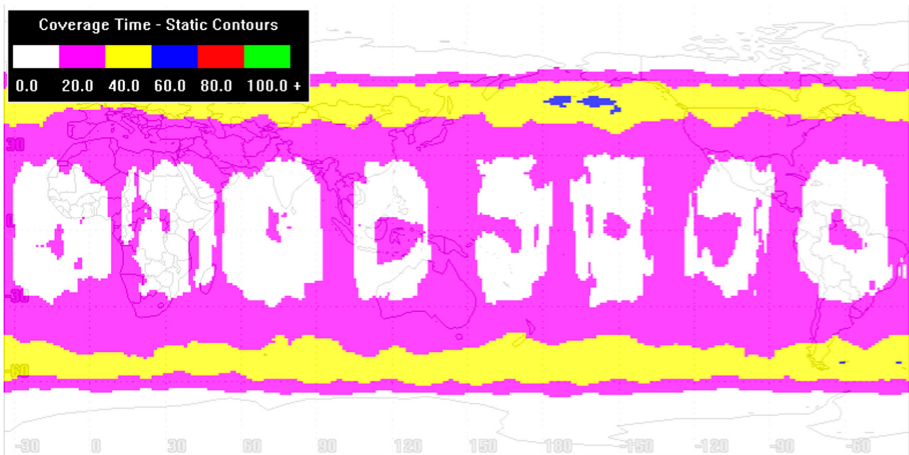


Fig. 2. Global three-satellite coverage of four satellite navigation systems

However, reference [11] shows that the positioning effect of single-orbit single-satellite time-division frequency measurement technology is affected by the frequency

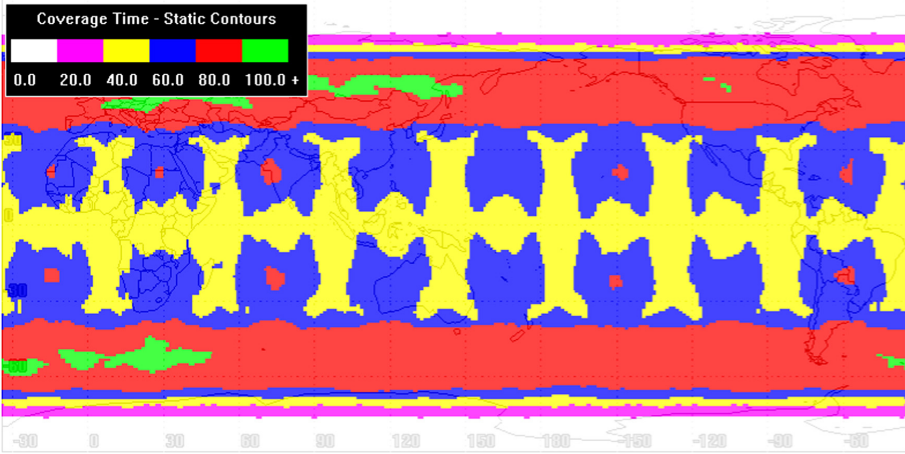


Fig. 3. Global single-satellite coverage of four satellite navigation systems

measurement interval, and the short frequency measurement interval will result in large positioning error. If the target source is located in the flight path of the observation satellite, the positioning error will be further increased. The time-division frequency measurement technique of single satellite in different orbits makes use of the single satellite flying through the visible region at different times to carry out independent observations of the same target source. The time interval between two adjacent frequency measurements is enlarged to improve the positioning accuracy. At the same time, because of the different flight direction and the distribution of sub-satellite points, the observation position selection of different orbits is more flexible.

4.2 Error Analysis

Simulation Parameter Setting

In this paper, the beidou navigation system is used for simulation. The initial time is 29 Nov 2018 21:00:00 (UTCG) and the end time is 29 Nov 2018 22:00:00 (UTCG).STK simulation software is used to obtain the ephemeris data of BeiDou Navigation Satellite System (BDS). If the target source is located in Nanjing (119 E, 32 N, elevation 0 km) and the lowest elevation angle of the user is 45, three satellites will pass through the target source’s visible range, namely No. 11, No. 27 and No. 28. The sub-satellite orbits of the three satellites at the global scale are shown in Fig. 4, the sub-satellite orbits of the satellites within the visual range of the target source are shown in Fig. 4. The orbital parameters are shown in Table 1, and during the set time period, all three satellites have 60 min visual time in the target source’s visual region (Fig. 5).

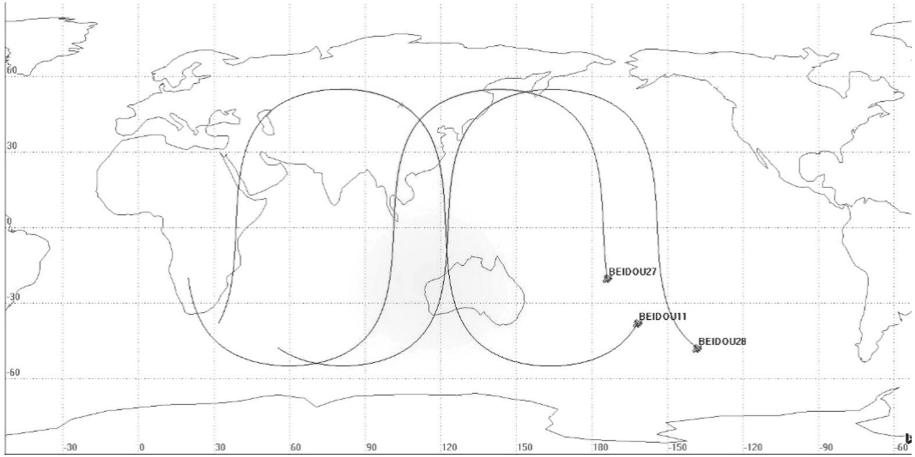


Fig. 4. The sub-satellite orbits of observation satellites

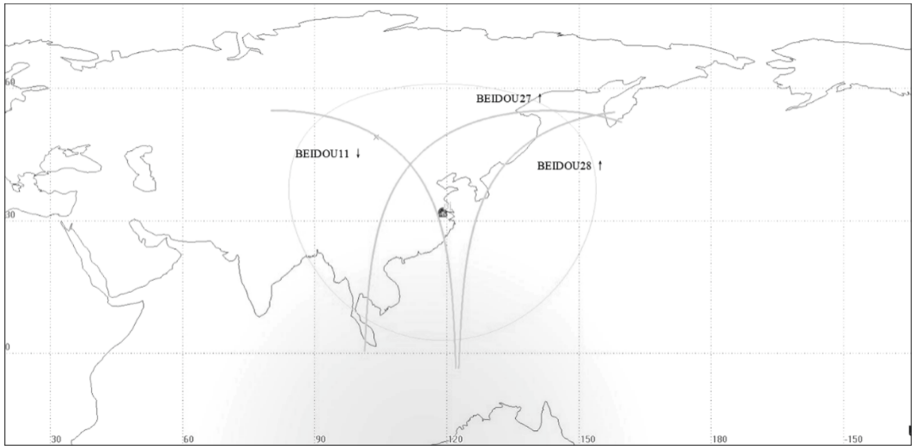


Fig. 5. The sub-satellite orbits of observation satellites in visible region

Table 1. Orbital parameters of satellites

Satellite number	Semi-major axis/km	Eccentricity	Inclination angle/ $^{\circ}$	Argument of the perigee/ $^{\circ}$	RAAN/ $^{\circ}$	True anomaly/ $^{\circ}$
11	21500	0	55	0	0	0
27	21500	0	55	0	120	253.333
28	21500	0	55	0	120	293.333

Comparison of Localization Techniques

If the observation interval is 2 min, the first two observing moments t_m, t_n are any two moments in the visible region of an observing satellite, where $m, n = 0, 2, \dots, 60, m, n$ are even numbers, $m \neq n$. If using the single-orbit single-satellite time-division frequency measurement and localization technique described in reference [10], the third satellite observation time is any time t_k , where $k = 0, 2, \dots, 60, k$ is an even number, $k \neq m \neq n$. If using the technique in this paper, the third satellite observation time is any time t_k in the visible region of the other observation satellite, where $k = 0, 2, \dots, 60, k$ is an even number. The signal carrier frequency f_c is 406.8 MHz, and the frequency measurement error is 0.2 Hz. The elevation error, the satellite velocity error and the satellite position error are ignored, the selection scheme of observation satellites and the corresponding minimum positioning error of the single-orbit single-satellite time-division frequency measurement technology proposed in reference [10] are shown in Table 2. The optimum scheme of observation position and the corresponding minimum positioning error of the single-satellite time-division frequency measurement and localization technology in different orbits are shown in Table 3.

Table 2. Selection scheme and minimum positioning error of single-satellite time-division frequency measurement localization in single orbit

Programme number	Observation satellite option	Minimum positioning error/km
1	Use satellite No. 11	40.550
2	Use satellite No. 27	25.595
3	Use satellite No. 28	61.128

In the selection scheme of observation satellites for single-satellite time-division frequency measurement localization in different orbits, the satellite written in the front represents the observation main satellite.

From the data in Table 2, it can be seen that the positioning effect of using satellite No. 27 as the observation satellite is the best among the three positioning schemes using the single-orbit single-satellite time-division frequency measurement localization technology proposed in reference [10]. The minimum positioning error is 25.595 km, and the positioning effect is not ideal.

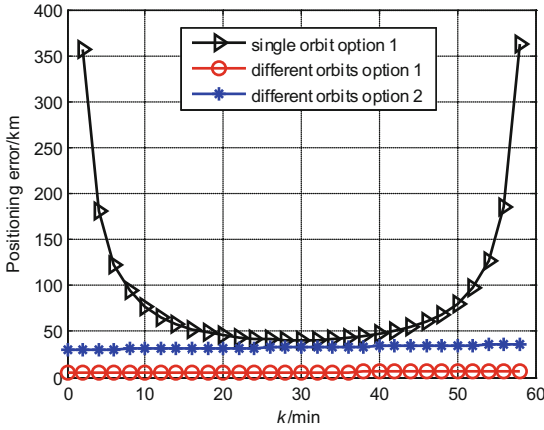
From the data in Table 3, it can be seen that the minimum positioning error of all the observation satellite selection schemes using single-satellite time-division frequency measurement and localization technology in different orbits is obviously lower than that of single orbit positioning scheme, the positioning accuracy is greatly improved.

Suppose the first two satellite observation time t_m, t_n are t_0 and t_m . Figure 6 is a comparison of single-satellite time-division frequency measurement and localization schemes between single orbit and different orbits. The positioning effects are compared with the third satellite observation time t_k , where k is the time interval between t_k and t_0 , and the programme numbers in the figure are those of the corresponding observation satellite options in Tables 2 and 3.

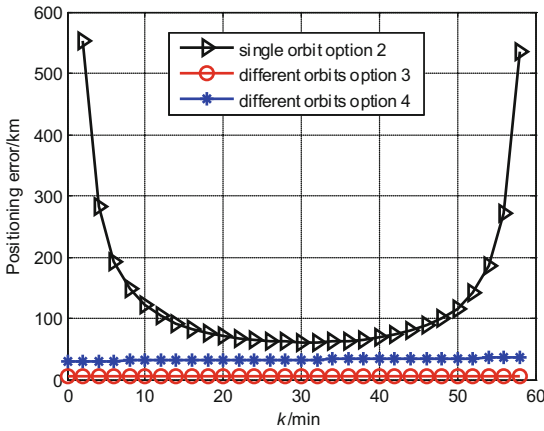
Table 3. Optimal observation satellite position scheme and its minimum positioning error for single-satellite time-division frequency measurement localization in different orbits

Programme number	Observation satellite option	The time of the observation satellite's position	Minimum positioning error/km
1	Use satellites No. 11 and No. 27	The observation time of satellite No. 11 is t_0 and t_M ; the observation time of satellite No. 27 is t_M	5.3165
2	Use satellites No. 11 and No. 28	The observation time of satellite No. 11 is $t_{M/2}$, and t_M ; the observation time of satellite No. 28 is t_M	6.0549
3	Use satellites No. 27 and No. 11	The observation time of satellite No. 27 is t_0 , and t_M ; the observation time of satellite No. 11 is t_M	5.0673
4	Use satellites No. 27 and No. 28	The observation time of satellite No. 27 is t_0 , and t_M ; the observation time of satellite No. 28 is t_M	6.0026
5	Use satellites No. 28 and No. 11	The observation time of satellite No. 28 is $t_{M/2}$, and t_M ; the observation time of satellite No. 11 is t_M	7.0755
6	Use satellites No. 28 and No. 27	The observation time of satellite No. 28 is t_0 , and t_M ; the observation time of satellite No. 27 is t_M	7.6542

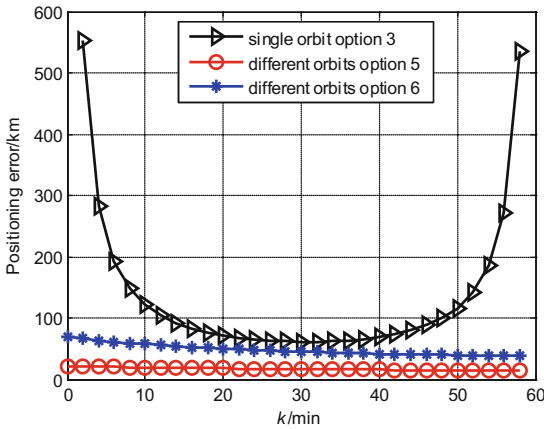
From the (a), (b), (c) of Fig. 6, it can be seen that the positioning accuracy can be improved by the introduction of another orbit, and the positioning error is stable. At the same time, the selection of satellite observation time is flexible. The simulation results show that the positioning accuracy is improved by 80.2% compared with the traditional satellite localization technology in single orbit, which can be effectively located by using the single-satellite time-division frequency measurement and localization technology in different orbits based on the optimal observation satellite position under the condition of limited view field.



(a) The observation main satellite is satellite No.11



(b) The observation main satellite is satellite No.27



(c) The observation main satellite is satellite No.28

Fig. 6. Comparison of different positioning methods

5 Conclusion

This paper is based on the technology of single-orbit single-satellite time-division frequency measurement and localization. Aiming at the problems of low success rate of multi-satellite localization and poor effect of single-satellite localization in single orbit with limited field of view, a single-satellite time-division frequency measurement and localization technique in different orbits based on the optimal observation position is proposed. Compared with the instantaneous three-satellite localization technology, the positioning accuracy is high and the positioning effect is stable. By comparing the influence of the satellite observation position in different orbits and at different observation times on the positioning accuracy, the selection scheme of the observation satellite is optimized. At the same time, the positioning accuracy of the single-satellite time-division frequency measurement and localization technology in different orbits is significantly improved. It is valuable for the design and application of the localization system in the scene with limited field of view.

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