# Multi-user Diffusive Molecular Communication Systems with A Passive Relay Node: Receiver Design and Performance Analysis

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**Abstract.** In diffusive molecular communications (DMC), the relay serves to address the challenge in long-distance DMC scenarios. Currently, the relay used in DMC mainly includes the amplify-and-forward relay and decode-and-forward relay. To facilitate information transmission, both of these relays are required to actively release information molecules, while consuming the energy stored by themselves and introducing a transmission delay. In this paper, a passive relay is proposed to support long-distance communications without excess energy consumption. Specifically, we consider a multi-user communication system with both near and far users, where it is assumed that specific types of molecules serve designated users and are reflected by other users during propagation. Due to this molecule reflection, the near user can function as a passive relay, aiding in long-distance transmission for the far user. Besides, two detectors—namely, the ideal and actual maximum likelihood detectors—are designed for the proposed system, and their detection performance is analyzed respectively. Moreover, energy consumption is studied for the proposed scheme. Numerical results indicate that the bit error rate (BER) performance can be greatly improved at a low synthesis cost.

**Keywords:** Diffusive molecular communication · passive relay · multi-user system · BER performance

## 1 Introduction

With the rapid advancements in nanotechnology and communication technology, molecular communication (MC) is widely anticipated to emerge as a predominant communication technology in future nano-scale communications. As a


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nature-inspired paradigm, MC typically encodes information onto the properties of molecules, such as concentration, release time, molecular type, and more [1]. Among all forms of MC, diffusive MC (DMC) stands out as one of the most practical methods for information transmission, as molecules propagate over the channel relying on the law of diffusion (such as Brownian motion) without the need for external energy [2,3]. It is well known that Brownian motion is characterized by its random and relatively slow speed, typically described at the scale of micrometers to nanometers. Consequently, one distinctive feature of DMC is its high sensitivity to communication distance [4]. Moreover, in DMC (typically, diffusion coefficients in the range of $10^{-8}$ to $10^{-10}$), the degradation and attenuation of molecules are more pronounced, severely limiting the transmission distance between target nodes [5]. On the other hand, the influence of inter-symbol interference (ISI) becomes increasingly dominant as the molecular concentration becomes flatter and lower with the increase in propagation distance [6]. Due to these reasons, researchers have concluded that one of the key challenges faced by DMC is the limited communication distance.

To facilitate long-distance transmission, the concept of molecular relay has been ingeniously introduced into the DMC field, as in wireless communication. In [7], the author first proposed the use of molecular relays in bacterial colonies, and a diffusion-based multi-hop network among bacteria colonies was analyzed, where each node of the network is formed by a population of bacteria. Afterwards, various molecular relays have been proposed, including the amplify-and-forward (AF) relay [8], decode-and-forward (DF) relay, and sense-and-forward (SF) relay [7]. Notably, the SF relay, a special approach in DMC, is capable of sensing the quantity of received molecules and subsequently retransmitting them. Besides, distinctive schemes for relaying are proposed. For instance, time-dependent molecular concentrations were utilized as the information carrier in a DF relay system in [9], and [10] relay collected the portion of information molecules (IMs) arriving first until a time $t$ and released them towards the receiver, such that the part of arriving first and the part of arriving later arrive almost at the same time at the receiver to enhance the whole communication performance. The deployment of multi-molecule relays is also crucial in multi-user systems. The joint optimizations of relay locations and decision threshold have been investigated for a multi-hop mobile DMC system by adopting an adaptive genetic algorithm (AGA) to minimize the average bit error probability in [11]. The authors of [12] introduced routing technology of molecular nanonetworks and selected the optimal relay path to maximize reliability and energy efficiency because of the high energy consumption in the molecular relay system.

Notice that all molecular relays mentioned earlier are active, i.e., the relay actively emits specific molecules, whether the same or not, upon receiving information from the transmitter. These relays undeniably offer strong performance, yet they come with associated costs, such as high energy consumption [13] and extended transmission delays [14]. Especially, [15] also calculated the energy required to be consumed on the relay, which means that introducing the relay brings more energy consumption. Besides, a DMC network is a popular trend, especially in constructing the Internet of Nano Things, where multi-user access is inevitable. While for multi-user communication across a broad range, deploying multiple active relays is impractical, which involves the aforementioned costs and also introduces external inter-link interference. Herein, a question arises: Must the relay be active rather than passive? Alternatively, whether it is feasible to introduce a new passive relay that can forward information without releasing new molecules. The authors of [16] present a potential solution to this question. Specifically, [16] investigated the directional signal propagation characteristics for a point source over a spherical surface and showed its directivity gain, especially with a reflective surface. Therefore, in this paper, we propose a multi-user system consisting of a transmitter, a near user, and a far user, where different types of IMs serve these users. We assume that each user only senses their corresponding molecules, while other types of molecules are fully reflected, due to no specific receptors. At this point, one can easily discover that the near user can act as a passive relay to assist communication between the transmitter and the far user. Given that the near user serving as a passive relay is more advantageous for supporting long-distance communication, the subsequent analysis primarily focuses on the far user equipped with a passive relay. Besides, we design an ideal maximum likelihood (IML) detector and an actual maximum likelihood (AML) detector for the proposed system and derive the theoretical bit error rate (BER) expressions for both detection methods. We further compare the BER in scenarios with/without a passive relay, along with the analysis of the impact of its placement arrangements on BER performance. Simulation results show that a virtual passive relay in the multi-user system can significantly enhance BER performance without extra cost.

## 2 System Model

In this section, we divide the whole system into two parts to introduce the specific transmission framework.

### 2.1 System Model for Multi-user DMC

First, we consider the system model for multi-user DMC. In this paper, a typical diffusion based multi-user communication system in a 3-D unbounded environment with three nodes consisting of a transmitter (Tx), a transparent receiver ($\mathrm{Rx}_1$), and an absorbing receiver ($\mathrm{Rx}_2$) is considered, as shown in Fig. 1. The absorbing receiver $\mathrm{Rx}_2$ is located at the origin and the Tx is located between $\mathrm{Rx}_2$ and $\mathrm{Rx}_1$. The Tx considered as a point source can transmit two types of molecules, namely type A molecules and type B molecules. According to the shorter distance of Tx to the surface of the $\mathrm{Rx}_2$ ($d_2 = |\mathbf{r_s}| - r_2$) than that to the $\mathrm{Rx}_1$ ($d_1 = |\mathbf{r} - \mathbf{r_s}|$), we consider the absorbing receiver $\mathrm{Rx}_2$ and transparent receiver $\mathrm{Rx}_1$ as a near user and a far user, respectively, both of which are sphere receivers with radii of $r_2$ and $r_1$, respectively. $\mathrm{Rx}_1$ and $\mathrm{Rx}_2$ have receptors for type A molecules and type B molecules on their surfaces, respectively, indicating that they only bind to corresponding type of molecules and exhibit certain

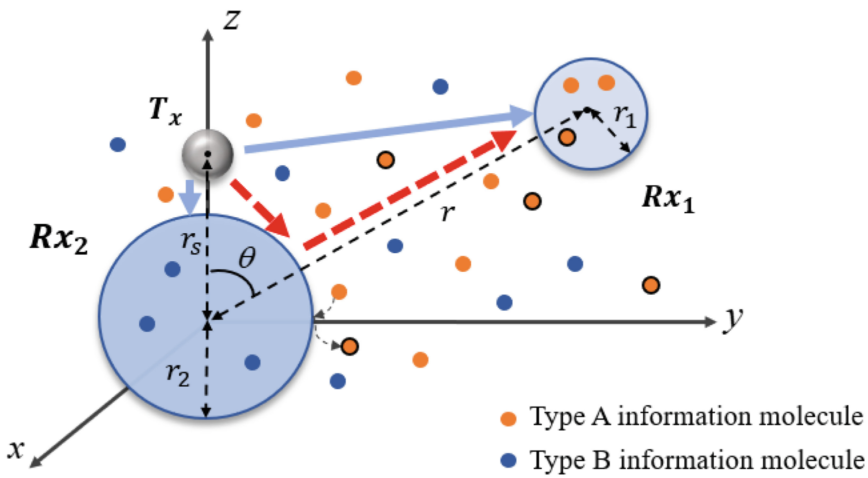


**Fig. 1.** A diffusion based multi-user communication system in a 3-D unbounded environment.

biological reactions. The Tx transmits the $j$th bit $b_j$ at time $t = (j-1)T$ by modulating the number of type A molecules with on-off keying (OOK) modulation, where $b_j = b_0, b_1, ..., b_j, ...$ is a binary information sequence, $b_j \in \{0, 1\}$, and the Tx emits an impulse of molecules to send $b_j = 1$, but releases no molecules for sending $b_j = 0$. $T$ is the symbol duration. The same is true for type B molecules. Therefore, we construct a multi-user DMC system where $Rx_1$ and $Rx_2$ use different types of molecules separately to communicate with Tx as shown by the thick solid line arrow in Fig. 1.

It is assumed that the released molecules diffuse to the receivers $Rx_1$ and $Rx_2$ through Brownian motion. The independent diffusion of molecules through the environment can be described by Fick's second law as [17]

$$\frac{\partial C(\mathbf{r}, t)}{\partial t} = D\nabla^2 C(\mathbf{r}, t) \tag{1}$$

where $D$ is the diffusion coefficient of the released molecules in $m^2/s$ and $C(\mathbf{r}, t)$ denotes the molecular concentration at location $\mathbf{r}$ and at time $t$. For the communication between Tx and the transparent receiver $Rx_1$, the observing concentration of the end-to-end system when transmitting a molecule at time $t = 0$ is given by [18]

$$C_1(\mathbf{r}, t) = \frac{1}{(4\pi Dt)^{3/2}} \exp\left(-\frac{|\mathbf{r} - \mathbf{r}_s|^2}{4Dt}\right) \tag{2}$$

where $\mathbf{r}_s$ and $\mathbf{r}$ are the vector distances from origin to the Tx and to the center of $Rx_1$, respectively. We assume that the concentration throughout the receiver's volume is uniform and equal to that at its center, which is justified if the Rx is sufficiently far from Tx. The probability of observing a molecule emitted by Tx at time $t = 0$ inside $V_{R_1}$ is given by

$$P_{o1}(\mathbf{r}, t) = C_1(\mathbf{r}, t) \times V_{R_1} \tag{3}$$

where $V_{R_1}$ is the volume of $\text{Rx}_1$. Moreover, for the communication between Tx and absorbing receiver $\text{Rx}_2$, we also have the corresponding expression of cumulative molecular hitting probability, which is given by [19]

$$P_{o2}(t) = \frac{r_2}{r_s}\text{erfc}\left[\frac{r_s - r_2}{\sqrt{4Dt}}\right] \tag{4}$$

where $\text{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-\eta^2} d\eta$.

### 2.2 System Model for the Passive Relaying DMC

Next, we consider the passive relaying DMC system as shown by the thick dashed line and the right thick solid line in Fig. 1. Based on the fact that $\text{Rx}_2$ does not have specific receptors for type A molecules, if released type A molecules move toward $\text{Rx}_2$, they will be perfectly reflected when they touch its surface. Afterwards, the reflected type A molecules continue to do the diffusion movement. At this point, the $\text{Rx}_2$ acts as a passive relay for type A molecules among the three-node DMC system. Therefore, the type A molecules detected at $\text{Rx}_1$ can be from two links: one link is that the type A molecules transmitted by the Tx directly diffuse to the $\text{Rx}_1$, and the other link is that the type A molecules touch $\text{Rx}_2$ and bounce back to receiver $\text{Rx}_1$. We assume that the passive relay $\text{Rx}_2$ is close enough to the Tx so that the time delay of the two links could be ignored. And thus when Tx emits a molecule at the point defined by vector $\mathbf{r}_s$ at time $t = 0$, the local concentration at $\text{Rx}_1$ defined by vector $\mathbf{r}$ and at time $t$ is updated from (2) to (5) as follows [16]

$$C(t, r, \mu) = \sum_{n=0}^{\infty} \frac{2n+1}{4\pi\sqrt{rr_s}} P_n(\mu) \int_0^{\infty} \frac{\mathcal{H} \times \mathcal{L}}{\mathcal{R}^2 + \mathcal{W}^2} e^{-D\gamma^{2t}\gamma} d\gamma \tag{5}$$

where $\mu = \cos\theta$ is when using the spherical coordinate system, $r$, $\theta$ and $\phi$ represent the radial, elevation and azimuth coordinates, respectively. $r$ and $r_s$ are the module lengths of $\mathbf{r}$ and $\mathbf{r}_s$, respectively, namely $r = |\mathbf{r}|$ and $r_s = |\mathbf{r}_s|$. $\mathcal{H} = [2\gamma a A_{n+\frac{1}{2}}(\gamma, r) - B_{n+\frac{1}{2}}(\gamma, r)]$, $\mathcal{L} = [2\gamma a A_{n+\frac{1}{2}}(r, r_s) - B_{n+\frac{1}{2}}(\gamma, r_s)]$, $\mathcal{R} = [2\gamma a J'_{n+\frac{1}{2}}(\gamma a) - J_{n+\frac{1}{2}}(\gamma a)]$ and $\mathcal{W} = [2\gamma a Y'_{n+\frac{1}{2}}(\gamma a) - Y_{n+\frac{1}{2}}(\gamma a)]$. The expressions of $A_v(\gamma, r)$ and $B_v(\gamma, r)$ are given in [16], and thus not presented here for brevity. Similarly, the probability of observing a molecule emitted by Tx inside $V_{R_1}$ at time $t$ is

$$P_{o3}(r, t) = C(t, r, \mu) \times V_{R_1}. \tag{6}$$

The comparison of the observing probabilities at $\text{Rx}_1$ of the two models will be shown in Sect. 5.

After transmitting the $m$-th bit based on OOK modulation, the number of molecules measured by the receiver within its detection space can be expressed as

$$y(t) = N \times \sum_{j=0}^{m} b_j P(t - jT) + n(t) \tag{7}$$

where the bit rate is $R = 1/T$ bits per second (bps), $P \in \{P_{o3}, P_{o1}\}$ depends on whether there is a passive relay or not, and $n(t)$ is the counting noise obeying Gaussian distribution, which will be presented later.

In summary, the $Rx_2$ holds two typical functions: one is to absorb type B molecules as an absorbing receiver, and the other is to reflect type A molecules as a passive relay. At this point, one can easily observe that a passive relay node $Rx_2$ is constructed based on the two-user DMC system. In other words, $Rx_2$ can play two different roles in the considered system without any conflict, i.e., the near user and the passive relay node for the far user.

## 3 Receiver Design

In this section, we derive the statistics of the received signal, based on which the IML and AML detectors are designed. Assume that the receivers synchronize with the transmitter and sample molecular concentration at the peak time $\hat{t}_d$ during each bit interval. Figure 2 gives the CIRs of the two different communication systems, where $t_{d1}$ and $t_{d2}$ represent the peak time of the two CIRs. We can find that the peak time of two different situations are not the same but similar. Thus, we use their respective peak times to approximate the same sampling time. Further, assume that the maximum length of the ISI is $L$. Here, the received signal for the $m$-th transmission can be written as

$$\begin{aligned} Y_m &= y(t = mT + \hat{t}_d) \\ &= \sum_{j=\max\{0,m-L\}}^{m} N \times b_j P((m-j)T + \hat{t}_d) + n(t). \end{aligned} \tag{8}$$

Then using the variable transform of $i = m - j$, we can express $Y_m$ of (8) in the form of

$$Y_m = \sum_{i=0}^{\min\{m,L\}} N b_{m-i} P_i + n(t) \tag{9}$$

where $P_i = P(iT + \hat{t}_d)$ and $n(t) \sim \mathcal{N}(0, \sum_{i=0}^{\min\{m,L\}} N b_{m-i} P_i)$. Notice that $Y_m$ can be approximated by a random variable obeying the Gaussian distribution when the Poisson parameter $\lambda$ is sufficiently large. The $\lambda$ indicates the average molecules observed at receiver when the $Y_m$ is considered as a Poisson random variable. On this basis, we can derive the distribution parameters including the mean and variance, denoted by $\mu_{Y_{b_m}}$ and $\sigma^2_{Y_{b_m}}$, respectively as

$$\mu_{Y_{b_m}} = Nb_m P_0 + \sum_{i=1}^{L} Nb_{m-i}P_i; \sigma^2_{Y_{b_m}} = \mu_{Y_{b_m}}$$
$$b_{m-L}, \ldots, b_{m-1}, b_m \in \{0,1\}. \tag{10}$$

The probability density function (PDF) of $Y_m$ is given by

$$P_Y(Y_m|\boldsymbol{b}_{\text{isi}}, b_m) = \frac{1}{\sqrt{2\pi}\sigma_{Y_{b_m}}} \exp\left(-\frac{(Y_m - \mu_{Y_{b_m}})^2}{2\sigma^2_{Y_{b_m}}}\right) \tag{11}$$

where $\boldsymbol{b}_{\text{isi}}$ is a vector of form $\boldsymbol{b}_{\text{isi}} = [b_{m-L}, ..., b_{m-1}]^T$, which contains the $L$ bits imposing ISI on $b_m$.

### 3.1 IML Detector

We first consider an IML detector, which assumes that the ideal knowledge of the previous $L$ bits, to evaluate the BER performance of the three-node communication and obtain the optimal detection threshold. The receiver decides whether $b_m = 1$ or $b_m = 0$ by maximizing the following decision function, i.e.,

$$\hat{b}_m = \underset{b_m \in \{0,1\}}{\arg\max}\, P_Y(Y_m|\boldsymbol{b}_{\text{isi}}, b_m) \tag{12}$$

where $\hat{b}_m$ is the detection of $b_m$. There are different means and variances for different ISI permutations at each bit decision interval. Therefore, by comparing the $P_Y(Y_m|b_{\text{isi}}, b_m)$ we can obtain the consequence, i.e.,

$$\hat{b}_m = \begin{cases} 0, & \text{if } P_Y(Y_m|\boldsymbol{b}_{\text{isi}}, b_m = 0) < \zeta_{\text{opt}} \\ 1, & \text{if } P_Y(Y_m|\boldsymbol{b}_{\text{isi}}, b_m = 1) > \zeta_{\text{opt}}. \end{cases} \tag{13}$$

When they are equal, the optimal decision threshold $\zeta_{\text{opt}}$ is obtained as

$$\zeta_{\text{opt}} = \sqrt{\frac{2\sigma^2_{Y_0}\sigma^2_{Y_1}\ln(\frac{\sigma_{Y_0}}{\sigma_{Y_1}}) - \mu^2_{Y_1}\sigma^2_{Y_0} + \mu^2_{Y_0}\sigma^2_{Y_1}}{\sigma^2_{Y_0} - \sigma^2_{Y_1}}} \tag{14}$$

where $\mu_{Y_{b_m}}$ and $\sigma^2_{Y_{b_m}}$ are given in (10).

### 3.2 AML Detector

For practical detection, it is difficult to obtain the exact ISI knowledge. Therefore, we consider an AML detector, which requires no knowledge of the exact ISI but exploits the expectation of ISI, denoted by $\bar{\mu}_Y$. It can be expressed as

$$\bar{\mu}_Y = \mathrm{E}\left\{\sum_{i=0}^{\min\{m,L\}} Nb_{m-i}P_i\right\} = \frac{1}{2}\sum_{i=0}^{\min\{m,L\}} NP_i. \tag{15}$$

Based on (14) and (15), the decision criterion of the AML detector can be derived following the same approach as that of the IML detector.

## 4 Performance Analysis

In this section, we evaluate the BER performance of the proposed system employing both IML and AML detectors. Subsequently, we quantify the energy consumption for transmitting information to $\text{Rx}_2$.

### 4.1 BER Analysis

The BER performance of $\text{Rx}_1$ with a passive relay is analyzed, and this analysis can also be applied to $\text{Rx}_2$. The BER expression for $\text{Rx}_1$ with OOK modulation is given by

$$P_b = \frac{1}{2}P(\hat{b}_m = 1|b_m = 0) + \frac{1}{2}P(\hat{b}_m = 0|b_m = 1) \tag{16}$$

where the probability of transmitting "1" and "0" are equal. Given a detection threshold $\zeta_{\text{opt}}$ in (14), we have

$$\begin{aligned} P(\hat{b}_m = 1|b_m = 0) &= P(Y_m > \zeta_{\text{opt}}|b_m = 0) \\ &= \sum_{\boldsymbol{b}_{\text{isi}} \in \mathcal{B}^{\text{ISI}}} P(\boldsymbol{b}_{\text{isi}})P(Y_m > \zeta_{\text{opt}}|b_{\text{isi}}, b_m = 0) \\ &= \frac{1}{2^L} \sum_{\boldsymbol{b}_{\text{isi}} \in \mathcal{B}^{\text{ISI}}} Q\left(\frac{\zeta_{\text{opt}} - \mu_{Y_0}}{\sigma_{Y_0}}\right) \end{aligned} \tag{17}$$

where $Q(x)$ is the Gaussian $Q$ function defined as $Q(x) = (2\pi)^{-1/2} \int_x^\infty e^{-t^2/2}dt$, $\mathcal{B} = \{0, 1\}$, and $P(\boldsymbol{b}_{\text{isi}})$ is the probability of the occurrence of a specific sequence of $\boldsymbol{b}_{\text{isi}}$ and $P(\boldsymbol{b}_{\text{isi}}) = 1/2^L$. Similarly, we have

$$P(\hat{b}_m = 0|b_m = 1) = 1 - \frac{1}{2^L} \sum_{\boldsymbol{b}_{\text{isi}} \in \mathcal{B}^{\text{ISI}}} Q\left(\frac{\zeta_{\text{opt}} - \mu_{Y_1}}{\sigma_{Y_1}}\right). \tag{18}$$

Then, by substituting both (17) and (18) into (16), the BER of $\text{Rx}_1$ can be obtained as

$$P_b = \frac{1}{2} + \frac{1}{2^L} \sum_{\boldsymbol{b}_{\text{isi}} \in \text{B}^{ISI}} \left(Q\left(\frac{\zeta_{\text{opt}} - \mu_{Y_0}}{\sigma_{Y_0}}\right) - Q\left(\frac{\zeta_{\text{opt}} - \mu_{Y_1}}{\sigma_{Y_1}}\right)\right). \tag{19}$$

### 4.2 Energy Consumption Analysis

In this subsection, we first assume that each IM released by Tx can be chosen as one type of protein, which is composed of $n_{aa}$ amino acids [13]. Here, the total cost to synthesize a molecule can be written as

$$E_{\text{molecules}} = 202.88(n_{aa} - 1) \text{ zJ}. \tag{20}$$

It is clear from (3) and (6) that when transmitting bit "1", additional $N_a$ molecules will arrive at $\text{Rx}_1$ compared to the link Tx $\rightarrow$ $\text{Rx}_1$, where $N_a =$

$[C(t, r, \mu) - C_1(\mathbf{r}, t)] \times V_{R_1} \times N$. This implies that in the proposed scheme, Tx can reduce the number of released molecules to achieve the same number of observed molecules at $\mathrm{Rx}_1$, i.e., saving

$$N_{save} = \frac{1}{2} \times N \left( \frac{C(t, r, \mu)}{C_1(\mathbf{r}, t)} - 1 \right) \tag{21}$$

molecules, where the probability of transmitting bit "1" and "0" is $\frac{1}{2}$. When considering the energy consumption, the proposed scheme can save

$$E_{\text{save}} = \frac{N}{2} \left( \frac{C(t, r, \mu)}{C_1(\mathbf{r}, t)} - 1 \right) \times 202.88(n_{aa} - 1) \text{ zJ} \tag{22}$$

in synthesizing molecules. Correspondingly, the energy consumption resulting from the transport of molecules through vesicles or similar mechanisms will also decrease.

## 5 Numerical Results

In this section, we analyze the results of the channel impulse response (CIR) under the multi-user system and make a comparison with the situation without a passive relay by particle-based simulations (PBS), as well as the BER performance of IML and AML detection with different lengths of ISI and various

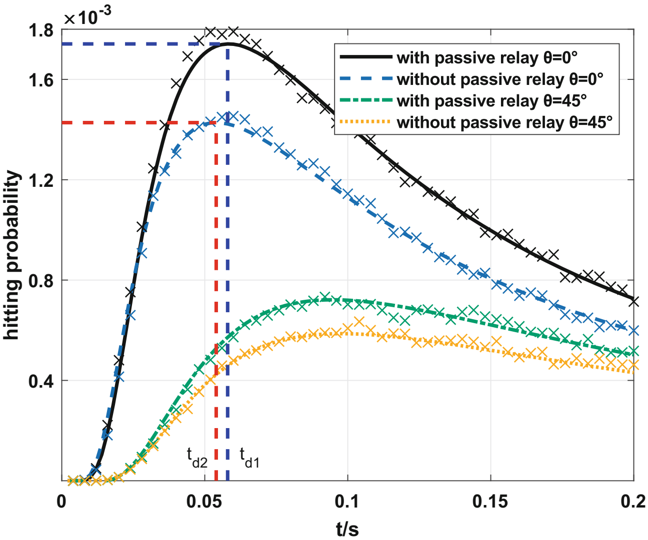


**Fig. 2.** Theoretical- (solid curves) and simulation results (markers) for $P_o(t)$ comparison at $\mathrm{Rx}_1$ with and without $\mathrm{Rx}_2$ ($r_2 = 12\mu m$, $r_s = 14\mu m$, $r = 32\mu m$ , $r_1 = 3\mu m$, $D = 10^{-9} m^2/s$ ).

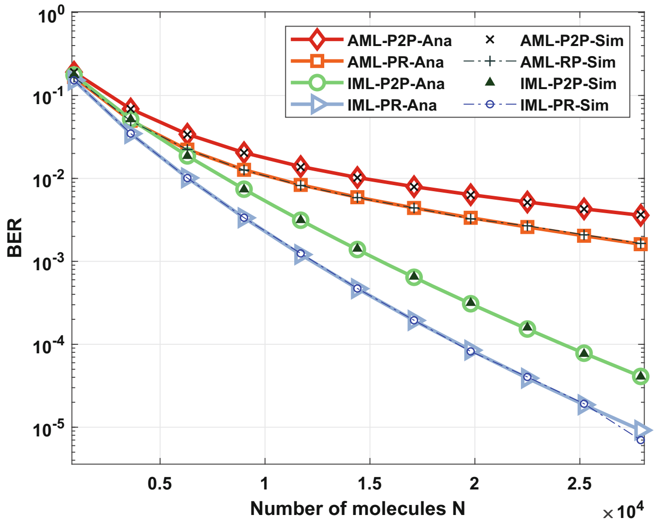


**Fig. 3.** Performance of the OOK modulation with various detectors and the validation of analytical results of the AML and IML detectors ($T = 0.4s$, $L = 4$).

position arrangements of a passive relay. During the PBS, the receiver $Rx_2$ is set to the origin and the position of Tx is $[0, 0, 14\mu m]$. Two different elevations of the $Rx_1$ are considered, i.e., $\theta = 0°$ and $\theta = 45°$. We also set the diffusion coefficient $D$ to $10^{-9}m^2/s$, the diffusion time step is $0.2\mu s$ in the time interval $0 \le t \le 20ms$, where $N = 1.6 \times 10^5$ molecules are released at each iteration and 10 iterations are performed for each case.

The Fig. 2 displays the theoretical and simulated results derived in this paper. It is clear from the figure that the theoretical hitting probability (solid curve) obtained from both (3) and (6) matches well with the simulation hitting probability (markers) under the scenarios with and without a passive relay $Rx_2$ by PBS where the type A molecules are reflected by $Rx_2$ when they touch its surface. According to the two sets of curves, it is also shown that the communication performance will be better when the near receiver can reflect the molecules detected by the far receiver than when only two receivers are working for their molecules. Considering the two elevations, it is observed that the increment of hitting probability under $\theta = 0°$ is larger than that under $\theta = 45°$, and the more they are aligned, the better the gain effect. However, the existence of the passive relay enhances the energy of received signals, but brings about a more severe ISI impact.

In Fig. 3, we compare the analytical BER results with their corresponding simulation counterparts of the AML and IML detectors, where "P2P" and "PR" express that the transmitter communicates with one of the receivers in multi-user

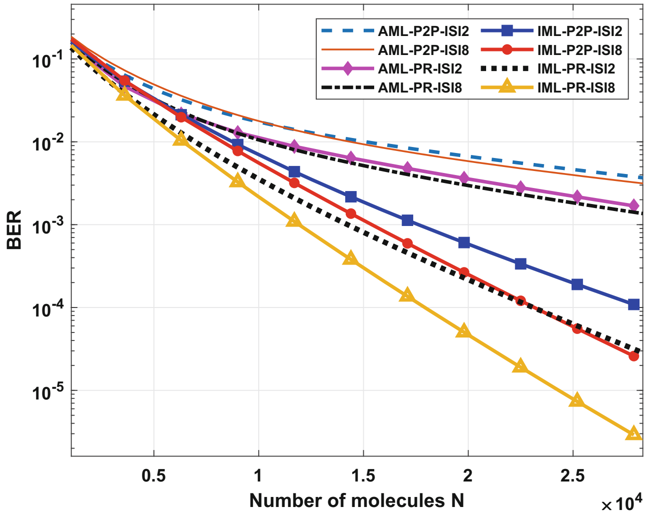


**Fig. 4.** BER comparison between a multi-user system with a passive relay and without a passive relay under various lengths of ISI, $L = 2$, and $L = 8$.

system without a passive relay node and with a passive relay node, respectively. The system parameters in Fig. 2 continue to be used with the bit interval $T = 0.4s$ and the length of ISI $L = 4$. For fairness, the BER performance is evaluated versus the number of emitted molecules since it can represent the ratio of signal to noise. It is observed that the analytical curves match well with the simulation counterparts in each case. The results also show that the BER performance of multi-user system with a passive relay is better than that without it no matter which detector. To achieve the same BER performance, the system with a passive relay can save more molecules released, which means energy saving without any cost. The BER gap when using the AML detector is smaller than that when using the IML detector, since simple OOK modulation cannot resist ISI, and the IML detector could bring better BER performance.

The Fig. 4 compares the impact of different lengths of ISI on the BER performance of the multi-user system. The BER performance of ISI length of 8 is not much better than that of ISI length of 2 when using the AML detector under both P2P and PR communication systems, because it considers statistical ISI rather than actual ISI. When using the IML detector, the more known ISI information is used the BER performance is better. It is also observed that the improvement in BER performance under "PR" condition is similar for different ISI lengths when using the AML detector, but the improvement is larger when using the IML detector.

In Fig. 5, we compare the BER performance at the far receiver $Rx_1$ when the passive relay node is in different positions under the same signal-to-noise ratio. The elevation $\theta$ takes 0, 30, 45, 60, and 90°C respectively, which is the

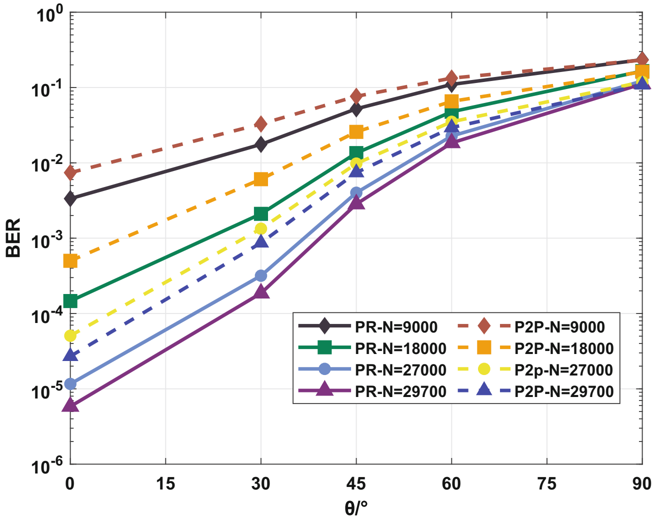


**Fig. 5.** BER comparison under different position arrangements between PR and P2P systems by using the IML detector. ($\theta = 0°, 30°, 45°, 60°, 90°$, $L = 4$).

angle between the line connecting the center of the receiver $Rx_1$ to the coordinate origin and the z-axis. The other system parameters remain unchanged. It can be observed that as $\theta$ increases, the BER performance decreases. Because the passive relay is modeled as a sphere, when the transmitter is very close to the passive relay, the surface closest to the transmitter can bounce the most information molecules, resulting in the maximum information molecule concentration gain in the direction of the line connecting the transmitter and relay center. Therefore, the correct selection of the location of passive relays also has a significant impact on the improvement of communication quality. The BER performance under the P2P communication system is also compared with the PR system.

The Fig. 6 shows the energy saved in the proposed scheme when the receiver observes the same number of molecules, where the number of molecules transmitted $N = 10000$, and we take the $n_{aa} = 51$, the same as the [13]. It is indicated that when the elevation $\theta$ is less than 90°C, the Tx can save high energy because the passive relay helps to enhance the observed molecules. However, when the $\theta$ is 90°C, the passive relay acts as an obstacle for the P2P system.

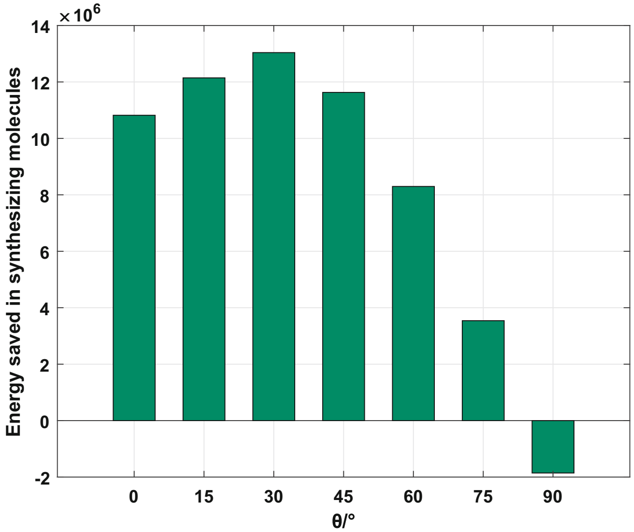


**Fig. 6.** Energy saved in synthesizing molecules. ($N$=1000, $n_{aa}$=51).

# 6 Conclusion

In this paper, we proposed a multi-user communication system model with a passive relay node. We explored that the existing passive relay node can effectively expand the range of the concentration of molecules. Meanwhile, the passive relay node can not only assist far users in communications without excess energy consumption but also act as a receiver to communicate with the transmitter, which saves the number of nodes for multi-user systems. At the far user, for passive relaying with OOK modulation, we showed that the BER performance with a passive relay node is better than that of without a passive relay node by using the IML and AML detectors under different lengths of ISI. We also investigated the impact of different positions of the passive relay node on communication reliability. Finally, results of energy consumption show that the scheme we proposed can save energy.

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