Detection for Uplink Massive MIMO System: A Survey

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Abstract. In this paper, we make a compressive survey for the research on detection in uplink Massive multiple input and multiple output (MIMO) system. As one key technology in Massive MIMO system, which is also one primary subject for the fifth generation wireless communications, this research is significant to be developed. As a result of large scaled antennas, the channel gain matrix in Massive MIMO system is asymptotic diagonal orthogonal, and it is an non-deterministic polynomial hard problem to obtain the optimum bits error rate (BER) performance during finite polynomial complexity time. The traditional detection algorithms for MIMO system are not efficient any more due to poor BER performance or high computational complexity. The exiting detection algorithms for Massive MIMO system are able to solve this issue. However, there are still crucial problems for them, including employing the deep learning technology for detection in Massive MIMO system, and not work for the millimeter wave Massive MIMO system in the strong spatial correlation environment even exiting keyhole effect, which is not rich scattering, as well as application in Hetnets wireless communications, and etc. Therefore, the research on detection for uplink Massive MIMO system is still in its early stage, there are lots of significant and urgent issues to overcome in the future.

Keywords: Massive MIMO · Low complexity detection · Optimum performance

1 Introduction

The technology of Massive multiple input and multiple output (MIMO) system was firstly proposed by Marzetta in 2010 [1–3]. As a hot research spot for 5G communications, the study on Massive MIMO system is getting increasing attention. In general, Massive MIMO system is defined as following: large scaled MIMO system whose base station is equipped with thousands and hundreds of antennas, which serve tens of user terminals simultaneously at the same frequency, as shown in Fig. 1. In order to provide ever-increasing throughput for

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assigned cell consecutively and steadily, from the initial point to point MIMO system, to the following multi-user (MU) MIMO system, the technology of multiple antennas equipped at both the base station and user terminals, has been widely applied. In order to satisfy the higher demand of big throughput and etc. for the coming 5G area, the technology of Massive MIMO system has great potential, and its corresponding research is urgent to be developed.

The outstanding advantages of Massive MIMO system are summarized as the following aspects. Firstly, though Massive MIMO technology, the system capacity is increased by even more than 10 times. Secondly, the energy efficiency achieves to be more than 100 times. Thirdly, Massive MIMO system is able to be built up with low power devices, and its required cost is much lower. Fourthly, considering anti-unintentional jamming and anti-deception jamming, Massive MIMO system has much higher robustness. Fifthly, Massive MIMO system is able to cut down the air interface delay obviously. At last, Massive MIMO technology is able to simplify the multiple access.

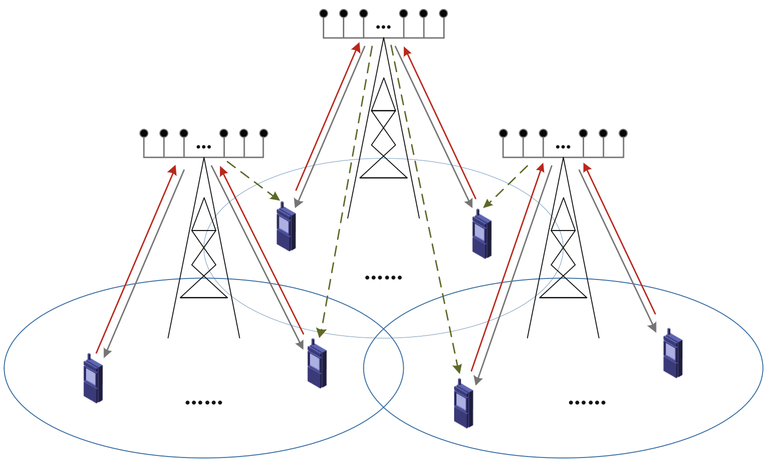


Fig. 1. Massive MIMO system

However, Massive MIMO system faces many challenges, which are generally introduced as following. Firstly, the uplink channel and the downlink channel of Massive MIMO system do not satisfy the reciprocity, which resulted that the time division duplexing (TDD) technology and etc. are not able to be realized or applied directly. Due to the large scaled antennas in Massive MIMO system, a large amount of pilot sequences are not orthogonal with each other any more. This generates the serious pilot pollution, which is one of the important and hard issue to overcome in Massive MIMO system. As a result of serving hundreds and thousands of users terminals simultaneously at the same frequency, the channel distribution in the propagation model is random and different from each other, which lead the channel estimation and capacity analysis method for

Massive MIMO system to be rather complex and hard to be solved. Due to the large scaled antennas, the traditional signal detection and estimation algorithms for MIMO system are not able to obtain the optimum bit error ratio (BER) performance of maximum likelihood (ML) algorithm during finite polynomial complex time, the research and develop on the low computational complexity detection algorithms in Massive MIMO system, is the premise of the communication in Massive MIMO system to be practice and realized, and is also one of the important and urgent problem to be solved [4]. Further more, the other emphasis research topics in Massive MIMO system include the trade-off problem between energy efficiency and spectral efficiency, beam forming problem, precoding technology, combination with millimeter wave (MMW) communication technology, application in Hetnets wireless communications and etc. We focus on and carry out the research on the low complexity detection algorithms in Massive MIMO system in this paper.

The reminder of this paper is organized as follows: The introduction to the research on detection algorithms for Massive MIMO system is discussed in Sect. 2. The crucial questions for the future research are presented in Sect. 3. Finally, the conclusion is introduced in Sect. 4.

2 Detection Algorithms for Massive MIMO System

The optimum detection algorithms including the ML detection algorithm by hard decision and Maximum a posteriori (MAP) detection algorithm by soft decision have the optimum BER performance. Essentially, ML detection algorithm is by means of ergodic method to obtain the optimum BER performance. The BER performance of ML detection algorithm is taken as the evaluation criterion of the optimum BER performance and the optimization objective for the detection algorithms both in MIMO system and in Massive MIMO system. However, the computational complexity of ML detection algorithm exponentially increased with the number of the transmitting antennas, and further pointed that it was an NP-hard problem for the detection algorithms in MIMO system to obtain the optimum BER performance during finite polynomial complexity time. Therefore, due to the large scaled antennas equipped in Massive MIMO system, the exponential computational complexity is too high, and it is impossible for ML detection algorithm to achieve the optimum BER performance over finite low complexity time. Neither ML detection algorithm nor the MAP detection algorithm is appropriate or able to employed in the practical detection for Massive MIMO system.

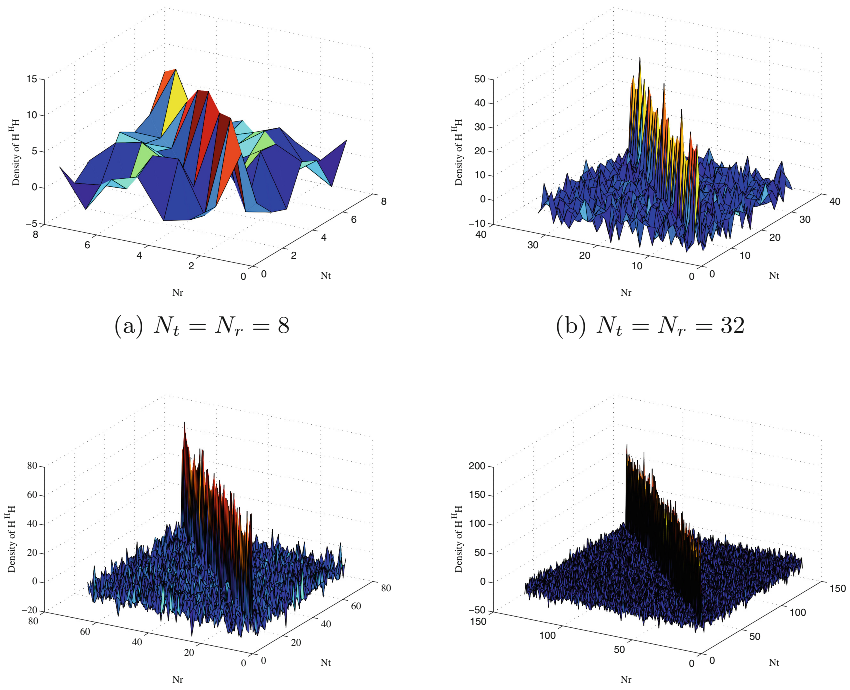
Then, considering the high computational complexity of the optimum detection algorithms in MIMO system, lots of researchers developed the study on the sub-optimum detection algorithms which have low computational complexity. The sub-optimum detection algorithms for MIMO system is divided into the linear detection algorithms and nonlinear detection algorithms.

Firstly, the linear detection algorithms include matched filtering (MF) detection algorithm, zero forcing (ZF) detection algorithm and minimum mean square error (MMSE) detection algorithm and etc. The computational complexity of linear detection algorithms polynomially increased with the number of transmitting antennas, compared with the exponential computational complexity of the optimum detection algorithms including ML detection algorithm and etc., the linear detection algorithms have much low computational complexity. However, the BER performance of linear detection algorithms is rather bad, and there was a big gap compared to the optimum BER performance. Under the demand of high quality detection performance in Massive MIMO system, the linear detection algorithms are required to be improved and optimized. Therefore, due to the bad BER performance, linear detection algorithms are not appropriate for directly applying in practical detection in Massive MIMO system.

Secondly, the nonlinear detection algorithms, which are divided into the following several categories. The fist category is represented as the interference cancellation based detection algorithms, including successive interference cancellation (SIC) detection algorithm and parallel interference cancellation (PIC) detection algorithm and etc., have the comparative polynomial computational complexity as that of linear detection algorithms, but obtain the bad BER performance which has a quite big gap compared to the optimum BER performance of ML detection algorithm. The second category is introduced as the sphere decoding (SD) detection algorithm, which is also included into the tree searching based detection algorithms, is able to obtain the approximate optimum BER performance of ML detection algorithm, so it usually substitute the ML detection algorithm and has been widely employed in practical system detection. However, under the condition of a medium fixed signal to noise ratio (SNR), the computational complexity of the SD detection algorithm still exponentially increases with the number of transmitting antennas, and not appropriate for Massive MIMO system equipped with large scaled antennas. The last category is denoted as the lattice reduction aided (LRA) detection algorithm, which has got much attention for the research on MIMO system recently. LRA detection algorithm is able to approach the optimum BER performance of ML detection algorithm under the condition of high SNR in small scaled MIMO system, the computational complexity of LRA polynomially increases with the number of transmitting antennas. The principle of the LRA detection algorithm is by means of orthogonalization the channel gain matrix. However, the channel gain matrix of Massive MIMO system is asymptotic orthogonal itself, which has been introduced and proved in the next section. Thus, the LRA detection algorithm does not work any more for the orthogonal channel gain matrix. The traditional LRA detection algorithm is based on and to improve the linear detection algorithms. For Massive MIMO system, the BER performance of the traditional LRA detection algorithm is not able to be better, and still has a big gap compared to the optimum BER performance of ML detection algorithm. With the worse BER performance, the traditional LRA detection algorithm is not appropriate for the practical detection in Massive MIMO system.

Further more, we make an essential analysis for the reason why the traditional detection algorithms for MIMO system is not efficient for Massive MIMO system any more.

The density of channel gain matrix $\mathbf{H}^{\mathrm{H}}\mathbf{H}$ versus the scale of antennas number $N_t = N_r$ in MIMO system is shown in Fig. 2, where $\mathbf{H} \in \mathbb{R}^{N_r \times N_t}$ is channel gain matrix, whose entry obeys to the complex Gaussian distribution with zero mean and unit variance, N_t and N_r denote the number of the transmitting antennas and that of the receiving antennas, respectively. From Fig. 2, the channel gain matrix of the MIMO system is becoming more and more diagonal gradually versus the increasing of the antennas numbers' scale in MIMO system. Considering the large scaled antennas, it is easy to conclude that the channel gain matrix in Massive MIMO system is asymptotic diagonal.



(a) $N_t = N_r = 8$

(b) $N_t = N_r = 32$

(c) $N_t = N_r = 64$

(d) $N_t = N_r = 128$

Fig. 2. The density of channel gain matrix $\mathbf{H}^{\mathrm{H}}\mathbf{H}$ versus the scale of antennas number $N_t = N_r$ in MIMO system

For arbitrary matrix, whether it is orthogonal or not, is decided by the criterion of orthogonality deficiency (od), as shown in the following,

$$\mathrm{od}(\mathbf{H}) = 1 - \frac{\det(\mathbf{H}^{\mathrm{H}}\mathbf{H})}{\prod_{n=1}^{N_t} \|\mathbf{h}_n\|^2} \tag{1}$$

where the channel gain matrix $\mathbf{H} = [\mathbf{h}_1, \cdots, \mathbf{h}_n, \cdots, \mathbf{h}_{N_t}]$, $\mathbf{h}_n$ is its nth column vector entry, and $n = \{1, 2, \cdots, N_t\}$. For arbitrary channel gain matrix $\mathbf{H}$, it is always satisfied that $0 \leq \text{od}(\mathbf{H}) \leq 1$. When $\text{od}(\mathbf{H}) = 1$, the channel gain matrix $\mathbf{H}$ is singular; When $\text{od}(\mathbf{H}) = 0$, the channel gain matrix $\mathbf{H}$ is orthogonal. Take the LRA detection algorithm as example, its objective in principle is to minimize the orthogonality deficiency of the channel gain matrix $\text{od}(\mathbf{H})$ until to be 0, thereby the orthogonal basis are obtained, and the original detection results are optimized and improved to be better. However, according to the theorem of Marčenko Pastur (M-P) law in large scaled random matrix theory, the matrix is asymptoticly diagonal little by little versus the growth of the matrix's dimension. As shown in Fig. 2, when the scale of the antennas number $N_t = N_r$ in MIMO system is increasing, the density of the diagonal entries is bigger and bigger than those of the off-diagonal entires in the channel gain matrix $\mathbf{H}^{\mathrm{H}}\mathbf{H}$, till to be totally diagonal. Furthermore, when $N_t = N_r \to \infty$, $\mathbf{H}^{\mathrm{H}}\mathbf{H} \approx N_r \mathbf{I}_{N_r}$, and $\|h_n\|^2 \approx N_r$. According to (1), it is easy to obtain that, $\text{od}(\mathbf{H}) \approx 0$.

Therefore, as a result of large scaled antennas, the channel gain matrix of Massive MIMO system is asymptotic diagonal orthogonal, which was introduced in detail in the following, and the traditional detection algorithms or their improvements for MIMO system are not appropriate for Massive MIMO system any longer. It is very imperative for the research on novel detection algorithms which are efficient for Massive MIMO system to be developed.

In recent years, the research on detection algorithms for Massive MIMO system has been developed, and is classed into and introduced by the several aspects, as shown in Fig. 3.

Examples for BER performance of the traditional detection algorithms for detecting 4-QAM signal in Massvie MIMO system are shown in Fig. 4, where the number of transmitting antennas and that of receiving antennas is $N_t = N_r = 128$ respectively, and the environment is rich scattering which means the channel is Rayleigh flat fading. Due to the high computational complexity of MLD algorithm in Massive MIMO system, the theoretical BER of single input and single output (SISO) system for detecting M-QAM signal in AWGN channel, is always taken as the lower bound for the BER performance of ML detection algorithm. Similarly, examples for the computational complexity versus the number of transmitting antennas N_t of the traditional detection algorithms for detecting 4-QAM signal in Massvie MIMO system are shown in Fig. 5, where we use the magnitude of the number of floating points operations (flops), which is denoted as $\mathrm{O}(\cdot)$ to evaluate the computational complexity in this paper.

Firstly, the local search based detection algorithms have been employed in Massive MIMO system. As one of the research hot spot in combination optimization technology, the local search method has been widely generalized and employed in many fields. In wireless communications feild, the local search algorithms are initially applied to detect multiuser in CDMA system, and then are gradually extended to be employed for detecting and estimating signal in MIMO system up to Massive MIMO system [5]. The local search detection algorithms for Massive MIMO system include the likelihood ascent search (LAS) detection algorithm [6], the randomized search (RS) detection algorithm and the reactive tabu search (RTS) detection algorithm [7] and etc. In Massive MIMO system equipped with large scaled antennas, the local search based detection algorithms are able to obtain the optimum BER performance of the ML detection algorithm during finite polynomial complexity time. However, both of the BER performance and computational complexity are decisively influenced by those of the initial solution vector. The traditional LAS based detection algorithm usually takes the detection result of linear detection algorithm as initial solution vector, such as that of MMSE detection algorithm, which is denoted as MMSELAS detection algorithm. Its BER performance and computational complexity is shown in Figs. 4 and 5, respectively. The computational complexity of MMSELAS detection algorithm is in the same magnitude order with that of MMSE detection algorithm. Furthermore, the MMSELAS detection algorithm is just efficient to detect low order modulation signal in Massive MIMO system, such as Binary Phase Shift Keying (BPSK) modulation signal or 4-Quadrature Amplitude Modulation (QAM) signal. Otherwise, for the high order modulation signal including 16-QAM signal or 64-QAM signal and etc., the BER performance of the MMSELAS detection algorithm is very poor and similar to that of the MMSE detection algorithm, which means that neither the MMSE detection algorithm nor the MMSELAS detection algorithm is appropriate to detect high order modulation signal in Massive MIMO system. When the RTS detection algorithm is employed for high order modulation signal in Massive MIMO system, its BER performance is getting worse and worse versus the increasing of either the antennas' number scale or the modulation order.

Secondly, the Markov Chain Monte Carlo (MCMC) [8] detection algorithm is also applied for Massive MIMO system. On account of the strong efficiency and flexible easy to operate, the MCMC method has been widely used in many research fields including financial industry and etc. In wireless communication field, the MCMC algorithm was extended to be employed for the receiver design in CDMA system and in MIMO system, until now to be applied to detect and estimate signal in Massive MIMO system. In Massive MIMO system, by means of the MCMC detection algorithm, as shown in Figs. 4 and 5, respectively, it is able to obtain the approximate optimum BER performance of ML detection algorithm, along with the low computational complexity which polynomially increases with the number of transmitting antennas. However, just like the LAS based detection algorithm, either the BER performance or the computational complexity of the MCMC detection algorithm is substantially effected by those of the initial solution vector. The exiting MCMC detection algorithm usually chooses the detection results of linear detection algorithms such as ZF detection algorithm or MMSE detection algorithm as initial solution vector, whose computational complexity is in the same order with that of linear detection algorithm, and BER performance approximates to the optimum one of ML detection algorithm for low modulation order signal, but is quite bad as that of linear detection algorithm for detecting high oder modulation signal. The traditional MCMC detection algorithm is not appropriate to detect high order modulation signal in Massive MIMO system. Although the multiple starts mixed Gibbs sampling improved MCMC detection algorithm provided the solution for high order modulation signal in medium small scaled MIMO system, its BER performance got rather worse versus the growth of the number of antennas or the modulation order, and is still not appropriate for detecting high order modulation signal in Massive MIMO system equipped with large scaled antennas.

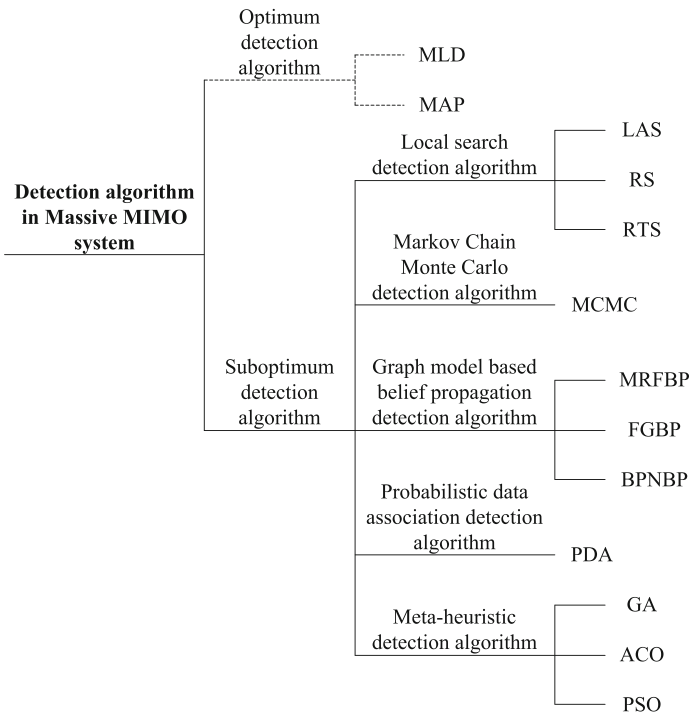


Fig. 3. The detection algorithms for Massive MIMO system

Thirdly, as one of the most important soft decision detection algorithm, the probabilistic data association (PDA) detection algorithm [9] has been also employed for the low complex detection in Massive MIMO system. At the beginning, the PDA detection algorithm was applied for multiple target tracking technology in remote sensing and radar fields. Along with the development of subjects fusion, the PDA algorithm has been gradually applied in communications filed, from the multiuser detection technology in CDMA wireless communication system, to be extended for the signal detection and estimation in MIMO system, until now for the signal detection and estimation in Massive MIMO system. In Massive MIMO system, as shown in Figs. 4 and 5, respectively, the PDA detection algorithm is able to obtain the approximate optimum BER performance over polynomial computational complexity. However, compared to other detection algorithms in Massive MIMO system, due to the PDA detection algorithm is one of the soft decision detection algorithms based on the bits domain, its computational complexity is more than one order higher than other low complexity detection algorithms. Further more, the BER performance of the PDA detection algorithm is poor for high order modulation signal. The PDA detection algorithm is still not appropriate to detect high order modulation signal in Massive MIMO system.

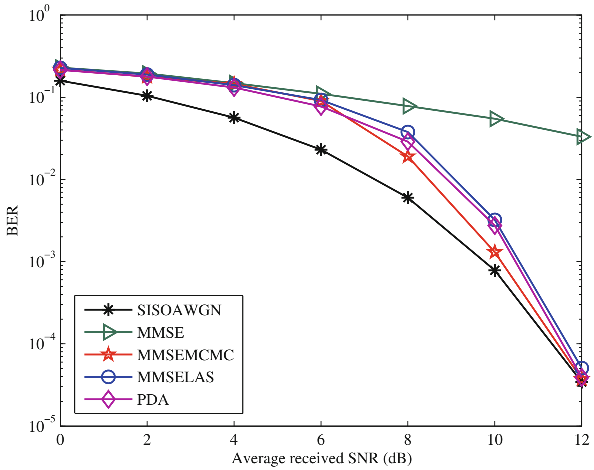


Fig. 4. Examples for BER performance of the traditional detection algorithms for detecting 4QAM signal in Massive MIMO system, where the number of transmitting antennas and that of receiving antennas is $N_t = N_r = 128$

Fourthly, the graph model based belief propagation (BP) detection algorithm is also one of the most important soft decision detection algorithms, and has been applied in the signal detection and decoding in Massive MIMO system. As the analysis method for the problem of probability distribution in complex system, the graph model based algorithm has been widely employed in lots of fields including engineering design and etc. The frequently-used major graph models include the Bayesian belief networks graph model, the Markov random fields (MRF) graph model and the factor graph (FG) model and etc. As one simple efficiency technology, the graph model based BP algorithm has been widely applied in many science domains such as the image processing, the computational biology and etc. In the communications field, various graph models based BP algorithm was initially employed in the multiuser detection in CDMA system, and then extended to be applied in signal detection and estimation in MIMO system, until now employed to solve the issue of signal detection and estimation in Massive MIMO system [10]. Along with polynomial computational complexity, the graph model based BP detection algorithm is able to achieve the approximate optimum BER performance of the ML detection algorithm. Whereby, the computational complexity of the FG model based BP detection algorithm linearly increases with the number of transmitting antennas as shown in Fig. 5, but due to its poor BER performance for high order modulation signal, the FG model based BP detection algorithm is not appropriate to detect high order modulation signal in Massive MIMO system.

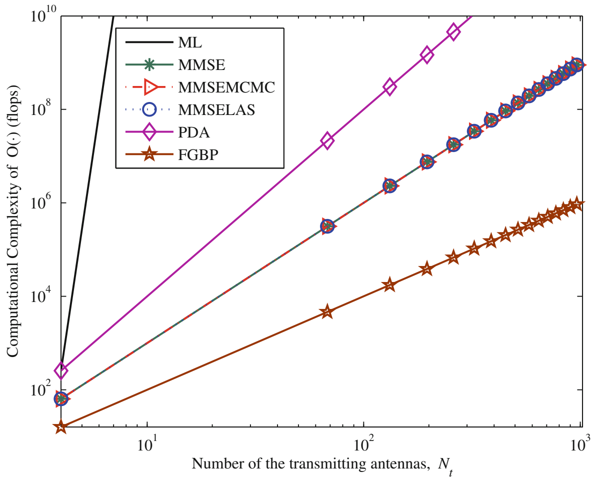


Fig. 5. Examples for computational complexity of the traditional detection algorithms for detecting 4QAM signal in Massive MIMO system

Finally, the meta-heuristic algorithm among the swarm intelligent technology based detection algorithm is another primary detection algorithm in Massive MIMO system, including the Genetic detection algorithm, ant colony optimization (ACO) detection algorithm [11] and particle swarm optimization (PSO) detection algorithm and etc. The BER performance of the meta-heuristic detection algorithm approaches the optimum one of ML detection algorithm during finite polynomial complexity time. However, the exiting research just presented the meta-heuristic based detection algorithm for media small scaled MIMO system. Along with the coming big data era, the artificial intelligent (AI) technology including the swarm intelligent technology is developing and improving in rapid speed. Therefore, the research on meta-heuristic based detection algorithms in Massive MIMO system has great and extensive application prospect and development potential.

Consequently, although the exiting research on the detection algorithms in Massive MIMO system has obtained essential developed, there are still lots of future work to be solved.

3 Crucial Questions for the Future Research

Through synthesizing the above discussion and analysis, the research on the detection algorithms for Massive MIMO system is still in a fledging period, there are lots of problems required to be solved and further improved in the future. Briefly, the crucial questions for the future research work are summarized and presented as follows.

3.1 Deep Learning Technology for Detection Algorithm in Massive MIMO System

Recently, the research on artificial intelligent (AI) is the leading edge science. As a research hot-spot in AI, the deep learning technology has got more and more attention, which has got many advantage achievements in the relevant fields. In 5G wireless communications, the deep learning technology has been employed for many aspects, including the channel estimation, the channel state information (CSI) feedback and reconstruction, the precoding [12,13], and etc. Further more, the inherent problem of the low complexity detection in Massive MIMO system is similar as that of big data processing, especially the one with high dimension. In consideration of the powerful processing ability of the deep learning technology for the high dimensional data, it has promising prospects to employ the deep learning technology for detection algorithm in Massive MIMO system, in order to obtain the optimum BER performance, and achieve the theoretical spectral efficiency with lower average receiving SNR, along with lower finite polynomial computational complexity, which is also a crucial and difficult question to be solved for the future research.

3.2 MMW Massive MIMO System: Strong Spatial Correlation Environment Even Existing the Keyhole Effect

Further more, the exiting detection algorithms for Massive MIMO system always take the transmission environment as rich scattering. However, in the more and more complex practical communications environment, this assumption is hard to be satisfied. For instance, the devices for Massive MIMO system are tending to be miniaturization by employing MMW technology, in order to be applicable for indoor communications. However, the channel space of Massive MIMO system in the indoor environment is usually strong spatial correlation at both between the transmitting and receiving antennas, and even generates the keyhole effect, which stops the signal transmission [14]. The exiting detection algorithms for Massive MIMO system are not able to detect signal in the strong space fading coherent environment and have no ability to eliminate the keyhole effect. Therefore, the research on detection algorithms for the strong space fading coherent environment even exiting the keyhole effect in MMW Massive MIMO system is difficult but important to carry out in the future.

3.3 Massive MIMO System in Hetnets Wireless Communications

When Massive MIMO system applied in outdoor mobile communications, many researches in 5G intend to combine the Massive MIMO technology with Hetnets wireless communications [15]. As a result of the wide coverage area, the channel propagation environment is random and diversity, which is different from the rich scattering environment where the traditional detection algorithms for Massive MIMO system usually assumed to state. Fortunately, the method of employing random matrix theory (RMT) both in the channel estimation and signal detection for Massive MIMO system in Hetnets makes it realizable. Thus, the research on low complexity detection algorithms for Massive MIMO system in Hetnets wireless networks, which is not limited by the channel conditions, is significant and challenge.

4 Conclusion

In this paper, a survey for the detection in Massive MIMO system is presented. From the introduction of Massive MIMO system, the research on low complexity detection algorithms is the hot spot among the key technologies. Through the comprehensive survey for the development of research on the low complexity detection algorithms in Massive MIMO system, as a result of the large scaled antennas, the channel gain matrix is asymptotic diagonal orthogonal, along with poor BER performance and high computational complexity, the traditional detection algorithms or their improvements for MIMO system are not appropriate for Massive MIMO system. The exiting detection algorithms for Massive MIMO system obtained the approximate optimum BER performance of ML detection algorithm over finite polynomial complex time. However, there are crucial problems to be solved, including the deep learning technology for the low complexity detection in Massive MIMO system, the detection research for the MMW Massive MIMO system which has strong spatial correlation environment even existing the keyhole effect, and the research on detection algorithms for Massive MIMO system in Hetnets wireless networks which is not limited by the channel conditions, and etc. In conclusion, the research on low complexity detection algorithms for Massive MIMO system is still in the initial beginning stage, lots of difficult but important issues are urgent to be solved, and the corresponding research work are significant to develop in the future.

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