

Application of Interpolation, Newton-Raphson's Method in Weather Forecasting

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Abstract. A distinct collection of known data points can be expanded upon using the Interpolation method to create additional data points within its range. For both regularly and unevenly spaced data points, various techniques have been devised to create usable interpolation equations. The Newton method is another name for the Newton's-Raphson approach. It bears the names of Joseph Raphson and Isaac Newton. This approach is a simple way to solve non-square and nonlinear problems as well as get an approximation of the real value's roots. Additionally, it tries to illustrate a novel method for calculating nonlinear equations that is quite similar to the straightforward Newton Raphson method. The inverse Jacobian matrix will be employed in various applications as well as for additional calculations. The Newton Raphson method's applications and constraints, the derivative Newton Raphson formula algorithm, and the self-derivative function in solving nonlinear equations with a scientific calculator are covered below.

Keywords: Interpolation, Newton-Raphson's.

1 Introduction

1.1 Interpolation

The calculus of finite differences serves as the foundation for the study of interpolation. We start by using the forward and backward differences of a function to derive two crucial interpolation formulas. These equations are frequently used in scientific and technical research [15]. Finding the value of g that correlates to any value of $h = h_i$ between h_0 and h_n is known as Interpolation. Thus, extrapolation is the process of determining a function's value outside of a given range, while interpolation is the method of estimating the value of a function for any intermediate independent variable value.

Let's say we have the following values of g for $f(h)$ for a range of h values:

$h:$	h_0	h_1	$h_2 \dots h_n$
$g:$	g_0	g_1	$g_2 \dots g_n$

Explicitly knowing the function $f(h)$ makes it easy to find the value of g that corresponds to any value of h . Conversely, in the majority of cases, where since, the form of $f(h)$ is unknown, figuring out what form $f(h)$ actually takes can be exceedingly challenging using a tabular set of numbers (h_i, g_i) . In these situations, the function $f(h)$ is substituted with a more straightforward function $\phi(h)$, which takes the same values as $f(h)$ at the tabulated set of points. $\phi(h)$, also referred to as the smoothing or interpolating function, can be used to compute any other value. The procedure is known as polynomial interpolation and $\phi(h)$ is the interpolating polynomial if it is a polynomial. Trigonometric interpolation is also available when $\phi(h)$ is a finite trigonometric series [3], [4].

2 Methodology

2.1 Newton-Raphson's Method

In contrast to other approaches, this one just needs to evaluate the derivative once every iteration. The secant approach is 1.389482397 times better than the Newton method, while the Newton-Raphson's method is 7.678622465 times better than the bisection method. It has been a challenge in previous years to find the solution to the set of nonlinear equations $f(x) = (f_1 \dots f_n) = 0$. One simple strategy to discover the solution to a nonlinear problem is to consider the Newton-Raphson's method.

This approach is a simple way to determine the nonlinear equation's roots; it produces good results with quick convergence speed, and it is also adopted by Matlab and C++. To perform this computation, a scientific calculator is utilized.

The roots of the true value are more precisely estimated using this method of determining roots. It makes use of the notion that a straight-line tangent to a continuous and differentiable function can be used to approximate it. The plan is to use mathematics to draw a tangent line where it intersects the x axis, starting with an estimate that is only somewhat near the true roots. It is possible to repeat this process and the x axis's junction point will produce a more accurate approximation of the original function root than the initial estimation [1], [4], [15].

2.1.1 Derivation

Let $g = f(h)$ has values $g_0, g_1, \dots, g_n$ which correspond to $h_0, h_1, \dots, h_n$ of h .

Assume that these h values are equally spaced so that,

$h_i = h_0 + i k \quad (i=0, 1, 2, \dots, n), \quad g(h)$ is a polynomial of degree n in h .

Assuming that,

$g(h_0) = g_0, g(h_1) = g_1, \dots, g(h_n) = g_n$.

We can write,

% Equation 1

$$g(h) = e_0 + e_1(h-h_0) + e_2(h-h_0)(h-h_1) + e_3(h-h_0)(h-h_1)(h-h_2) + \dots + e_n(h-h_0)(h-h_1) \dots (h-h_{n-1})$$

putting $h = h_0, h_1, h_2, \dots, h_n$ successively in Equation (1)

we get,

$$g_0 = e_0, \quad g_1 = e_0 + e_1(h_1 - h_0), \quad g_2 = e_0 + e_1(h_2 - h_0) + e_2(h_2 - h_0)(h_2 - h_1), \\ \dots, \text{ so on.}$$

From these, we find that.

$$e_0 = g_0, \quad \Delta g_0 = g_1 - g_0 = e_1 k$$

$$\therefore e_1 = \frac{1}{k} \Delta g_0$$

$$\text{Also, } \Delta g_1 = g_2 - g_1 = e_1(h_2 - h_1) + e_2(e_2 - e_0)(e_2 - e_1) = e_1 k + e_2 k k = \\ \Delta g_0 + 2k^2 e_2$$

$$\therefore e_2 = \frac{1}{2k^2} (\Delta g_1 - \Delta g_0) = \frac{1}{2!k^2} \Delta^2 g_0$$

$$\text{Similarly, } e_3 = \frac{1}{3k^3} (\Delta g_1 - \Delta g_0) = \frac{1}{3!k^3} \Delta^3 g_0$$

Substituting these values in Equation (1)

$$g(h) = g_0 + \frac{1}{k} \Delta g_0 (h - h_0) + \frac{1}{2!k^2} \Delta^2 g_0 (h - h_0)(h - h_1) + \frac{1}{3!k^3} \Delta^3 g_0 (h - h_0)(h - h_1)(h - h_2) \dots [15]$$

2.1.2 Numerical solution

Provided function:

$z^3 - z - 1 = 0$, is differentiable.

Initially, the derivative of $f(z)$ is $f'(z) = 3z^2 - 1$

Let's now determine the approximate value.

$$f(1) = 1 - 1 - 1 = -1 \text{ and } f(2) = 8 - 2 - 1 = 5$$

The root is therefore located through the interval $[1, 2]$. Therefore, let's assume that the function's first guess root is $z_1 = 1.5$

$$f(z) = z^3 - z - 1$$

Now,

$$f(1.5) = 1.5^3 - 1.5 - 1 = 0.875$$

$$f'(1.5) = 3 \cdot 1.5^2 - 1 = 5.750$$

Applying Newton's Iteration Formula:

$$z_2 = z_1 - f(z_1)/f'(z_1) = 1.5 - 0.875/5.750 = 1.34782600$$

The iteration for $z_3, z_4, \dots$ is done similarly.

Table 1. Newtons's iteration

n	x_n	$f(x_n)$
1	1.34782608696	0.100682173091
2	1.32520039895	0.002058361917
3	1.32471817400	0.000000924378
4	1.32471795724	0.000000000000
5	0.000000000000	

As a result, the intended root, adjusted to nine decimal places of the provided function is $x = 1.324717957$. Only $x = 1.3252$ is produced by the MATLAB software.

However, this value can be increased by increasing the permitted error supplied.

2.1.3 Matlab code

```
% Program Code of Newton-Raphson's Method in MATLAB
clc
clear
e = input('Enter the Function f(x) : ','s');
z(1) = input('Enter Initial Value of x : ');
err = input('Enter Error : ');

t = inline(e)
dffff = diff(str2sym(e));
d = inline(dffff);

    for k = 1:100
        z(k+1) = z(k) - ((f(z(k)))/d(z(k)))
        errr(k) = abs((z(k+1)-z(k))/z(k))
        if errr(k)<err
            break
        end
    end
root = z(k) [1],[15]
```

2.1.4 Result

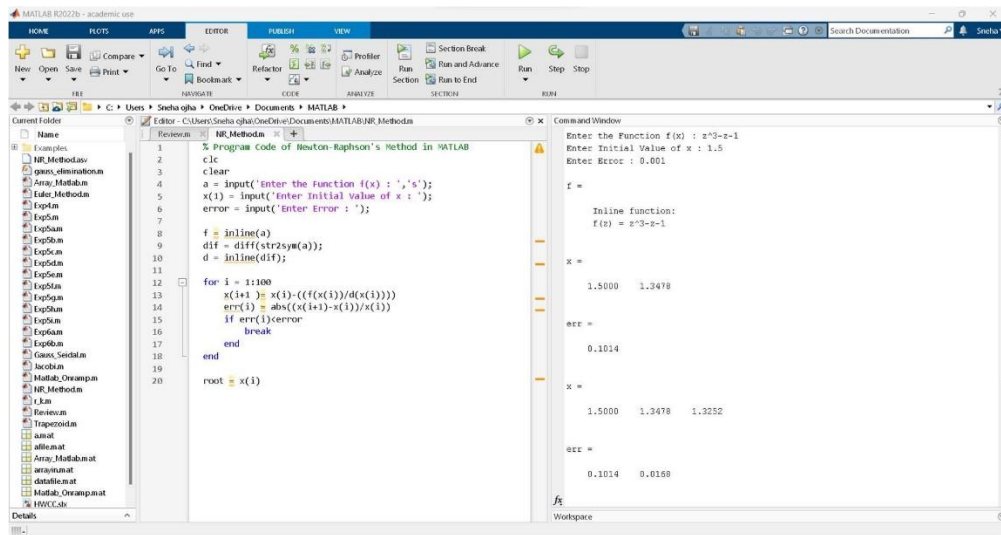


Fig. 1 (a). Code and 1st half of output

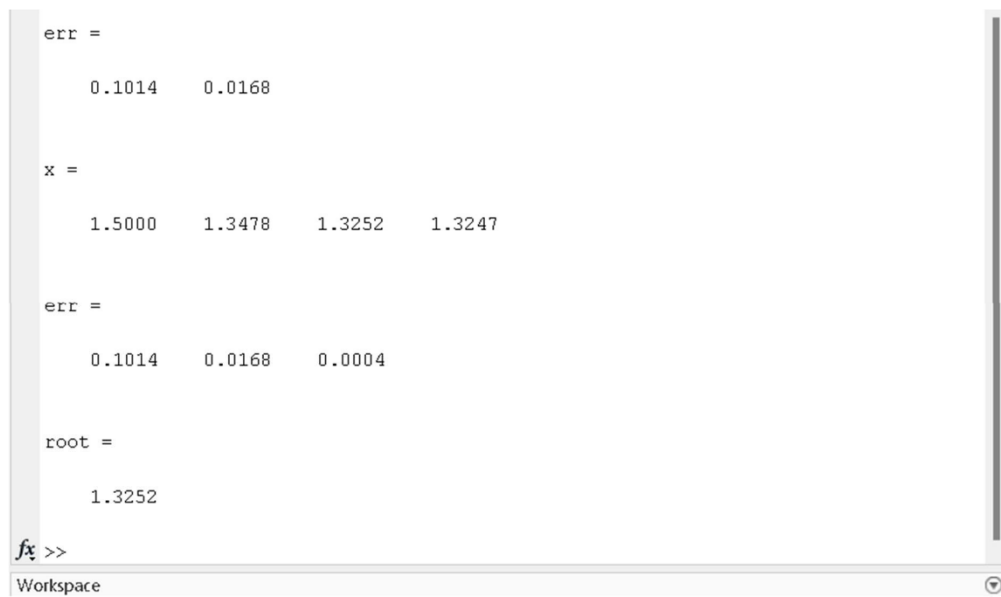


Fig. 1 (b). 2nd half of output

3 Application

3.1 Brief

A mathematical method called interpolation is used to estimate values that aren't stated clearly in a data collection. It is frequently used to produce missing data points or make predictions in a variety of fields. Here are a few examples of interpolation's practical uses:

Weather Forecasting: In areas without direct observation, meteorologists forecast the weather using interpolation. Based on data made close by, they employ interpolation techniques to estimate meteorological variables like temperature, wind speed, and precipitation at places [2], [3], [8], [9].

Making use of MATLAB code to assist in interpolating temperature values between observed data [1].

3.2 MATLAB code

```
clc
clear
```

```
%Numerical Analysis via Interploation
%Use of interp1 to interpolate between the provided data
```

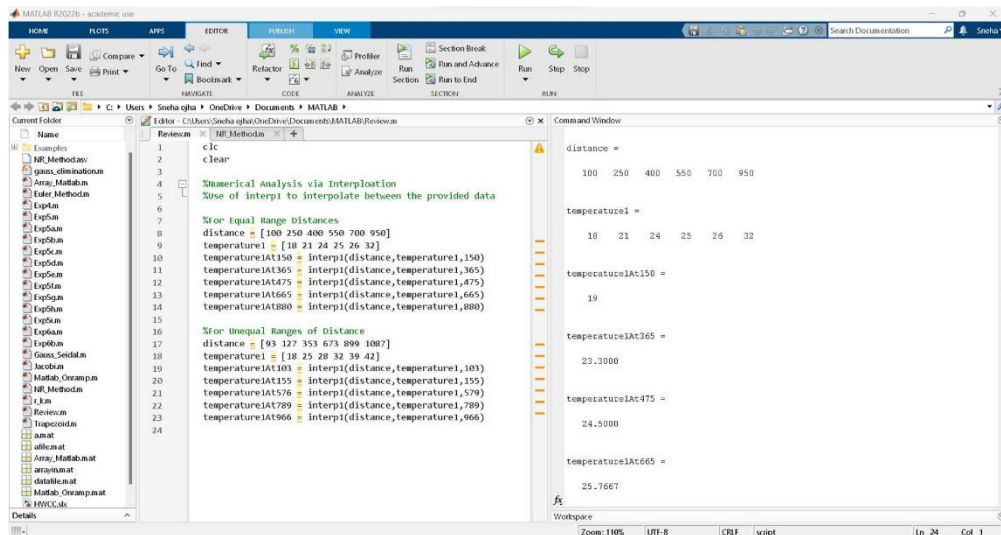
```
%For Equal Range Distances
```

```
distance = [100 250 400 550 700 950]
temperature1 = [18 21 24 25 26 32]
temperature1At150 = interp1(distance,temperature1,150)
temperature1At365 = interp1(distance,temperature1,365)
temperature1At475 = interp1(distance,temperature1,475)
temperature1At665 = interp1(distance,temperature1,665)
temperature1At880 = interp1(distance,temperature1,880)
```

```
%For Unequal Ranges of Distance
```

```
distance = [93 127 353 673 899 1087]
temperature1 = [18 25 28 32 39 42]
temperature1At103 = interp1(distance,temperature1,103)
temperature1At155 = interp1(distance,temperature1,155)
temperature1At576 = interp1(distance,temperature1,579)
temperature1At789 = interp1(distance,temperature1,789)
temperature1At966 = interp1(distance,temperature1,966)
```

3.3 Outcome



The figure shows the MATLAB R2020b interface. The code editor on the left contains the following MATLAB code:

```
1 clc
2 clear
3
4 %Numerical Analysis via Interpolation
5 %Use of interp1 to interpolate between the provided data
6
7 %For Equal Range Distances
8 distance = [100 250 400 550 700 950]
9 temperature1 = [18 21 24 25 26 32]
10 temperature1At150 = interp1(distance, temperature1, 150)
11 temperature1At365 = interp1(distance, temperature1, 365)
12 temperature1At475 = interp1(distance, temperature1, 475)
13 temperature1At665 = interp1(distance, temperature1, 665)
14 temperature1At880 = interp1(distance, temperature1, 880)
15
16 %For Unequal Ranges of Distance
17 distance = [93 127 353 673 899 1087]
18 temperature1 = [18 25 28 32 39 42]
19 temperature1At103 = interp1(distance, temperature1, 103)
20 temperature1At155 = interp1(distance, temperature1, 155)
21 temperature1At576 = interp1(distance, temperature1, 576)
22 temperature1At789 = interp1(distance, temperature1, 789)
23 temperature1At966 = interp1(distance, temperature1, 966)
24
```

The Command Window on the right shows the output of the code:

```
distance =
    100    250    400    550    700    950

temperature1 =
     18     21     24     25     26     32

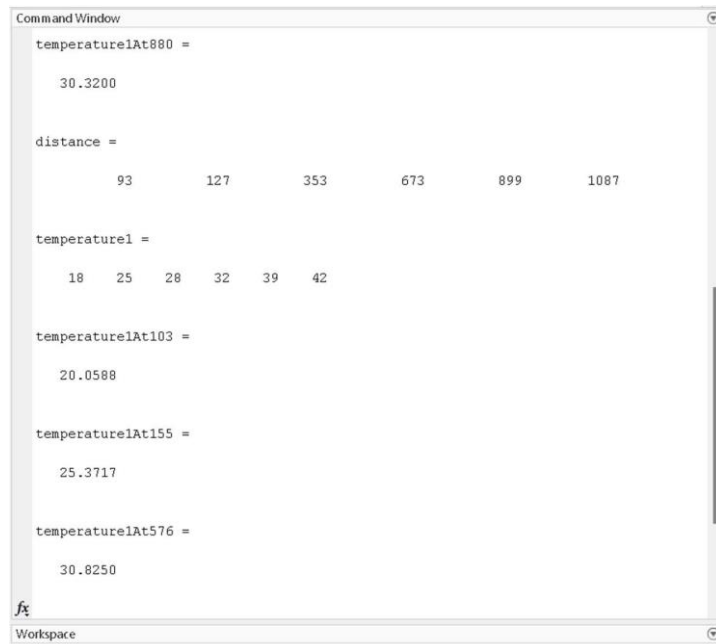
temperature1At150 =
     19

temperature1At365 =
    23.3000

temperature1At475 =
    24.5000

temperature1At665 =
    25.7667
```

Fig. 2(a). MATLAB code and output 1st half



The Command Window shows the continuation of the output from the MATLAB code:

```
temperature1At880 =
    30.3200

distance =
     93    127    353    673    899   1087

temperature1 =
     18     25     28     32     39     42

temperature1At103 =
    20.0588

temperature1At155 =
    25.3717

temperature1At576 =
    30.8250
```

Fig. 2(b). Output 2nd half

```
temperature1At789 =  
    35.5929  
  
temperature1At966 =  
    40.0691  
  
fx >>  
Workspace
```

Fig. 2(c). Output 3rd half

4 Result and discussion

The approximate temperature range for a certain place, where we have data from the nearby area as our initial data that helps us identify the missing data, has been found using the interpolation method [7], [10], [13].

Using MATLAB's library function `interp1`, we implemented this method in a code that finds data within a one- dimensional array's ranges by means of interpolation.

Additionally, we can apply the interpolation method in the following areas to improve the results of our daily data [6], [11], [15]:

- I. Geographic Information Systems (GIS): In a spatial dataset, GIS software estimates values for missing or incomplete data points through the use of interpolation. It can be used, for instance, to calculate the population density in places without official census data.
- II. Medical Imaging: To create missing data points in medical images, interpolation is employed in this field. This is especially helpful for imaging methods when images are produced from a small number of data points, imaging techniques include Computed-Tomography (CT) Scans & Magnetic-Resonance- Imaging (MRI).
- III. Finance: To ascertain the worth of securities that are not traded on the market, interpolation is employed in financial modeling. Analysts can approximate the yield curve, for instance, for bonds with maturities that are not offered on the market by using interpolation.
- IV. Audio and video processing: Interpolation is a technique used to boost digital signal resolution in audio and video processing. Applications like upscaling low-resolution video to high-resolution video benefit greatly from this.

5 Conclusions

In contrast to earlier techniques The Newton Raphson approach is quicker and more precise. This strategy is not helpful in this case if the tangent and the x axis are just parallel to one another. This approach is mostly utilized while calculating small molecules.

It is not employed when calculating large molecules. It is simple to convert various dimensions with this method. It starts poorly and iterates in a stagnant manner. It is also possible to state

that the intrinsic value based on measured permittivity can be ascertained with great benefit from the Newton Raphson approach. One less than is the polynomial's degree of the total count of pairs of observations. You can utilize the polynomial that correlates with the given set of numerical information.

If the two extreme values of the independent variable fall inside the range, for interpolation. The interpolation technique presented here can be effectively applied to inverse interpolation as well.

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