



# A Coded Modulation Scheme for IoT System

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**Abstract.** The Internet of Things (IoT) gives huge challenges in computation complexity, response latency and transmission reliability. In this paper, we propose a coded modulation scheme for short packet applicable to IoT system. The structure of the proposed scheme is similar to traditional convolutional codes. The states by encoding the information bits are immediately modulated to achieve joint design of coding and modulation. We further theoretically give the optimal generator matrix by maximizing the minimum distance among the branch modulated symbols. Finally, we give some numerical simulations to demonstrate that the proposed scheme is applicable to IoT system.

**Keywords:** IoT · Channel coding · Coded modulation · Convolutional codes

## 1 Introduction

The Internet of Things (IoT) has been attracting many attentions from all over the world recently owing to its huge capacity to connect billions of devices and providing wide-range services for them [1, 2]. In many application scenarios, the low response latency for IoT services is considerable challenging from system architecture to wireless physical layer technologies [3]. One of them is channel coding that plays an important role on physical layer. It dominates the transmission reliability, the response latency and receiving computation delay [4].

In IoT system, limited by transmission packet size, the codes must be very short, which will critically deteriorate the performance [5, 6]. The eligible codes in IoT system should have the capability to support short packet with low computation complexity, low latency and high reliability [7]. Polar codes have successfully applied in commercial communication system, such as 5G cellular communication system [8]. It can achieve the capacity when the length of code is large

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enough [9]. Moreover, it also performs well in finite length regime. The new developed polar codes called polarization-adjustment convolutional (PAC) codes can achieve the bound of finite-length codes at some specific parameters [10]. Both of them are promising codes in application in IoT system [11]. Unfortunately, their performances rely heavily on the constructions, which are usually specific based on various code size and rate [12, 13]. It limits the adaptability of codes for various scenarios.

Joint design of coding and modulation can asymptotically achieve high-order discrete memoryless channel (DMC) and additional white Gaussian noise channel (AWGN) [14]. In practical, they are competitive in low signal-to-noise ratio (SNR) channel and strong interference channel as well [15]. They also have advantages in flexible parameter design and construction [16]. However, the traditional coded modulation scheme are usually considered independently for coding and modulation cause they are usually effective for long codes [17]. In this paper, we propose a coded modulation scheme that is similar to traditional convolutional codes. The codeword is recursively generated by encoding the information bits. Different from the traditional convolutional codes, the encoded states are further one-by-one modulated to be modulation symbols. At the receiver, some traditional convolutional suboptimal decoding algorithms [18–20], such as sequential decoding and list decoding, etc., can be efficiently used to decode the code. We further improve the minimum distance of the branch modulated symbols by theoretically give the optimal generator matrix. By numerically comparing with some high-performance short codes, such as polar codes and PAC codes, we finally demonstrate the proposed scheme is promising in application of IoT system.

## 2 Construction of the Proposed Code

### 2.1 Encoding and Decoding of the Proposed Code

The diagram of the proposed coding is shown in Fig. 1. The information bits are written in two-dimensional form as  $\mathbf{b}_i = \{b_{i,1}, b_{i,2}, \dots, b_{i,\kappa}\}$ ,  $i = 1, 2, \dots, n$ . Then a state  $\mathbf{s}_i = \{s_{i,1}, s_{i,2}, \dots, s_{i,n}\}$ ,  $i = 1, 2, \dots, n$  can be recursively calculated as

$$\mathbf{s}_i = [\mathbf{s}_{i-1} \ \mathbf{b}_i] \mathbf{G} \quad (1)$$

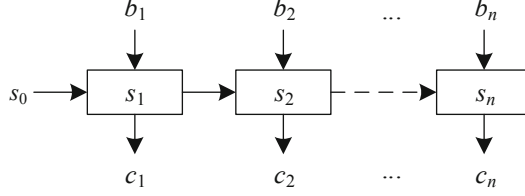
where  $\mathbf{G}$  is the binary generator matrix with dimension  $(n + \kappa) \times n$ .  $\mathbf{s}_0$  is initially set to a random binary vector with length of  $n$ . The modulation symbol  $c_i$ ,  $i = 1, 2, \dots, n$  is calculated as

$$c_i = \frac{\lambda}{2^n} \left( 2 \sum_{\tau=1}^n 2^{\tau-1} s_{i,\tau} - 1 \right) \quad (2)$$

So far, we have introduced the basic construction of the proposed code. It is a code with code length  $n$ , basic rate  $R = \kappa$ . The rate can be any adjusted by inserting known bits in  $\mathbf{b}_i$ .

$\mathbf{G}$  can be further decomposed into the following form:

$$\mathbf{G} = [\mathbf{G}_L \ \mathbf{G}_R]^T \quad (3)$$



**Fig. 1.** The diagram of the proposed coding.

where  $\mathbf{G}_L$  and  $\mathbf{G}_R$  are with dimensions of  $n \times n$  and  $\kappa \times n$ , respectively. Obviously,  $\mathbf{G}_L$  is the generator matrix that encodes the information from the previous state  $\mathbf{s}_{i-1}$ , and  $\mathbf{G}_R$  is responsible for encoding the current information bits  $\mathbf{b}_i$ . Then Eq.(1) can be rewritten as

$$\mathbf{s}_i = \mathbf{s}_{i-1} \mathbf{G}_L \oplus \mathbf{b}_i \mathbf{G}_R = \delta_i \oplus \mathbf{b}_i \mathbf{G}_R \quad (4)$$

where  $\delta_i = \mathbf{s}_{i-1} \mathbf{G}_L$  represents the state encoded from all of the past information bits.

We suppose the modulated symbols  $c_i$  are transmitted through the AWGN channel as

$$y_i = c_i + n_i \quad (5)$$

where  $n_i$  is the noise with mean 0 and variance  $\sigma^2$ . The decoding procedure is similar to the traditional convolutional code. The goal of decoding is to find a minimum global metric from all of the possible transmitting modulated symbols. The global metric formula is calculated as

$$M_{i,j2^\kappa+t} = M_{i-1,j} + m_{i,t} \quad (6)$$

where  $M_{i-1,j}$  means the  $j$ th,  $j = 1, 2, \dots, \Omega$ , global metric related to the  $(i-1)$ th received symbol and  $\Omega$  a parameter related to decoding complexity.  $m_{i,t}$  represents the  $t$ th,  $t = 1, 2, \dots, 2^\kappa$  branch metric related to the  $i$ th received symbol. And it is calculated as

$$m_{i,t} = (c_{i,t} - y_i)^2 \quad (7)$$

where  $c_{i,t}$ ,  $t = 1, 2, \dots, 2^\kappa$  means the possible transmitting symbols related to the  $i$ th received symbol. Since  $\Omega$  grows exponentially along with  $n2^\kappa$ , we adopt some suboptimal decoding algorithms for convolutional code [18–20], such as sequential decoding and list decoding, etc. to restrict the complexity.

Usually,  $\mathbf{G}_L$  is constructed in a random way to randomizing the encoding. However,  $\mathbf{G}_R$  with small row size dominating the current modulation symbol  $c_i$  performance can be improved by carefully designing.

## 2.2 Design of $\mathbf{G}_R$

As shown in Fig. 2, while the previous state is randomly transformed to be  $\delta_i$  by  $\mathbf{s}_{i-1}$  times  $\mathbf{G}_L$ , it outputs  $2^\kappa$  states  $\mathbf{s}_{i,j}$ ,  $i = 1, 2, \dots, n, j = 1, 2, \dots, 2^\kappa$  and  $2^\kappa$  modulated symbols  $c_{i,j}$  corresponding to  $\kappa$  information bits  $\mathbf{b}_{i,j}$ . Since the designing generator matrix  $\mathbf{G}_R$  is the same for all of the encoding stage, we ignore the subscript  $i$  in the following. Let superscript ' and '' means any pair of states, symbols or information bits.

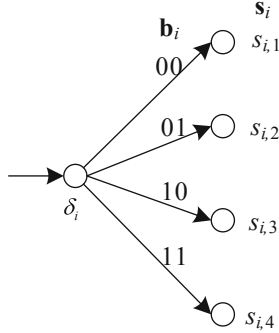


Fig. 2. An example of the branch states.

We define the distance of a pair of current branch modulation symbols ( $c'$ ,  $c''$ ) as

$$\begin{aligned}
 d(c', c'') &= |c' - c''| \\
 &= \left| \frac{\lambda}{2^n} \left( 2 \sum_{\tau=1}^n 2^{\tau-1} s'_\tau - 1 \right) - \frac{\lambda}{2^n} \left( 2 \sum_{\tau=1}^n 2^{\tau-1} s''_\tau - 1 \right) \right| \\
 &= \frac{\lambda}{2^{n-1}} \left| \sum_{\tau=1}^n 2^{\tau-1} (s'_\tau - s''_\tau) \right|
 \end{aligned} \tag{8}$$

where  $\lambda$  represents the power factor to adjust the average transmission power. When  $\delta$  and  $\mathbf{G}_R$  are given,  $d(c', c'')$  is determined by information bits  $\mathbf{b}'$  and  $\mathbf{b}''$ . So for convenience, we use  $d(\mathbf{b}', \mathbf{b}'')$  to replace  $d(c', c'')$  in the following. The minimum distance of  $d(\mathbf{b}', \mathbf{b}'')$  can be written as

$$d_m = \min_{\mathbf{b}', \mathbf{b}'', \mathbf{b}' \neq \mathbf{b}''} d(\mathbf{b}', \mathbf{b}'') \tag{9}$$

We optimize  $\mathbf{G}_R$  by maximizing the minimum distance  $d_m$  as

$$D_m = \max_{\mathbf{G}_R} \min_{\mathbf{b}', \mathbf{b}'', \mathbf{b}' \neq \mathbf{b}''} d(\mathbf{b}', \mathbf{b}'') \tag{10}$$

In the following, we give two propositions.

**Proposition 1.** When  $\mathbf{G}_R = [\mathbf{I}_\kappa \ \mathbf{O}]$ , where  $\mathbf{I}_\kappa$  is the identity matrix of dimension  $\kappa$ ,  $\mathbf{O}$  is the all-zero matrix of  $\kappa \times (n - \kappa)$ , for any  $\delta$ , the minimum distance of  $d(\mathbf{b}', \mathbf{b}'')$  is equal to  $\lambda/2^{\kappa-1}$ .

*Proof.* There are  $\mathbf{b}'\mathbf{G}_R = \mathbf{b}'[\mathbf{I}_\kappa \ \mathbf{O}] = [\mathbf{b}' \ \mathbf{O}]$ , and  $\mathbf{b}''\mathbf{G}_R = \mathbf{b}''[\mathbf{I}_\kappa \ \mathbf{O}] = [\mathbf{b}'' \ \mathbf{O}]$ . Then  $\mathbf{s}' = \delta \oplus \mathbf{b}'\mathbf{G}_R = \delta \oplus [\mathbf{b}' \ \mathbf{O}]$  and  $\mathbf{s}'' = \delta \oplus \mathbf{b}''\mathbf{G}_R = \delta \oplus [\mathbf{b}'' \ \mathbf{O}]$  whose elements are

$$s'_\tau = \begin{cases} \delta_\tau \oplus b'_\tau & , \tau \leq \kappa \\ 0 & , \tau > \kappa \end{cases} \quad (11)$$

and

$$s''_\tau = \begin{cases} \delta_\tau \oplus b''_\tau & , \tau \leq \kappa \\ 0 & , \tau > \kappa \end{cases} \quad (12)$$

Then

$$\begin{aligned} d(\mathbf{b}', \mathbf{b}'') &= \frac{\lambda}{2^{n-1}} \left| \sum_{\tau=1}^n 2^{\tau-1} (s'_\tau - s''_\tau) \right| \\ &= \frac{\lambda}{2^{n-1}} \left| \sum_{\tau=1}^{\kappa} 2^{n-\kappa-\tau} (\delta_\tau \oplus b'_\tau - \delta_\tau \oplus b''_\tau) \right| \\ &= \frac{\lambda}{2^{\kappa-1}} \left| \sum_{\tau=1}^{\kappa} 2^{\tau-1} (b'_\tau - b''_\tau) \right| \\ &\geq \frac{\lambda}{2^{\kappa-1}} \end{aligned} \quad (13)$$

iff  $b_1! = b''_1, b'_\tau = b''_\tau, \tau > 1, d(\mathbf{b}', \mathbf{b}'') = \lambda/2^{\kappa-1}$ .

**Proposition 2.** There doesn't exist  $\mathbf{G}_R$  for any  $\delta$  such that  $d_m > \lambda/2^{\kappa-1}$ .

*Proof.* Assuming that there is a  $\mathbf{G}_R$  such that  $d_m > \lambda/2^{\kappa-1}$ , since the elementary transformation of matrices will not reduce the minimum distance, we can carry out the elementary transformation  $\mathbf{G}_R$  to the systematic code form as  $\mathbf{G}_R = [\mathbf{I}_\kappa \ \varepsilon]$  where  $\varepsilon$  is a binary matrix of  $\kappa \times (n - \kappa)$ . Obviously, according to Proposition 1,  $\varepsilon$  must be a nonzero matrix. Otherwise, its minimum distance is equal to  $\lambda/2^{\kappa-1}$ , then  $[\mathbf{b}' \ \mathbf{G}_R] = [\mathbf{b}' \ \varphi']$ , and  $[\mathbf{b}'' \ \mathbf{G}_R] = [\mathbf{b}'' \ \varphi'']$ ,  $\varphi'$  and  $\varphi''$  are a nonzero vector of length  $n - \kappa$ , and they are not equal, then

$$\begin{aligned} d(\mathbf{b}', \mathbf{b}'') &= \\ &= \frac{\lambda}{2^{n-1}} \left| \sum_{\tau=1}^{\kappa} 2^{n-\kappa+\tau-1} (b'_\tau - b''_\tau) + \sum_{\tau=1}^{n-\kappa} 2^{\tau-1} (\delta_\tau \oplus \varphi'_\tau - \delta_\tau \oplus \varphi''_\tau) \right| \\ &= \frac{\lambda}{2^{n-1}} |\Delta_1 + \Delta_2| \end{aligned} \quad (14)$$

Clearly,  $|\Delta_1| \leq 2^{n-\kappa}$ . In order to make the minimum distance of  $d(\mathbf{b}', \mathbf{b}'')$  greater than  $\lambda/2^{\kappa-1}$ , it means that when  $\Delta_1$  taking the minimum value,  $\Delta_2$  must

be greater than 0. Assuming that there is a vector  $\delta$  such that  $\Delta_2$  is greater than 0, then with the negation of  $\delta$ , we have

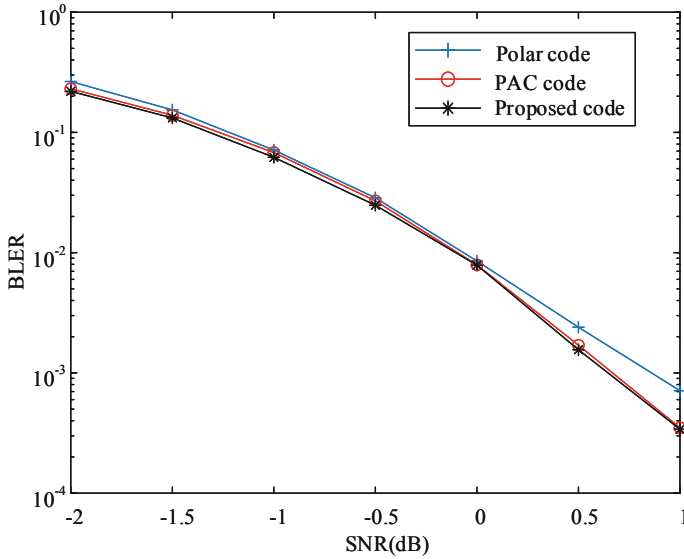
$$\begin{aligned}\bar{\Delta}_2 &= \sum_{\tau=1}^{n-\kappa} 2^{\tau-1} (\bar{\delta} \oplus \varphi' - \bar{\delta} \oplus \varphi'') \\ &= - \sum_{\tau=1}^{n-\kappa} 2^{\tau-1} (\delta \oplus \varphi' - \delta \oplus \varphi'') \\ &= -\Delta_2\end{aligned}\tag{15}$$

In this case,  $\bar{\Delta}_2$  is less than 0. So there is no  $\mathbf{G}_R$  for any  $\delta$  such that  $d_m > \lambda/2^{\kappa-1}$ .

It concludes that the maximum distance equaling  $\lambda/2^{\kappa-1}$  can be achieved by simply setting  $\mathbf{G}_R = [\mathbf{I}_\kappa \mathbf{O}]$ .

### 3 Numerical Results

In this section, we will give some simulations to show the advantage of the proposed code. We choose a polar code and a PAC code for the comparison. The basic simulation parameters including the length of the codeword and the length of the information bits are 128 and 40 for the three codes. The channel we choose is AWGN channel. In particular, the construction methods of the polar code and the PAC code are Gaussian approximation and Reed-Muller Gaussian approximation, respectively. For the polar code, we use 4-bit CRC aided to improve the performance. Correspondingly, a CRC-aided list successive cancellation algorithm with list length of 32 is used to decode the code. For the PAC code, the decoding algorithm is list successive cancellation. For the proposed code, to keep the same of code length and rate, we let  $\kappa = 1$  and set 40 zeros by equal interval among the first 80 information bits and append 48 zeros at the end of information bits as tail bits. The decoding algorithm we choose is list decoding algorithm. The final simulation results in terms of block error rate (BLER) are shown in Fig. 3. From this figure, we can see that the three codes show very close performance in the low SNR regime. But in high SNR regime, say, 0.5dB  $\sim$  1dB, the polar code performs slightly worse than the others. And the proposed code has almost the same performance with the PAC code. But the proposed code is with advantage in flexible construction and parameter design. Any codes with various lengths and rates can be constructed in real time, which is very competitive in IoT system.



**Fig. 3.** BLER performance comparison among the polar code, the PAC code and the proposed code.

## 4 Conclusions

To meet the requirement of channel coding in IoT system, we study a coded modulation scheme for short packet with high reliability and flexible construction. Different from the traditional coded modulation schemes, the modulated symbols are generated one-by-one directly from the encoded states. The state is generated similar to the traditional convolutional codes. Accordingly, some traditional convolutional suboptimal decoding algorithms, such as sequential decoding and list decoding, etc., can be used to decode the code. By theoretically design the optimal generator matrix, we further improve the minimum distance among the branch modulated symbols. Finally, we give some numerical results to compare BLER performance with some high-performance short codes, such as polar code and PAC code and demonstrate the applicability of the proposed scheme in IoT system.

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