Complex Network Articulation Points Detection and Centrality Measures

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Abstract. To clearly understand how network structure and function interact is a basic difficulty in the study of large networked systems. An old-fashioned idea from graph theory, called articulation points, may be used to do this. In a network, a node If removing it causes the network to become disconnected or causes more network components to get linked, it is an articulation point (AP). Single points of collapse are represented as articulation points in networks. The major goal of this research is to provide a method for identifying the articulation points and centrality measures. We can locate the articulation points considerably more quickly and effectively by using TARJAN'S Algorithm, which uses depth-first search. It must fulfill two requirements to qualify as an articulation point. For the root node of a DFS traversal to be an articulation point, it must contain at least two offspring nodes that are members of various sub graphs. It has been discovered that articulation points (APS) are crucial for maintaining the reliability and connection of several real-world networks. By assigning each node in the graph a scalar value based on an assumption, centrality metrics may be used to quantify each node's significance. A fundamental centrality metric is node degree. In terms of node neighbors, it is equivalent. Hence, the more neighbors a node has, the more central and densely linked it is, and the more it affects the network by having more neighbors.

Keywords: Articulation Points · Complex networks · TARJAN's Algorithm · Depth-First Search · Node · Graph Theory · Centrality · Degree Centrality · Betweenness Centrality

1 Introduction

To clearly understand how network model and functionality interact is a basic difficulty in the investigation of big networked systems. To address this issue, we look at articulation points, a fundamental concept in graph theory. If removing a node from a network

P. Pareek et al. (Eds.): IC4S 2024, LNICST 599, pp. 447–463, 2025.
https://doi.org/10.1007/978-3-031-77081-4_35

causes it to become disconnected or results in more linked components, the node is an articulation point (AP) [1]. A depth-first search-based linear-time approach can quickly locate certain APs. It has been discovered that APs are crucial for maintaining the reliability and connection of several network systems. For instance, if an AP is attacked or interrupted in an infrastructure network like a power grid or an air traffic network, the infrastructure is seriously risked. Data transfer from one network component to another will be halted in wireless sensor networks if APs fail. Lethal mutations are more prevalent in the group of closely related proteins known as APs in the yeast protein-protein interaction network [2–3]. To methodically explore the structure and operation of real-world networks, analysis of APs offers us a new perspective. We still don't fully comprehend the functions played by APs in many complicated systems, despite the significance of APs in assuring the stability and connection of several network systems. We are motivated to create by this reality. [4].

1.1 Graph

A significantly more detailed definition of a graph exists in mathematics. The capital letter G is typically used to denote a graph, which is a set. You might recall from high school maths that a set is essentially a collection of objects, some of which may be ordered and some of which may be other sets (a set can have sets as members). In the case of graphs, nodes and edges are the entities within the collection. Consequently, a graph is a set that has two sets as members, a set of node and a set of edges shown in Eq. 1.

$$\mathbf{G} = \mathbf{V},\ \mathbf{E} \tag{1}$$

Typically, the collection of nodes reflects the participants in a real-world social network. Graphs, which describe the actual social network, are shown using point and line diagrams, such as those in Fig. 1. Nodes (representing actors) are shown in these diagrams as circles. Although actors in social network analysis are frequently either individuals or organisations, nodes can represent anything that connects to other similar entities in a larger system in broader applications of network imagery in the physical and biological sciences (typically referred to as network science). This includes buildings that provide electricity and residences, computers and servers, ecosystem species, towns, and essentially anything substantial that we can define a relationship on or from which some form of content may be said to be transferred.

Edges show if there is a link or other connection between two nodes. These often involve some sort of social relationship in social network research. In a subsequent chapter, we'll define social bonds, discuss the variety of sorts that exist, and discuss their characteristics. For the time being, we may state that in social network analysis, these connections are inter-node links, and edges in a graph are used to represent them. In graph theory, edges are best understood as a group of node pairs, where the nodes that make up each pair are those that are engaged in the focus connection. Therefore, if node A and node B are connected by a relationship R, then AB is an edge in the relevant graph. The edges can stand in for electricity lines in the context of dwellings and power plants. In the meanwhile, internet cables and wi-fi connectivity connect servers and computers, and roads connect communities. Edges indicate the ability for content to flow, whether that material be electrical power, digital data, or moving humans in automobiles. The material that moves between two nodes in social networks typically reflects some sort of relationship. Three nodes and two edges make up this graph. The players A, B, and C represented by the nodes A, B, and C are those whose real-world social ties we are interested in examining. Simple lines connecting A and B, as well as B and C, are edges on a graph that show the existence of a relationship. These edges are designated as AB and BC. Nodes A and C don't have a relationship in reality, which is reflected in the absence of an edge between them [4].

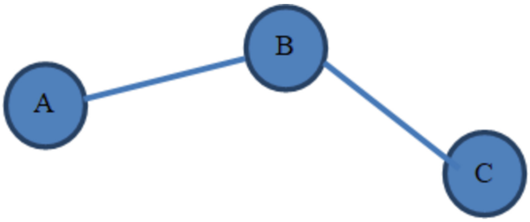

Fig. 1. An example of a network having three nodes [2]

1.2 Network

A collection of units or entities that up a network. Different types of networks are shown in Fig. 2. These are sometimes referred to as nodes or vertices, but what turns them into a network is the fact that at least some pairs of them are connected by a set of connections or ties. These are also occasionally referred to as edges. Therefore, a network is just a collection of nodes, some of which are connected. In social networks, the nodes

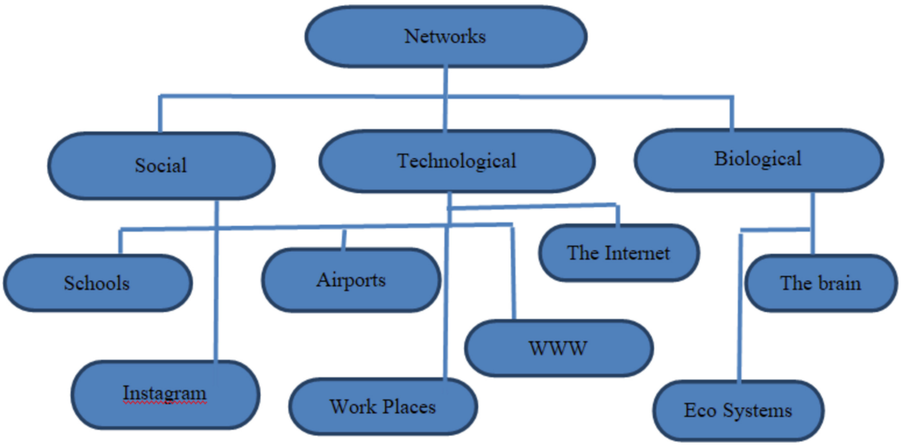

Fig. 2. Different types of networks

are individuals, and the connections are any kind of positive or negative social bond (such as friendship, hostility, or co-employment) between those individuals. Networks are occasionally shown as images in a point-and-line graphic. The norm in this case is that the nodes (i.e., persons) are represented by circles (or another polygon, such a triangle or a square), and the connections between the nodes are represented by lines or, occasionally, arrows. The term “graph” is occasionally used to describe this method of expressing networks [4].

2 Problem Statement

Finding every vertex whose removal from the graph, together with all of its incident edges, would result in the graph being disconnected is our goal. Designing dependable networks requires the elimination of articulation points. An articulation point for a disconnected undirected graph is the removal of a vertex that increases the number of linked components. There are a lot of connections between different sites. In a complicated network, it will be challenging for a user to locate and eliminate the articulation points. Hence, articulation point identification will also make it simple for users to disconnect networks by eliminating articulation points in complicated networks if something goes wrong. Centrality measures are scalar values assigned to every graph node to assess its significance according to a presumption.

3 Proposed Solution

In this research, we developed a method for finding the articulation sites using TARJAN’S Algorithm and Depth-First search. The method is based on two conditions. When a DFS traversal’s root node may be an articulation point or must be an articulation point, it must contain at least two offspring nodes from separate subgraphs. Finding precise articulation locations will be simple with the aid of centrality in graph theory. The method will provide the most precise articulation points in the complicated networks for this project when it is performed on a dataset of node points.

4 Methdology

The process involved in this project work is depicted in the flowchart shown in Fig. 3 below.

4.1 Complex Network

The topologies of these networks are quite like one another despite the variations between complex systems found in nature or society since they are regulated by the same principles. Then, we may investigate these systems using the same set of mathematical and computational methods. A network, in general, is a system that may be seen as a graph, made up of components known as nodes or vertices and a collection of connecting links

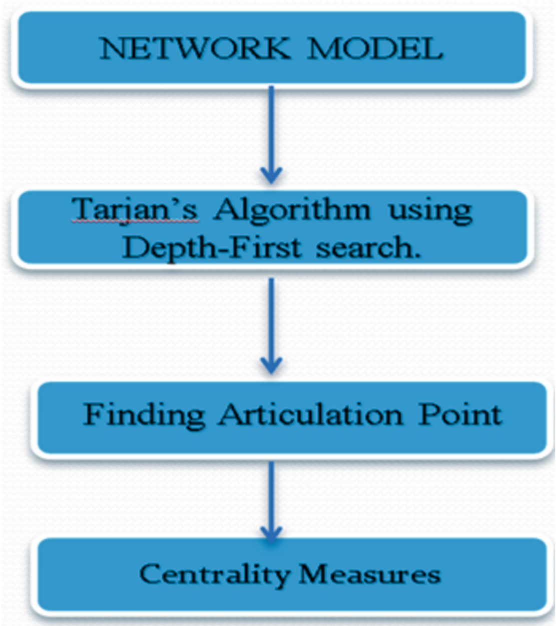

Fig. 3. Flow chart of the proposed model

(edges) that indicate the interactions between them. Problems become easier to understand and manage when a system is modelled as a graph. In terms of mathematics, a network may be represented by an adjacency matrix A. An adjacency matrix of N X N exists for a network with N vertices. Components are the simplest type of network group. Each node in a linked component can be reached by another node by some path. Most network datasets only contain a single sizable, linked component with a few isolates, however certain special datasets may have three or four sizable, separate components. There are two types of connections between components in a directed graph: weak and strong [5]. Figure 4 shows the Complex Network of Zachary's karate club.

4.2 Centrality Measures

[6] Centrality measure actually known to how centralized actors were in a network's design. It has become important as a term from its topological origins and now it refershow important actors are to a network balance. Topological centrality always has a clear definition, but many operationalizations. Network importance has many definitions and many operationalizations. We will further explore the possible meanings and operationalizations of centrality in this paper. The degree and betweenness as well as the closeness, and eigenvector are the four best centrality measurements; each has advantages and disadvantages. The major issue we wanted to highlight is that the analytical utility of each depends critically on the network context, the type of relation being investigated, and the underlying network topology. The only thing we want you to take away

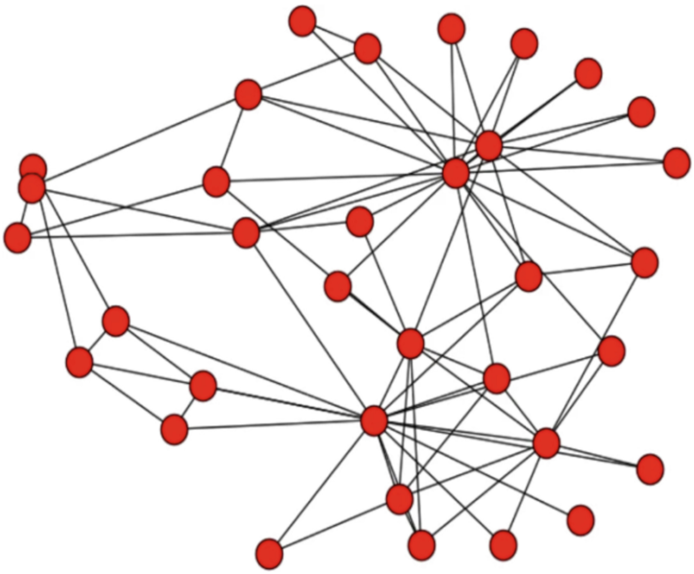

Fig. 4. Complex Network of Zachary's karate club [1]

from this is that one can better suit your study objectives than another, not that one is superior than the other. Figure 5 shows the Centrality Measure of a complex network.

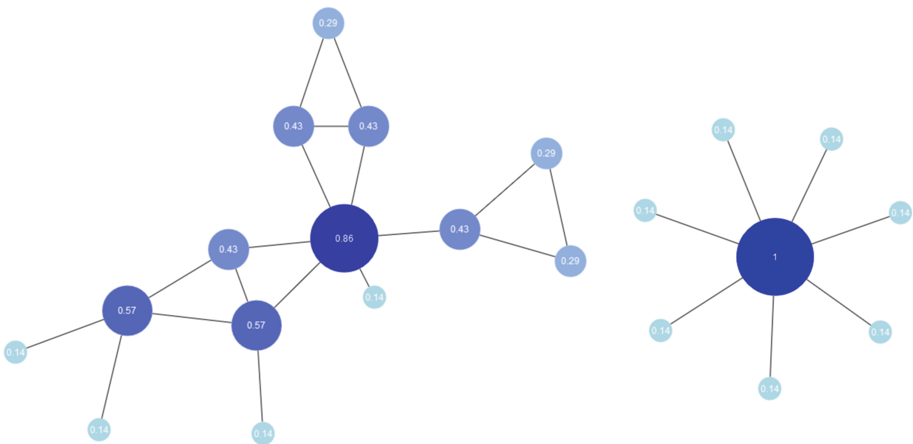

Fig. 5. Centrality Measure of a complex network. [2]

4.2.1 Degree Centrality

To identify popular nodes in a network, utilise the Degree Centrality technique. Depending on how a connection projection is oriented, degree centrality counts the number of entering or exiting (or both) relationships from a node. Both weighted and unweighted graphs are compatible with it. For each graph node, the method calculates the total of all affirmative weights of neighbouring connections in the weighted case. Weights that are not positive are disregarded. Although it may be used with heterogeneous networks, the approach does not compute degree centrality for each kind of interaction. Instead, it will approach the network as homogeneous, as suggested by the characteristics of the method [6]. Figure 6 and Fig. 7 shows agraph with nodes labelled with their out-degree centrality.

Degree Centrality:

$$\boldsymbol{C_D}_{\boldsymbol{A_{ij}=weighted\ matrices\ between\ i\ and j}}(\boldsymbol{J}) = \sum\nolimits_{J=1}^{n} \boldsymbol{A_{IJ}} \tag{2}$$

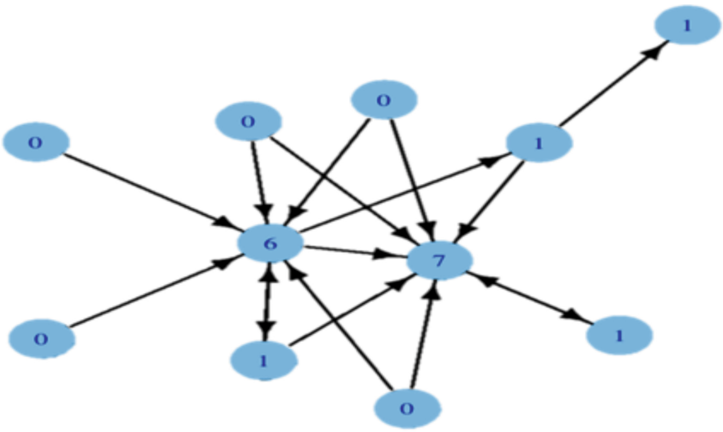

Fig. 6. A graph with nodes labelled with their in-degree [3]

4.2.2 Betweenness Centrality

The measure of a node's impact over a graph's information flow known as betweenness centrality may be used to identify this influence. It is frequently used to identify nodes that connect two distinct regions of a network. In a graph, the method determines the quickest route between every pair of nodes. Based on the number of quickest routes that travel through a node, each node is given a score. Higher betweenness centrality ratings will be achieved by nodes that regularly reside on the shortest pathways between other nodes. The concept of betweenness centrality is applied to graphs with or without weights. The approximation unweighted graph technique of Brande's serves as the foundation for the GDS execution. Several simultaneous Dijkstra algorithms are utilized for weighted graphs [7]. The Fig. 8 shows a graph with nodes labelled with Betweenness centrality.

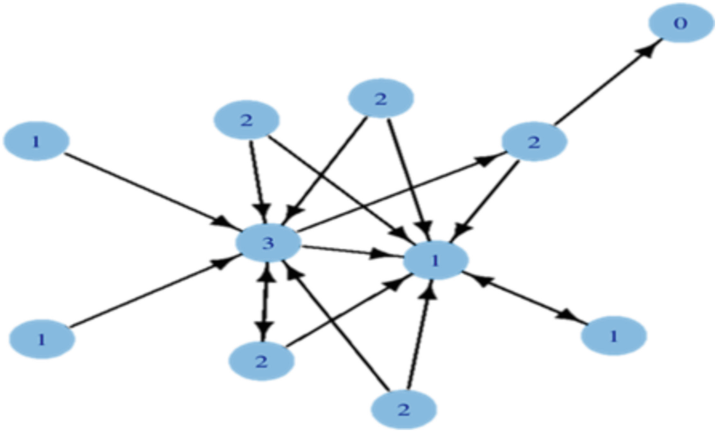

Fig. 7. A graph with nodes labelled with their out-degree centrality [3]

Betweenness centrality for weighted nodes:

$$
\begin{aligned}
S_i &= \sum\nolimits_{j=1}^{n} a_{ij} w_{ij} \\
a_{ij} &= \textit{weighted matrices between i and j} \\
w_{ij} &= \textit{weighted matrices between i and j}
\end{aligned}
\tag{3}
$$

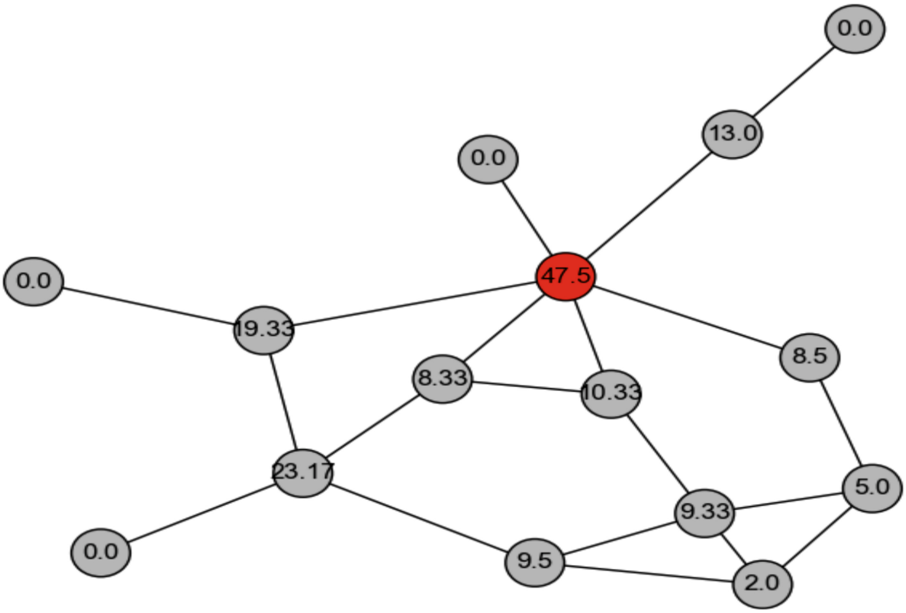

Fig. 8. A graph with nodes labelled with Betweenness centrality [2]

4.2.3 Closeness Centrality

An approach to identify nodes that may efficiently disseminate information throughout a graph is through their closeness centrality. The average distance (inverse distance) a node is from all other nodes is measured by its closeness centrality. The lengths between nodes that have a high proximity score are the shortest. Based on determining the shortest pathways among all nodes in a pair, the Closeness Centrality method determines the total of each node's distances to all other nodes for each node u. To compute the closeness centrality value for just that node, the resultant total is then reversed. Instead of representing the total of the shortest pathways, this score is often normalised to indicate their average duration. Figure 9 shows a graph with nodes labelled with Closeness Centrality [8].

Closeness Centrality:

$$\boldsymbol{C}(\boldsymbol{x}) = \frac{\mathbf{1}}{\sum_{y} \boldsymbol{d}(x, y)} \tag{4}$$

Where d (y, x) is the distance between vertices x and y

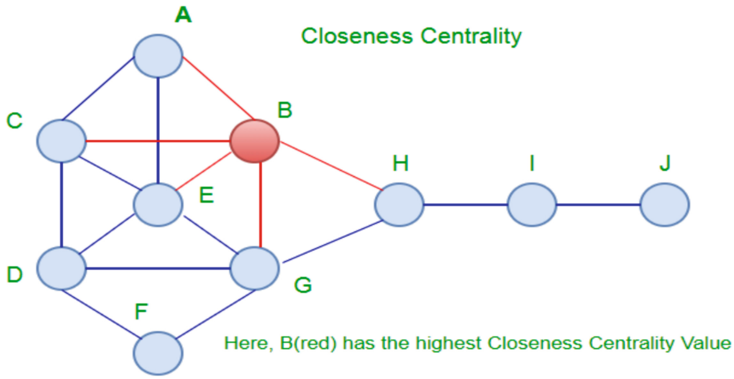

Fig. 9. A graph with nodes labelled with Closeness Centrality [5]

4.3 Tarjan's Algorithm

To determining the strongly connected components (SCCs) of a directed acyclic graph, TARJAN'S procedure is a graph theoretical algorithm. As an alternate technique, it matches the time constraint for Kosaraju's algorithm and the path-based strong component algorithm since it runs in linear time. If every vertex in a directed graph can be reached from every other vertex in the graph, the graph is said to be highly linked. This implies that there would be a path connecting each pair of nodes. A partition or subgraph that is not a subgraph of another highly connected component constitutes a strongly connected component for a directed graph. This implies that the largest subgraph meeting the criteria must be a strongly linked component [9, 10].

There are two ways to find articulation points.

4.3.1 The Simple Approach

The simplest strategy is this one. The main concept is to repeatedly delete one node at a time from the network to identify which nodes compartmentalise the graph. This approach is efficient in locating the articulation point, as it should be, but it is time-consuming because each node in the tree must be explored individually [11, 12].

The time complexity = O (v*(v + e))

The second approach:

DFS is used to first generate a depth tree. In DFS, if vertex u discovers vertex v, v is regarded as u's child. Based on the subsequent circumstances, a vertex may be regarded as an articulation point.

1) A root node with two or more offspring is referred to as a vertex.
2) No vertices on sub - tree rooted with child v does have a back edge to an ancestor of u, even if u is not the root of the depth first tree.

4.3.2 Depth First Search Approach

The SCC subtrees are deleted from the nodes and recorded when they are encountered using a DFS that is performed over them [13, 14].

For every user, a pair of variables called dfs num(u) and dfs low(u) are kept. When a node is first investigated, its counter is given the value dfs num(u). The lowest dfs num that is accessible from u and is not a part of another SCC is stored in the variable dfs low(u).

The studied nodes are placed into a stack as they are examined. To update dfs low(u), the unexplored children of a node are investigated. When a node is discovered with dfs low(u) = = dfs num(u), it is the first examined node in its strongly connected component, and all nodes in the stack above it are popped out and given the appropriate SCC number. Figure 10 shows DFS Spanning of a graph node.

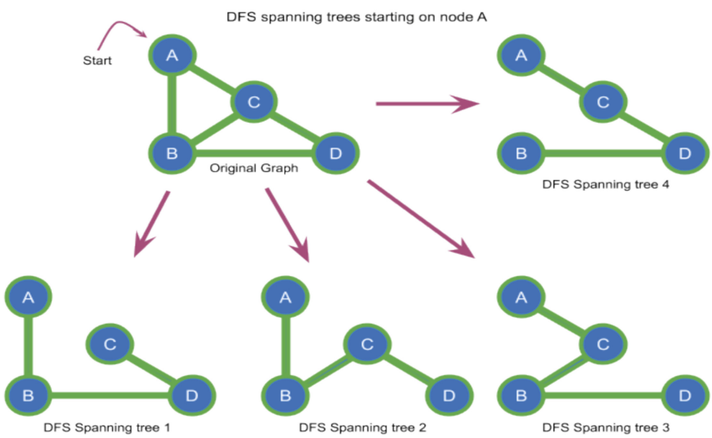

Fig. 10. DFS Spanning of a graph node

4.3.3 Network X

The Network X Python library is used to generate, manipulate, and research the structure, dynamics, and functions of complex graph networks [15]. The Python library Network X is used for complicated graph network analysis. You must to have a basic understanding of graphs before you can comprehend Network X features. A variety of linkages and processes in physical, biological, social, and informational systems are modeled using graphs, which are Mathematical structures. A graph is made up of nodes or vertices (which represent the system's entities) and edges (which indicate the connections among those entities), which together make up the system's entities. In order to find and comprehend complicated associations and/or to optimise the pathways between connected data in a network, working with graphs requires browsing edges and nodes. The study of relationships in social networks, the detection of online threats, and the identification of consumers most likely to purchase a product based on shared tastes are just a few examples of the various applications of graph network analysis. Nodes in reality can be individuals, teams, locations, or objects like clients, members, cities, shops, airports, ports, gadgets, cell phones, molecules, or websites. Any object that is hashable—that is, one whose value never changes—can be a Network X node. Text strings, pictures, XML objects, whole graphs, and customised nodes can all be included. Numerous methods to create, read, and write graphs in a variety of formats are included in the basic package. With over ten million vertices and a billion edges, Network X can function on very huge graphs. The core package, which is free software distributed under the BSD license, contains data structures for expressing several graph types, including directed graphs, self-looping and parallel edge graphs, as well as basic graphs. Additionally, Network X has a sizable development community that supports both the ecosystem of third parties and the core package [16, 17].

4.3.4 Zachary's Karate Club

An Knowledge Flow Theory for Conflicts and Fission, also in Small Groups," Wayne W. Zachary utilised the well-known dataset Zachary's Karate Club to characterise the connections in a college karate club. The data set is renowned for its accurate representation of a community framework, which happens when vertices in the network can be classified into closely linked clusters. In the case of Zachary's Karate Club, the network can be divided into two groups that are led by Mr. Hi, the organization's karate instructor, and John A, its president. The network successfully forecasts how the karate the team will split into two separate clubs after an argument over pay causes a rift among Mr. Hi and John A. The network predicts which individuals will join which new club in 33 of 34 situations by tracking group meetings that take place outside of the club. Since the 1977 publication of the original work, a large number of Zachary's Karate Club visualisations have been produced. Each node (member of the club) is arranged in a circular arrangement in the network's original visualisation, and the edges (relationships between nodes that are not part of the club) are depicted as straight lines connecting the nodes. Figure 11 shows Complex network of Zachary's karate club [18, 19].

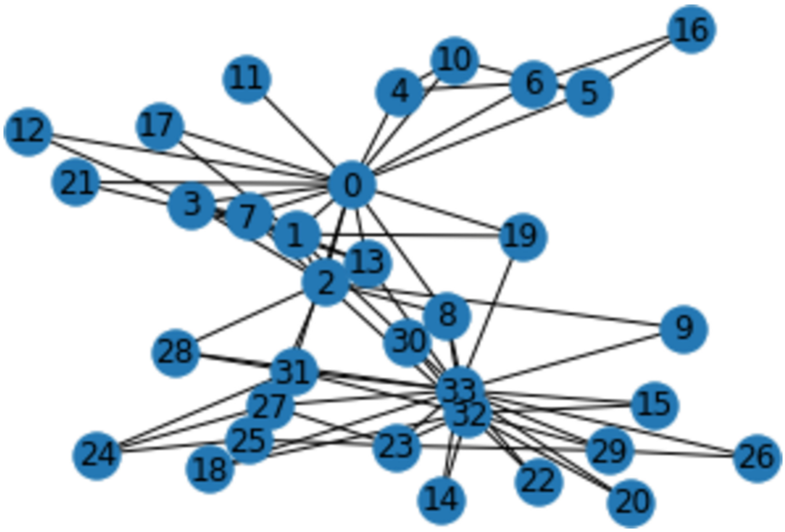

Fig. 11. Complex network of Zachary's karate club

4.3.5 Articulation Points

A vertex in a graph that, if removed together with its incident edges, would disconnect the graph is known as an articulation point (or cut vertex) [20]. One typical approach for finding articulation points in a graph is to do a depth-first search (DFS) on the graph while keeping track of the earliest visited times of each vertex and the earliest visited times of the vertices that can be reached from the present vertex through a back edge. The present vertex is an articulation point if the earliest visited time of the vertex that may be reached through a back edge is lower than the earliest visited time of the current vertex [21, 22]. The Fig. 12 and Fig. 13 shows articulation points.

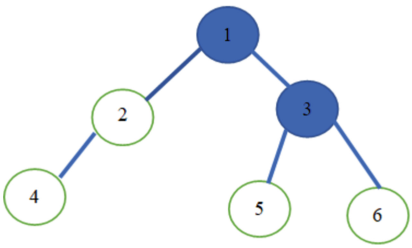

Fig. 12. Connected network having six nodes

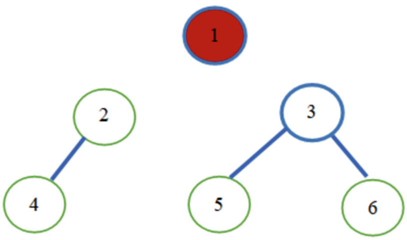

Fig. 13. Vertex 1 is an articulation point

5 Results

Articulation points can be detected using TARJAN'S algorithm. The network contains the connected components. The data is taken in the text format which connects the data as the graph. Figure 14 shows articulation points identification in a complex network.

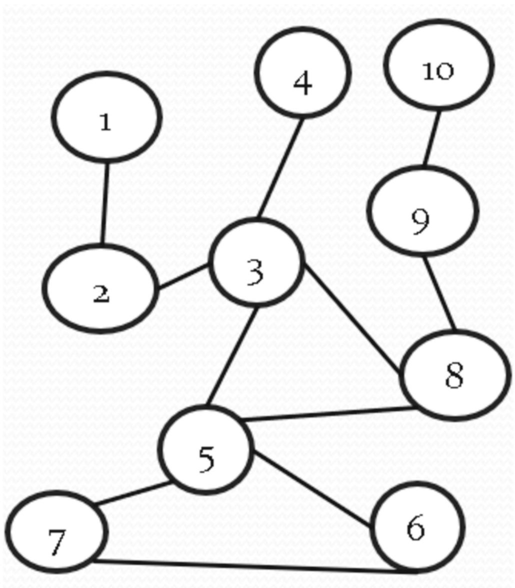

Fig. 14. Complex network to find the articulation points.

From the above complex network by using the TARJAN'S algorithm which uses the DFS.

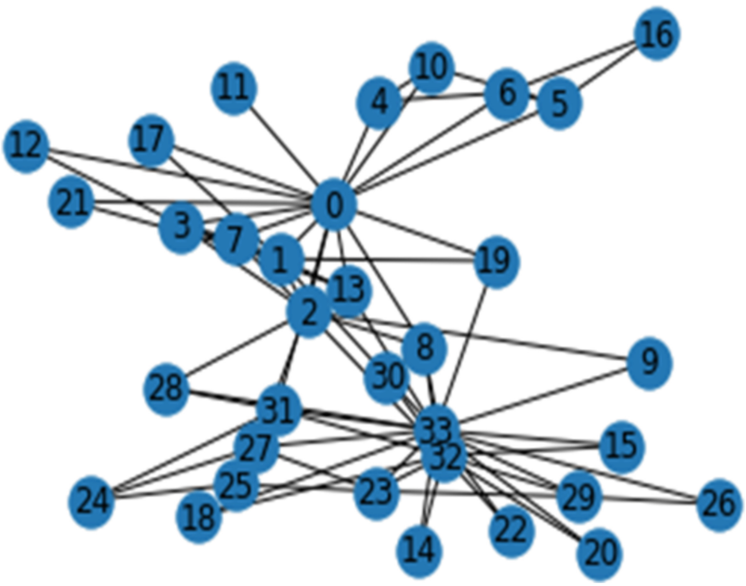

Fig. 15. Network to find the centrality measures

Articulation Points: 2 3 5 8

The karate club at Zachary's is centralised. Based on the relationship between centrality measurements like degree, betweenness, and closeness centrality, we may discover how they are related to one another. There are 33 nodes in this network that are all linked. The centrality measures of a network as shown in below Fig. 15.

5.1 Degree Centrality

To determine the degree of centrality of Zachary's Karate Club, which consisted we may utilise NetworkX in this case and that is shown in Fig. 16.

Degree centrality measure which are most connected nodes they are 0 1 2 32 33.

0:0.48484848484848486, 1: 0.2727272727272727, 2: 0.30303030303030304, 3: 0.18181818181818182, 4: 0.09090909090909091, 5: 0.12121212121212122, 6: 0.12121212121212122, 7: 0.12121212121212122, 8: 0.15151515151515152, 9: 0.06060606060606061, 10: 0.09090909090909091, 11: 0.030303030303030304, 12: 0.06060606060606061, 13: 0.15151515151515152, 14: 0.06060606060606061, 15: 0.06060606060606061, 16: 0.06060606060606061, 17: 0.06060606060606061, 18: 0.06060606060606061, 19: 0.09090909090909091, 20: 0.06060606060606061, 21: 0.06060606060606061, 22: 0.06060606060606061, 23: 0.15151515151515152, 24: 0.09090909090909091, 25: 0.09090909090909091, 26: 0.06060606060606061, 27: 0.12121212121212122, 28: 0.09090909090909091, 29: 0.12121212121212122, 30: 0.12121212121212122, 31: 0.18181818181818182, 32: 0.36363636363636365, 33: 0.5151515151515151

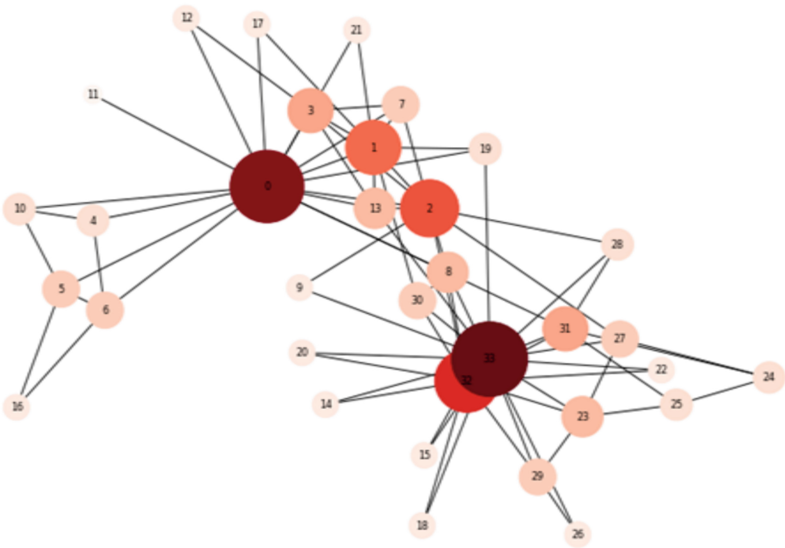

Fig. 16. Betweenness centrality measures of Zachary's karate club

5.2 Betweenness Centrality

To determine the Betweenness of centrality of Zachary's Karate Club, which consisted we may utilise NetworkX in this case. Figure 17 shows Betweenness centrality measures of Zachary's karate club.

0:0.43763528138528146, 1: 0.053936688311688304, 2: 0.14365680615680618, 3: 0.011909271284271283, 4: 0.0006313131313131313, 5: 0.02998737373737374, 6: 0.029987373737373736, 7: 0.0, 8: 0.05592682780182781, 9: 0.0008477633477633478, 10: 0.0006313131313131313, 11: 0.0, 12: 0.0, 13: 0.04586339586339586, 14: 0.0, 15: 0.0, 16: 0.0, 17: 0.0, 18: 0.0, 19: 0.03247504810004811, 20: 0.0, 21: 0.0, 22: 0.0, 23: 0.017613636363636363, 24: 0.0022095959595959595, 25: 0.0038404882154882154, 26: 0.0, 27: 0.02233345358345358, 28: 0.0017947330447330447, 29: 0.0029220779220779218, 30: 0.014411976911976909, 31: 0.13827561327561325, 32: 0.145247113997114, 33: 0.30407497594997596

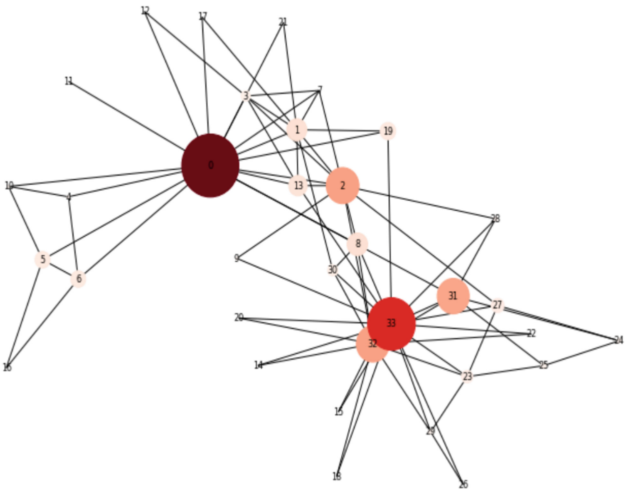

Fig. 17. Betweenness centrality measures of Zachary's karate club

Betweenness centrality measure which are between the most nodes they are 0 2 31 32 33.

5.3 Closeness Centrality

To determine the Closeness of centrality of Zachary's Karate Club, which consisted we may utilise NetworkX in this case. Figure 18 shows Closeness centrality measures of Zachary's karate club.

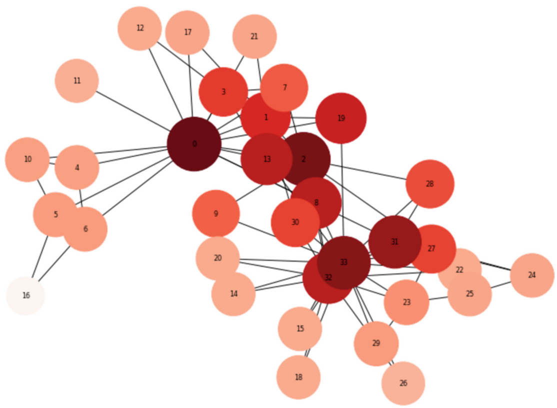

Fig. 18. Closeness centrality measures of Zachary's karate club

Closeness centrality measure which are related to the most nodes they are 0 1 2 3 7 8 9 13 19 27 28 30 31 32 33.

6 Conclusion

Here we conclude that detection of articulation points using TARJANS algorithm through DFS approach can helps to disconnect the network using these articulated nodes, if there is any emergency attacks like terrorist attacks, to stop any risks in the society. Centrality measures which use Network X as an algorithm to find the graph properties directly without writing any type of code. These centrality measures like Degree centrality, Closeness centrality, Between ness centrality can used to know more most connected nodes and find the shortest distance the nodes. These are mostly used to find the central and major nodes which make the routes very important.

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