Towards the Future Data Market: Reward Optimization in Mobile Data Subsidization

Zehui Xiong[1,2], Jun Zhao[2], Jiawen Kang[3(✉)], Dusit Niyato[2], Ruilong Deng[4], and Shengli Xie[5,6]

[1] Alibaba-NTU Joint Research Institute, NTU, Singapore, Singapore
[2] School of Computer Science and Engineering, NTU, Singapore, Singapore
[3] Energy Research Institute, NTU, Singapore, Singapore
kavinkang@ntu.edu.sg
[4] College of Control Science and Engineering, School of Cyber Science and Technology, Zhejiang University, Hangzhou, China
[5] Guangdong-Hong Kong-Macao Joint Laboratory for Smart Discrete Manufacturing, Guangzhou, China
[6] Joint International Research Laboratory of Intelligent Information Processing and System Integration of IoT, Ministry of Education, Guangzhou, China

Abstract. Mobile data subsidization launched by network operators is a promising business model to provide some economic insights on the evolving direction of the 4G/5G and beyond mobile data market. The scheme allows content providers to partly subsidize mobile data consumption of mobile users in exchange for displaying a certain amount of advertisements. The users are motivated to access and consume more content without being concerned about overage charges, yielding higher revenue to the data subsidization ecosystem. For each content provider, how to provide appropriate data subsidization (reward) competing with others to earn more revenue and gain higher profit naturally becomes the key concern in such a ecosystem. In this paper, we adopt a hierarchical game approach to model the reward optimization process for the content providers. We formulate an Equilibrium Programs with Equilibrium Constraints (EPEC) problem to characterize the many-to-many interactions among multiple providers and multiple users. Considering the inherent high complexities of the EPEC problem, we propose to utilize the distributed Alternating Direction Method of Multipliers (ADMM) algorithm to obtain the optimum solutions with fast-convergence and decomposition properties of ADMM.

Keywords: Data subsidization · Next-generation mobile data · Network economics · Game theory

Corresponding author: Jiawen Kang

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1 Introduction

With the rapidly growing demand of network data volume in mobile 5G and beyond, *mobile data subsidization* is envisioned as a promising scheme to bring in additional revenue for network operator in wireless networks. This scheme allows users to consume content (e.g., watch video content from HBO, Hulu, and Netflix) subsidized by content providers without eating into their data allowances. Therefore, the scheme motivates users to access and consume more content without being concerned about overage charges, yielding higher revenue to the data subsidization ecosystem. Data subsidization has become an appealing area of research since its emergence. There are numerous application examples of data subsidization programs. Some network operators in the US such as AT&T and Verizon have proposed the *Data Sponsoring Plan* and *FreeBee Sponsored Data Plan*, respectively, that permit content providers to partly pay for the data consumption fees instead of mobile users themselves [1]. Under the data subsidization scheme, the mobile users, i.e., the subscribers of network content services can offload their data usage fees to content providers. In particular, the data usage of mobile users is allowed to be partly subsidized, but in return the mobile users need to receive and view a certain amount of advertisements. Although the content providers bear the cost of subsidization, the advertisement earnings increases consequently. Clearly, the data subsidization potentially leads to a mutual benefit for both participants in wireless networks. Meanwhile, this scheme will guarantee the continuity and efficiency of cellular data service in 5G and beyond, thereby improving the connectivity of massive number of mobile devices to heterogeneous cellular networks [6].

The subsidization can be treated as the reward offered to users, and how to provide appropriate subsidization, i.e., the reward optimization, is naturally a key concern for the content providers. Furthermore, the data subsidization scheme can foster greater reward competition among the content providers. However, the competition for earning from the users among content providers has not been formally studied in the literature. Moreover, most of the existing works [2,14,15,17,21] on mobile data subsidization consider that the users can only consume the content with advertisement (i.e., *sponsored content*) and passively accept the corresponding reward. Therein, the only active strategies of users are to choose how much sponsored content that they demand. However, the users are able to reject the sponsored content and still consume the *normal content* paying full data usage fees without advertisement. The users aim to maximize their individual payoff in a self-interest manner by finding the balance between sponsored content and normal content. The strategic behaviors of users further make it more challenging to explore the reward competition among providers. Essentially, when there are tremendous numbers of providers and users in future mobile data market, the constrained optimization [7] and traditional game techniques [19,21] are practically inapplicable in terms of the complexity and scalability. Nevertheless, this has not been well-addressed in existing literature that motivates the study of this paper.

In this paper, we study the reward optimization for content providers in the framework of mobile data subsidization by analyzing the rational behaviors of both content providers and users. We discern the fact that game theory is a suitable analytical tool to address such a two-sided interaction problem [9,10,18]. Therefore, we investigate the interactions among the content providers and mobile users by formulating a hierarchical Stackelberg game model. The game model is developed to jointly maximize the profits of content providers, and the utilities of mobile users. In the game, the content providers are the leaders that determine the reward offered to users first. Then, the users are the followers that decide on how much sponsored content and normal content to consume based on the reward claimed by the leaders. Specifically, the major contributions of this paper are summarized as follows:

1. A hierarchical Stackelberg game is established to model the strategic interactions among multiple content providers and multiple users in the data subsidization system, where the profits of content providers and the utilities of users are jointly maximized.
2. To explore the reward competition among providers, we then formulate an Equilibrium Programs with Equilibrium Constraints (EPEC) problem to characterize the many-to-many interactions among multiple providers and multiple users. Considering the inherent high complexities of EPEC problem, we propose to employ the distributed Alternating Direction Method of Multipliers (ADMM) algorithm to tackle the EPEC problem. Taking advantage of the fast-convergence property and high scalability of ADMM, we derive the optimum solutions with reasonable complexity in a distributed manner.
3. Numerical simulations are conducted to demonstrate the analytical results and evaluate the system performance in the proposed Stackelberg game-based schemes. The results confirm that with the proposed scheme, the optimization of the profits of content providers and the optimization of the utility of users can be jointly attained.

The rest of the paper is structured as follows. Section 2 shows the system description characterizing the mobile data subsidization system and develops a hierarchical game framework. We formulate and study the multi-provider multi-user Stackelberg game in Sect. 3. Section 4 presents the performance evaluation, and Sect. 5 concludes the paper.

2 Problem Formulation

In this section, we propose the model of mobile data subsidization, and we utilize a hierarchical game approach to characterize the model and analyze the reward optimization for content providers therein.

2.1 Mobile Data Subsidization Model

As illustrated in Fig. 1, we consider a 4G/5G system with mobile data subsidization in which there are M heterogeneous Content Providers (CPs) labeled as $o_1, o_2, \ldots, o_M$, such as the video CPs: Youtube, Netflix, Hulu, and Vimeo. The CPs can subsidize the mobile data usage of N heterogeneous Mobile Users

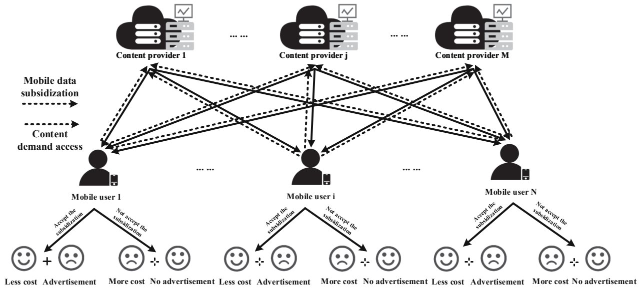

Fig. 1. Schematics of mobile data subsidization model.

(MUs) labeled as $s_1, s_2, \ldots, s_N$. The MUs can access and consume the content from different CPs, and the content can be downloaded directly through the network infrastructure such as 4G/5G base stations. The data usage of MUs can be partly subsidized by CPs via displaying a certain amount of advertisements. The CP earns the advertisement revenue when the MU accepts the offered data subsidization, i.e., the reward. Meanwhile, the MUs are able to reject viewing the advertisements, and still access and consume the content paying the full data usage fees without subsidization.

We denote y as the content (volume) demand from an MU, and $\sigma g(y)$ as its utility derived from enjoying the content, where $\sigma > 0$ is a factor representing the utility coefficient of MU, e.g., a particular valuation between MU and content. Similar to [5,16], we first define the following function, $g(y) = \frac{1}{1-\alpha} y^{1-\alpha}$, where $0 < \alpha < 1$ is a given coefficient. Specifically, $g(\cdot)$ is a non-decreasing and concave function with decreasing marginal satisfaction, representing the decreasing marginal preference of MUs to content. In traditional content consumption, the usage-based pricing is popular, i.e., the network operator charges MU s_i a certain unit price p_i for the volume of content downloaded. Here, the unit price is the same for all MUs for fairness, i.e., $p_i = p, \forall i \in \{1, 2, \ldots, N\}$. Therefore, the utility formulation of the MU with content demand y is expressed by $v(y) = \sigma g(y) - py$.

With the data subsidization scheme, the data usage fees for accessing the content can be partly subsidized by the CP. Denote $\theta \in [0, 1]$ as a reward factor of content subsidized by the CP, i.e., θ units of the content is subsidized. Therefore, if the MU accepts the subsidization, it pays for the rest $(1 - \theta)y$ units of content, with the cost $(1 - \theta)py$ incurred to the MU [16]. However, the MU's utility of enjoying the content is discounted as it needs to view a certain amount of advertisement displayed by the CP. We denote l_a as the amount of advertisement imposed by the CP per volume of content. In what follows, we assume that l_a is constant for all content. For example, Hulu plays the same amount of advertisement regularly between videos for all of its subscribers (users). We consider a normalized $l_a \in [0, 1]$ since the CPs have the amount of advertisements strictly less than that of the provided content. For the ease of derivation, we define an auxiliary variable as $\tau = \frac{1}{1+l_a}, \tau \in \left[\frac{1}{2}, 1\right]$, representing a discounting factor in terms of viewing advertisement. The larger the length of the advertisement per content, the smaller of τ. Therefore, the utility of the MU which has the content demand y that accepts the subsidization from the CP is formulated as follows:

$$\hat{u}(y) = \tau\sigma g(y) - (1-\theta)py. \tag{1}$$

2.2 Game Formulation

Without loss of generality, we first assume that the MU, i.e., the buyer, decides on a fraction of the content demand to access that accepts the data subsidization (*sponsored content*), denoted by x, where $x \in [0, 1]$. Then, the fraction of the content demand of that not accepts the data subsidization, i.e., the *normal content* demand of the MU is $1-x$ accordingly. Note here that the MUs' actions (the content volume to be purchased) are normalized to sum up to one, e.g., one video content [16]. Nevertheless, the analytical results will not structurally change even if the content demand is not normalized [16].

Mobile Users in Stage II. Let $x_{i,j}$ and $\theta_{i,j}$ denote the content demand of MU s_i that accepts the data subsidization from CP o_j, and the reward factor of CP o_j for MU s_i, respectively. Given the reward factors $\boldsymbol{\theta}_j = \{\theta_{1,j}, \theta_{2,j}, \dots, \theta_{N,j}\}$ determined by each CP $o_j, j \in \{1, 2, \dots, M\}$, each rational MU $s_i, i \in \{1, 2, \dots, N\}$ decides the strategies of content demand from different CPs that maximizes its utility. Specifically, each MU aim to maximize its utility by finding the balance between the content with subsidization (sponsored content) and that without subsidization (normal content). Let $\boldsymbol{x}_i = \{x_{i,1}, x_{i,2}, \dots, x_{i,M}\}$ be the content demand strategies of MU s_i from all CPs, $\boldsymbol{x}_{-i}$ be the strategies of all other MUs except MU s_i, and $\boldsymbol{\Theta} = \{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \dots, \boldsymbol{\theta}_M\}$ be the reward strategies of all CPs, i.e., the reward factors. Therefore, the sum utility of MU s_i obtained from all CPs is formulated as follows:

$$\begin{aligned} u_i(\boldsymbol{x}_i; \boldsymbol{x}_{-i}, \boldsymbol{\Theta}) = \sum_{j=1}^{M} \omega_{i,j} \Big(& \tau\sigma_i f(x_{i,j}) - (1-\theta_{i,j}) x_{i,j} p \\ & + \sigma_i f(1-x_{i,j}) - (1-x_{i,j})p \Big), \\ & \forall i \in \{1, 2, \dots, N\}, \end{aligned} \tag{2}$$

where

$$f(x_{i,j}) = \frac{1}{1-\alpha} x_{i,j}^{1-\alpha}, \tag{3}$$

and $\omega_{i,j}$ is the probability for MU s_i to choose CP o_j to access the content, and $\sum_{j=1}^{M} \omega_{i,j} = 1$, $\forall i = \{1, 2, \dots, N\}$. For example, $\omega_{i,j} = 1$ (or $\omega_{i,j} = 0$) means that MU s_i only access (or not access) the content from CP o_j.

Consequently, each MU s_i needs to choose its optimal strategies $\boldsymbol{x_i}$ based on the strategies of all other MUs except MU s_i (i.e., $\boldsymbol{x}_{-i}$) and the reward factors announced by all CPs (i.e., $\boldsymbol{\Theta}$), by solving the following optimization problem:

Problem 1. (The MU s_i sub-game)

$$\begin{aligned} \underset{\boldsymbol{x}_i}{\text{maximize}} \quad & u_i(\boldsymbol{x}_i; \boldsymbol{x}_{-i}, \boldsymbol{\Theta}), \forall i \in \{1, 2, \dots, N\}, \\ \text{subject to} \quad & x_{i,j} \in [0, 1]. \end{aligned} \tag{4}$$

Content Providers in Stage I. Let μ_j denote the advertisement revenue coefficient of CP o_j, and hence $\mu_j h(x)$ represents the advertisement revenue obtained from MUs that watch the advertisements, in which $h(x)$ is defined by using the α-fair function [16,19], as follows:

$$h(x) = \frac{1}{1-\gamma} x^{1-\gamma}, \tag{5}$$

where $0 < \gamma < 1$ which is a coefficient. Each CP o_j aims to maximize the obtained total profit, i.e., the gained advertisement revenue minus the cost of subsidization offered to MUs, which can be formulated as follows:

$$\Pi_j(\boldsymbol{\theta}_j; \boldsymbol{\theta}_{-j}, \boldsymbol{X}, \boldsymbol{\omega}) = \sum_{i=1}^{N} \omega_{i,j} \Big(\mu_j h(x_{i,j}) - \theta_{i,j} p x_{i,j} \Big), \tag{6}$$

$\forall j \in \{1, 2, \dots, M\}$, where $\boldsymbol{\omega} = \langle \omega_{i,j} \rangle, i \in \{1, 2, \dots, N\}, j \in \{1, 2, \dots, M\}$ denotes the pairing probability between CPs and MUs, $\boldsymbol{\theta}_{-j}$ denotes the optimal strategies of all other CPs except CP o_j ($\boldsymbol{\theta}_{-j} = \boldsymbol{\Theta} \backslash \boldsymbol{\theta}_j$), and $\boldsymbol{X}$ denotes the optimal strategies of all MUs in terms of content demand, i.e., $\boldsymbol{X} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_N\}$.

By adopting the incentive mechanism method in [20], we can set the probability of each MU s_i choosing CP o_j as

$$\omega_{i,j} = \frac{\theta_{i,j}}{\sum\limits_{k=1}^{M} \theta_{i,k}}. \tag{7}$$

By observing from (7), we know that the value of $\omega_{i,j}$ is larger on condition that the reward factor offered by CP o_j increases given the fixed reward strategies of other CPs. This indicates that MU s_i obtains the greater data subsidization from CP o_j. As such, in order to attract more MUs, each CP o_j ($j \in \{1, 2, \dots, M\}$), tends to provide greater subsidization (i.e., higher reward factors) to MUs. The reason is that MUs are more likely to access and consume the content from CP o_j when $\omega_{i,j}$ increases. However, each CP o_j, $\forall j \in \{1, 2, \dots, M\}$, cannot maintain the value of reward factor $\theta_{i,j}$ that is too high to reduce the subsidization cost. Moreover, when each CP o_j determine its reward factors $\boldsymbol{\theta}_j$ for different MUs, the CP needs to consider the reward factors offered by other CPs (i.e., $\boldsymbol{\theta}_{-j}$) as well as the strategies of all MUs (i.e., $\boldsymbol{X}$). This thereby leads to the reward competition among CPs. Therefore, the optimization problem for each CP is defined as follows:

Problem 2. (The CP o_j sub-game)

$$\begin{aligned} \underset{\boldsymbol{\theta}_j}{\text{maximize}} \quad & \Pi_j(\boldsymbol{\theta}_j; \boldsymbol{X}, \boldsymbol{\theta}_{-j}, \boldsymbol{\omega}), \forall j \in \{1, 2, \ldots, M\}, \\ \text{subject to} \quad & \theta_{i,j} \in [0, 1]. \end{aligned} \tag{8}$$

Considering the inherent leader-follower relations among the CPs and MUs, we hence model the two-sided interaction problem as a hierarchical game. In particular, we model the problem as a multi-leader multi-follower Stackelberg game, in which the CPs are leaders and the MUs are followers. Consequently, **Problems 1** and **2** jointly form a Stackelberg game with the objective of finding the Stackelberg equilibrium. The Stackelberg equilibrium is defined as a point where the payoffs of the leaders are maximized given that the followers adopt their best responses [3]. In the following, we define the Stackeberg game.

Definition 1. *Let $\boldsymbol{X}^*$ and $\boldsymbol{\Theta}^*$ denote the optimal content demand strategies of MUs (followers) and the optimal reward strategies of CPs (leaders), respectively. Let $\boldsymbol{x}_i$ be the strategy of MU s_i, $\boldsymbol{x}_{-i}$ be the strategies of all other MUs except MU s_i, $\boldsymbol{\theta}_j$ be the strategy of CP o_j, and $\boldsymbol{\theta}_{-j}$ be the strategies of all other CPs except CP o_j. Then, the point $(\boldsymbol{X}^*, \boldsymbol{\Theta}^*)$ is the Stackelberg equilibrium of the multi-leader multi-follower game provided that the following conditions,*

$$\Pi_j(\boldsymbol{\theta}_j^*, \boldsymbol{\theta}_{-j}^*, \boldsymbol{X}^*) \geq \Pi_j(\boldsymbol{\theta}_j, \boldsymbol{\theta}_{-j}^*, \boldsymbol{X}^*), \forall j, \tag{9}$$

and

$$u_i(\boldsymbol{x}_i^*, \boldsymbol{x}_{-i}^*, \boldsymbol{\Theta}^*) \geq u_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}^*, \boldsymbol{\Theta}^*), \forall i, \tag{10}$$

are satisfied, where $\boldsymbol{X} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N\}$ and $\boldsymbol{\Theta} = \{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_M\}$.

In the context of game theory, each of the leaders (CPs) or the followers (MUs) is rational and autonomous making the decision in a distributed manner [8]. In the following sections, we investigate the Stackelberg equilibrium by analyzing the optimal strategies of the followers and leaders in the game.

3 Multi-CP Multi-MU Game as EPEC

In this section, we investigate the general multi-leader multi-follower game that incorporates M CPs and N MUs, i.e., the many-to-many interaction. In such a multi-CP and multi-MU scenario, each MU has multiple CP choices to access the content, and each CP is able to provide subsidization to multiple MUs to earn more advertisement revenue. Accordingly, each individual CP competes with others for the equilibrium, which is constrained by the lower equilibrium among the MUs. This leads to the Equilibrium Programs with Equilibrium Constraints (EPEC) problem formulation.

For the EPEC problem, we also follow the basic idea of backward induction and consider the sub-game problem among MUs with the fixed CP strategies first. Clearly, we have the following lower equilibrium condition among MUs:

$$\begin{aligned}\boldsymbol{x}_i^* &= \arg\max_{\boldsymbol{x}_i} u_i(\boldsymbol{x}_i; \boldsymbol{x}_{-i}, \boldsymbol{\Theta}) \\ &= \arg\max_{\boldsymbol{x}_i} \sum_{j=1}^{M} \omega_{i,j} \Big(\tau\sigma_i f(x_{i,j}) - (1-\theta_{i,j}) x_{i,j} p \\ &\quad + \sigma_i f(1-x_{i,j}) - (1-x_{i,j})p \Big), \forall i \in \{1, 2, \ldots, N\}. \end{aligned} \tag{11}$$

With the anticipation of all MUs' behaviors as indicated in (11), each CP o_j, $j \in \{1, 2, \ldots, M\}$, aims to set its reward factors so as to receive the optimal profit, which is given by

$$\Pi_j = \sum_{i=1}^{N} \omega_{i,j} \Big(\mu_j h(x_{i,j}^*) - \theta_{i,j} p x_{i,j}^* \Big), \forall j \in \{1, 2, \ldots, M\}, \tag{12}$$

where $\omega_{i,j}$ is related to the setting reward factors of all CPs, as indicated in (7). Therefore, in order to obtain the maximum profit, each CP also needs to consider the strategies of all other CPs. Since each CP can provide the data subsidization for multiple MUs simultaneously, we include the following constraint to indicate the limited total budget in terms of data subsidization held by each CP o_j:

$$\sum_{i=1}^{N} P_{i,j}\theta_{i,j} - Q_j \le 0, j \in \{1, 2, \ldots, M\}, \tag{13}$$

where all $\{P_{i,j} | i = 1, 2, \ldots, N\}$ and Q_j are real, scalar constants. In summary, the CPs' optimization problems are formulated as the following EPEC problems:

$$\begin{aligned} &\underset{\boldsymbol{\theta}_j}{\text{maximize}} \quad \Pi_j = \sum_{i=1}^{N} \omega_{i,j} \Big(\mu_j h(x_{i,j}^*) - \theta_{i,j} p x_{i,j}^* \Big), \\ &\text{subject to} \quad \begin{cases} \sum\limits_{i=1}^{N} P_{i,j}\theta_{i,j} - Q_j \le 0, \\ 0 \le \theta_{i,j} \le 1, \\ \boldsymbol{x}_i^* = \arg\max_{\boldsymbol{x}_i} u_i(\boldsymbol{x}_i; \boldsymbol{x}_{-i}, \boldsymbol{\Theta}), \\ \text{subject to} \begin{cases} x_{i,j} \ge 0, \\ x_{i,j} \le 1, \end{cases} \end{cases} \end{aligned} \tag{14}$$

for all $i \in \{1, 2, \ldots, N\}$, and $j \in \{1, 2, \ldots, M\}$.

The EPEC describes the hierarchical optimization problems that contain equilibrium problems at both the upper and lower levels [11,23]. As aforementioned, the CPs are independent and rational entities, which aim to maximize their individual profit. However, maximizing Π_j for CP o_j affects the profits of other CPs and the utilities of all MUs. Likewise, the utility maximization of MU s_i affects the profits of all CPs. In practice, when the number of CPs and the number of MUs are large, the centralized optimization in terms of the profits and the utilities of all CPs and all MUs, respectively, is difficult to achieve the optimal solutions simultaneously. Furthermore, in the multi-CP multi-MU scenario, the coordination of multiple conflicting payoffs leads to high complexity to achieve the optimal result. As such, we propose to utilize the distributed Alternating Direction Method of Multipliers (ADMM) algorithm with the fast convergence property for the above large-scale optimization problem, which is guaranteed to converge to the optimum results [12,13,23]. ADMM is an efficient large-scale optimization tool for solving convex or even nonconvex functions.

3.1 ADMM-Form Optimization Concepts

Before presenting the ADMM implementation for solving the EPEC problem, we briefly introduce a typical ADMM-form optimization problem. To facilitate the narrative, we first focus on a simple system with a single provider and N users. Therein, the objective of the provider is expressed as follows:

$$\begin{aligned} \underset{\boldsymbol{y}}{\text{minimize}} \quad & L(\boldsymbol{m}) = \sum_{i=1}^{N} l_i(m_i) \\ \text{subject to} \quad & \sum_{i=1}^{N} G_i m_i - T_i = 0, \end{aligned} \tag{15}$$

where $\boldsymbol{m} = \{m_1, \dots m_i, \dots, m_N\}$, and $l_i(m_i)$ represents the cost of provider j if its strategy is m_i. Specifically, G_i and T_i are real scalar constants, and m_i is a real scalar variable. $l_i(m_i)$ is convex on m_i.

With t denoted as the iteration index, the provider iteratively updates the value of $\boldsymbol{m}$ such that

$$\boldsymbol{m}(t+1) = \arg\min(H(\boldsymbol{m})) - \sum_{i=1}^{N} \lambda_i(t) G_i m_i + \Psi, \tag{16}$$

where

$$\Psi = \frac{\rho}{2} \sum_{i=1}^{N} \|G_i m_i - T_i\|_2^2. \tag{17}$$

$\rho > 0$ is a damping factor, and $\|\cdot\|_2^2$ represents the Frobenius-2 norm. Likewise, the dual variable λ is iteratively updated by

$$\lambda_i(t+1) = \lambda_i(t) - \rho \left(\sum_{i=1}^{N} G_i m_i(t+1) - Q_i \right). \tag{18}$$

If $l_i(m_i)$ is separable and convex, the ADMM algorithm will eventually converge to the set of stationary solutions [12,13,22,23]. It is worth noting that for the case of non-convex objective functions, the convergence of ADMM can still be ensured in certain cases [4].

3.2 Multi-CP Multi-MU Based ADMM for Solving EPEC

In what follows, we elaborate the iteration process of the Multi-CP Multi-MU based ADMM that is leveraged to optimize the profits of CPs and the utilities of MUs in the framework of mobile data subsidization. Specifically, each iteration is composed of two-layer optimization as follows:

(1) **Utility Optimization in lower layer:** In the lower layer, each MU s_i ($i = 1, 2, \ldots, N$) observes the announced reward factors $\theta_{i,j}^{(q)}$ from CPs at the start of each iteration q, and decides on the content demand strategies towards different CP o_j, $x_{i,j}$ (within the strategy space $[0, 1]$), maximizing its utility $u_i(\boldsymbol{x}_i)$. Note that the superscript (q) represents the q^{th} iteration of the ADMM in the external loop. The objective of each MU s_i is to maximize its individual utility $u_i(\boldsymbol{x}_i)$, and obtain the optimal values of $\boldsymbol{x}_i$. This forms the internal loop of the ADMM. Hence, the value of $\boldsymbol{x}_i$ is updated by each MU s_i at each iteration of the internal loop as follows:

$$\boldsymbol{x}_i^{(q)}(t+1) = \arg\max\left(u_i(\boldsymbol{x}_i^{(q)})\right). \tag{19}$$

During each iteration of the external loop q, the MUs are able to derive a set of values of content demand, $x_{i,j}^{(q)}$, which maximize their utilities at the end of the internal loop. t is the index of iteration in the internal loop. At the same time, these values can be predicted by all CPs, which will be employed to update the values of $\theta_{i,j}$ in the higher layer.

(2) **Profit Optimization in higher layer:** In this layer, the CPs are aware of the behaviors of MUs due to the first-moving advantage and hence can predict the content traffic to be transferred to MUs. Specifically, each CP o_j ($j = 1, 2, \ldots, M$) controls the values of $\boldsymbol{\theta}_j$ within the strategy space $[0, 1]$ to maximize its profit by invoking ADMM as follows:

$$\boldsymbol{\theta}_j^{(q)}(t+1) = \arg\max\left(\mathit{\Pi}_j(\boldsymbol{\theta}_j^{(q)})\right) + \sum_{i=1}^{N} \lambda_i^{(q)}(t) P_{i,j}\theta_{i,j} + \mathit{\Psi}, \tag{20}$$

where

$$\mathit{\Psi} = \frac{\rho}{2}\sum_{i=1}^{N}\left\| \sum_{m=1, m\neq j}^{M} x_{i,m}^{(q)}(\tau) + P_{i,j}\theta_{i,j} - Q_j \right\|_2^2, \tag{21}$$

and $\tau = t$ if $m > j$, and $\tau = t+1$ if $m < j$. Specifically, t is the index of iteration in the internal loop. $\rho > 0$ is the damping factor, and λ is the dual variable which will be updated as follows:

$$\lambda_i^{(q)}(t+1) = \lambda_i^{(q)}(t) + \rho\left(\sum_{i=1}^{N} P_{i,j}\theta_{i,j}^{(q)}(t+1) - Q_j\right). \tag{22}$$

The updated reward factors $\theta_{i,j}^{(q+1)}$ are then broadcasted to the MUs for the next iteration, i.e., the $(q+1)^{\text{th}}$ iteration. This forms the external loop of the algorithm. The external loop will not terminate until the condition

$$\left\| \sum_{j=1}^{M} \Pi_j(\boldsymbol{p}_j^{(q)}) - \sum_{j=1}^{M} \Pi_j(\boldsymbol{p}_j^{(q-1)}) \right\| \leq \epsilon \tag{23}$$

holds, where ϵ is a pre-determined small-valued threshold.

The detailed steps of the above ADMM algorithm is presented in Algorithm 1. Moreover, we can obtain the following theorem after analyzing the utility and profit functions of MU and CP, respectively.

Theorem 1. *The utility function of each MU s_i in (2), and the profit function of each CP o_j in (6) are strictly concave, where $i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, M$.*

Proof. The first-order and second-order derivatives of (2) with respect to $x_{i,j}$ can be expressed by

$$\frac{\partial u_i}{\partial x_{i,j}} = \sum_{j=1}^{M} \omega_{i,j} \Big(\tau\sigma x_{i,j}^{-\alpha} - \sigma(1 - x_{i,j})^{-\alpha} + \theta p \Big), \tag{24}$$

$$\frac{\partial^2 u_i}{\partial x_{i,j}^2} = \sum_{j=1}^{M} \omega_{i,j} \Big(- \alpha\tau\sigma x_{i,j}^{-\alpha-1} - \alpha\sigma(1 - x_{i,j})^{-\alpha-1} \Big) < 0. \tag{25}$$

We can hence easily conclude with the negativity of (25).

Moreover, we have the profit function of CP o_j

$$\begin{aligned} \Pi_j &= \sum_{i=1}^{N} \omega_{i,j} \Big(\mu_j h(x_{i,j}) - \theta_{i,j} p x_{i,j} \Big) \\ &= \sum_{i=1}^{N} \frac{\theta_{i,j}}{\sum_{k=1}^{M} \theta_{i,k}} \Big(\mu_j h(x_{i,j}) - \theta_{i,j} p x_{i,j} \Big) \\ &= \sum_{i=1}^{N} \left(\frac{\theta_{i,j}}{\sum_{k=1}^{M} \theta_{i,k}} \mu_j h(x_{i,j}) - \frac{\theta_{i,j}^2}{\sum_{k=1}^{M} \theta_{i,k}} p x_{i,j} \right), \end{aligned} \tag{26}$$

$\forall j \in \{1, 2, \ldots, M\}$. To demonstrate the concavity of Π_j on $\theta_{i,j}$, we need to ensure the negativity of $\frac{\partial^2 \Pi_j}{\partial \theta_{i,j}^2}$. We expand the first-order and second-order derivatives of (26) with respect to $\theta_{i,j}$ in (27), and (28), respectively:

$$\frac{\partial \Pi_j}{\partial \theta_{i,j}} = \sum_{i=1}^{N}\left(\frac{\mu_j h(x_{i,j}) \sum\limits_{k=1,k\neq j}^{M} \theta_{i,k}}{\left(\sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} + \theta_{i,j}\right)^2} - p x_{i,j} \frac{2\theta_{i,j}\left(\sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} + \theta_{i,j}\right) - \theta_{i,j}^2}{\left(\sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} + \theta_{i,j}\right)^2}\right)$$
$$= \sum_{i=1}^{N}\left(\frac{\mu_j h(x_{i,j}) \sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} - 2\theta_{i,j} p x_{i,j} \sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} - \theta_{i,j}^2 p x_{i,j}}{\left(\sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} + \theta_{i,j}\right)^2}\right), \quad (27)$$

and

$$\frac{\partial^2 \Pi_j}{\partial \theta_{i,j}^2} = \sum_{i=1}^{N}\left(\frac{\left(-2p x_{i,j} \sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} - 2\theta_{i,j} p x_{i,j}\right)\left(\sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} + \theta_{i,j}\right)^2}{\left(\sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} + \theta_{i,j}\right)^4}\right)$$
$$- \sum_{i=1}^{N}\left(\frac{\left(2\sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} + 2\theta_{i,j}\right) \mathrm{A}}{\left(\sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} + \theta_{i,j}\right)^4}\right)$$
$$= -2\sum_{i=1}^{N}\left(\frac{p x_{i,j}\left(\sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} + \theta_{i,j}\right)^2 + \mathrm{B}}{\left(\sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} + \theta_{i,j}\right)^3}\right)$$
$$= -2\sum_{i=1}^{N}\left(\frac{p x_{i,j}\left(\sum\limits_{k=1,k\neq j}^{M} \theta_{i,k}\right)^2 + \mu_j h(x_{i,j}) \sum\limits_{k=1,k\neq j}^{M} \theta_{i,k}}{\left(\sum\limits_{k=1,k\neq j}^{M} \theta_{i,k} + \theta_{i,j}\right)^3}\right) < 0, \quad (28)$$

where

$$\mathrm{A} = \mu_j h(x_{i,j}) \sum_{k=1,k\neq j}^{M} \theta_{i,k} - 2\theta_{i,j} p x_{i,j} \sum_{k=1,k\neq j}^{M} \theta_{i,k} - \theta_{i,j}^2 p x_{i,j}, \quad (29)$$

and

$$\mathrm{B} = \mu_j h(x_{i,j}) \sum_{k=1,k\neq j}^{M} \theta_{i,k} - 2\theta_{i,j} p x_{i,j} \sum_{k=1,k\neq j}^{M} \theta_{i,k} - \theta_{i,j}^2 p x_{i,j}. \quad (30)$$

We can then deduce that $\frac{\partial^2 \Pi_j}{\partial \theta_{i,j}^2}$ is negative, and hence validate the concavity of Π_j on $\theta_{i,j}$.

Algorithm 1. Multi-CP Multi-MU Based ADMM for Solving EPEC problem

1: **Input:**
Initial input $\theta_{i,j} \in [0,1]$, where $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, M$, a pre-determined small-valued threshold ϵ, $q = 1$;
2: **repeat**
3: Utility optimization for MUs using ADMM (Internal Loop): MUs observe the announced reward factors $\theta_{i,j}$, and decide on their content demand strategies $x_{i,j}^{(q)}$ by evaluating their derived utilities, $u_i(\boldsymbol{x}_i)$;
4: Profit optimization for CPs using ADMM (External Loop): CPs predict the MU behaviors $x_{i,j}$, and invoke ADMM to perform the maximization for their individual profit Π_j. The optimal reward factor $\theta_{i,j}^{(q)}$ is obtained by maximizing their profits;
5: $q = q + 1$;
6: **until** $\left\| \sum_{j=1}^{M} \Pi_j(\boldsymbol{\theta}_j^{(q)}) - \sum_{j=1}^{M} \Pi_j(\boldsymbol{\theta}_j^{(q-1)}) \right\| \leq \epsilon$
Output: The optimal strategies of content demand $\boldsymbol{x}_i^* = \boldsymbol{x}_i^{(q)}$, where $i = 1, 2, \ldots, N$; The optimal reward factors $\boldsymbol{\theta}_j^* = \boldsymbol{\theta}_j^{(q)}$, where $j = 1, 2, \ldots, M$.

According to [12,13,23], if the optimization problems faced by MUs and CPs are both convex, the ADMM can converge to the optimum results, i.e., $\boldsymbol{x}_i^*$, $i \in \{1, 2, \ldots, N\}$ and $\boldsymbol{p}_j^*$, $j \in \{1, 2, \ldots, M\}$ in a distributed manner. We further confirm the convergence of the ADMM in the next section.

4 Performance Evaluation

In this section, we employ numerical simulations to justify the analytical results and evaluate the system performance metrics in the mobile data subsidization model, with default network parameters set as follows: $\alpha = 0.8$, $\gamma = 0.9$, $l_a = 0.5$, $\tau = 1/(1 + 0.5)$, $p = 100$, $M = 3$, and $N = 3$.

Before evaluating the system performance with the proposed scheme, we first confirm the convergence of the distributed ADMM algorithm in a data subsidization system with 3 CPs ($\mu_1 = 10, \mu_2 = 20, \mu_3 = 60$) with 3 MUs ($\sigma_1 = 5, \sigma_2 = 15, \sigma_3 = 30$). The results are presented in Figs. 2 and 3, where the EPEC problem is solved in an iterative manner. In particular, the results in Fig. 2 show the convergence of the competition among MUs to achieve the lower-layer equilibrium, and the results in Fig. 3 show the convergence of the competition among CPs to achieve the higher-layer equilibrium. Different MUs and CPs finally achieve different payoffs at the convergence point. We find that the MU with the higher value of σ_i will obtain the higher utility as it derives more benefit from viewing the content. Moreover, the CP with the higher value of μ_j has greater competitiveness as it extracts more advertisement revenue from content traffic, and hence it is able to offer more data subsidization to attract MUs.

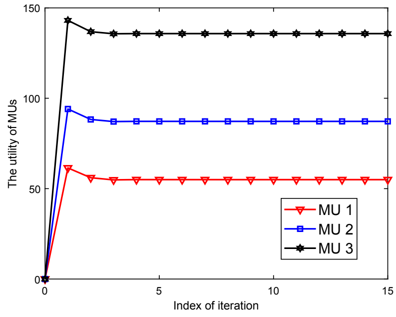

Fig. 2. The utilities of MUs vs. the index of iteration.

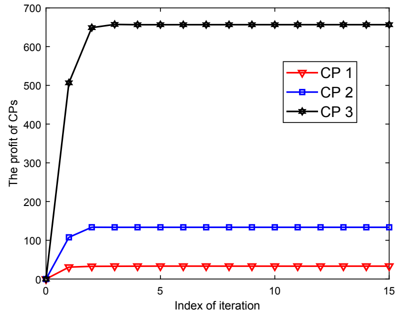

Fig. 3. The profits of CPs vs. the index of iteration.

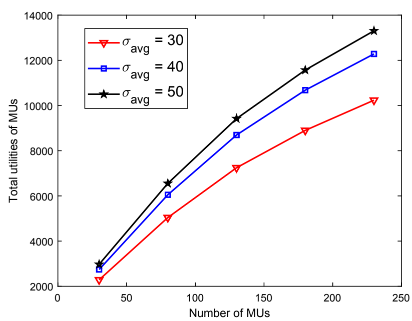

Fig. 4. Total utilities of MUs vs. the number of MUs

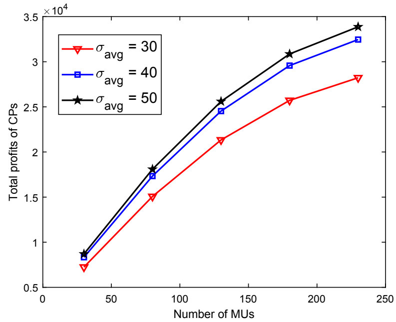

Fig. 5. Total profits of CPs vs. the number of MUs

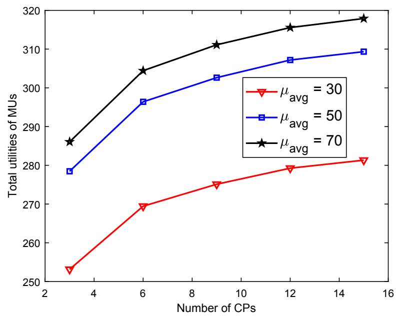

Fig. 6. Total utilities of MUs vs. the number of CPs.

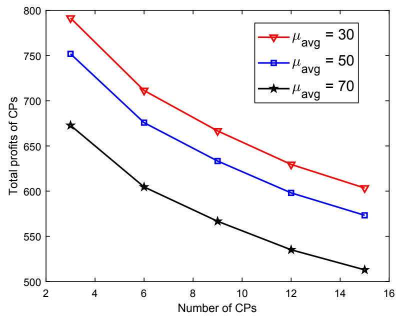

Fig. 7. Total profits of CPs vs. the number of CPs.

Then, we study the system performance of a general data subsidization system with M CPs and N MUs under the proposed scheme. We assume the utility coefficients of MUs σ_i and advertisement revenue coefficients of CPs μ_j follow the normal distribution $\mathcal{N}(\sigma_{\text{avg}}, 2)$, and $\mathcal{N}(\mu_{\text{avg}}, 2)$, respectively. Specifically, we evaluate the performance with different numbers of MUs and CPs, as depicted in Figs. 4, 5, 6 and 7. From Figs. 4 and 5, we can observe that the total utilities of MUs and the total profits of CPs both increase with the increase of the number of MUs. The reason is that the more MUs will consume more "sponsored content", which results in more profits of CPs. Nevertheless, as the number of MUs increases, the marginal increase of the total utilities of MUs and total profits of CPs both decrease. This is due to the limited data subsidization constraint. As the total reward is limited, the CP cannot provide enough subsidization for all MUs when the number of MUs is high. Therefore, the increasing rate of the total utilities of MUs decreases as the number of MUs increases. The restrained subsidization from CPs cannot continuously achieve greater profits for themselves. Consequently, the increasing rate of total profit of CPs decreases as well. Furthermore, by comparing different values of σ_{avg}, we find that the increase of the average utility coefficients leads to the increase of total utilities of MUs and the increase of total profits of CPs. The increase of σ_{avg} improves the valuation derived from viewing the content, which hence promotes the higher willingness of MUs to access and enjoy the content. This in turn increases the total profits of CPs.

In addition, we observe from Fig. 6 that the total utilities of MUs increase but the total profits of CPs decrease as the number of CPs increases. The reason is that the competition among CPs becomes sharper when there are more CPs in the system. Each CP competes with other CPs to promote its content traffic by providing more subsidization for MUs. Therefore, the total profits of CPs decrease in presence of sharper competition. Meanwhile, the competition among CPs results in the greater subsidization to MUs, leading to the increase of total utilities of MUs. Moreover, we find that the increase of the average advertisement revenue coefficients μ_{avg} improves the total utilities of MUs but reduces the total profits of CPs. The reason is that the higher value of μ_{avg} intensifies the reward competition among CPs in data subsidization system since the CPs can obtain greater advertisement revenue from the given "sponsored content" traffic. On one hand, the CPs provide greater data subsidization that incurs more cost, and hence the total profits are reduced. On the other hand, the greater subsidzation improve the total utilities of MUs.

5 Conclusion

In this work, we have established a hierarchical Stackelberg game to model the interactions among content providers and mobile users in the framework of mobile data subsidization scheme. We have characterized the many-to-many interactions among multiple providers and multiple users by formulating an Equilibrium Programs with Equilibrium Constraints (EPEC) problem. Moreover, we have employed the distributed Alternating Direction Method of Multipliers (ADMM) algorithm to tackle the EPEC problem by utilizing the fast-convergence properties of ADMM. Numerical results have been presented to confirm the analytical solutions and evaluate the system performance with the proposed schemes.

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