

# Efficient and Fair Rate Allocation Scheme for Multi-user over Heterogeneous Wireless Access Networks

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**Abstract**—Allocating bandwidth to multiple users sharing multiple heterogeneous access networks is an important issue in the next generation wireless networks. In this paper, we propose a bandwidth allocation scheme named as Efficient and Fair Allocation in Rate (EFAR), which is formulated as a convex optimization problem that maximizes the utility of each user and determines how to split the total rate among the multiple access networks. Compared with the scheme that minimizes the total expected distortion of all the video streams and other schemes that only consider the system efficiency or the user fairness, the EFAR scheme considers both efficiency and fairness, and achieves a more balanced utility among all the users.

**Keywords**- multi-user; heterogeneous networks; utility; bandwidth allocation

## I. INTRODUCTION

One of the most important features of the next generation wireless networks will be the heterogeneous wireless access in which the multi-homed terminals are able to connect several wireless access networks simultaneously. This would take full advantage of the limited resources of various wireless access networks. Therefore, the radio resource management will face new challenges, and the bandwidth allocation scheme should be designed to manage the wireless resources more efficiently and fairly.

The bandwidth allocation problem for multi-user in a single network is important and has been well studied in many literatures [1-5]. In [1], the bandwidth is allocated for each user to maximize the sum of the utility of all the users. In [2], joint routing and rate allocation algorithm for multiple video streams in ad hoc networks is addressed. The bandwidth allocation based on the game theoretic frameworks is described in [3-5]. In [3], since each user is considered as a follower, while the Service Provider is considered as a leader, a Stackelberg game model is used to allocate bandwidth. Non-cooperation game theory is used to allocate the network capacity among users sharing a single wireless network in [4]. A bandwidth allocation scheme for users to share the network resource fairly using cooperative game theoretic model is presented in [5]. However, only one wireless access network is considered in these bandwidth allocation schemes which do not address the problem of the simultaneous use of heterogeneous networks. The bandwidth allocation for heterogeneous wireless networks is investigated from the network point of view in [6-8]. The bandwidth allocation problem is formulated as an  $N$ -Person cooperative game in [6] and non-cooperative game in [7]. A

utility-based bandwidth allocation scheme for multiple services in the heterogeneous networks is presented in [8], which allocated the bandwidth to users based on the utility fairness of networks. Multi-user multi-network bandwidth allocation problem is also addressed in [9-10]. The problem is studied from the perspective of the  $H_\infty$  optimal control of linear dynamic system in [9], while it is described from the perspective of minimizing the total distortion of the system in [10]. However, the existing rate allocation schemes for multi-user multi-network mainly focus on the efficiency issue, and not take the proportional fairness among the users into account.

Considering both the system efficiency and the user fairness, we propose a bandwidth allocation scheme named as the Efficient and Fair Allocation in Rate (EFAR) for multi-user over heterogeneous wireless access networks in this paper. The EFAR scheme determines the total allocated rate of each application flow by maximizing the utility of each user, and distributes the traffic among multiple available wireless access networks simultaneously according to the available network bandwidth. Since the different characteristics of users and networks are taken into account, the EFAR scheme will guarantee the fairness among the users and provide a good system performance.

Our main contributions are summarized as follows: In order to allocate bandwidth efficiently and fairly, we first define the user's utility which takes both bandwidth and fairness into account, and then propose a solution to maximize each user's utility. Moreover, we determine the tradeoff parameter and the bandwidth allocation weight for the EFAR scheme.

## II. SYSTEM MODEL

We consider a heterogeneous environment, in which there are a lot of disparate wireless access networks, such as IEEE 802.11 WLAN, WCDMA and IEEE 802.16 WMAN, as shown in Fig. 1. The geographical area is covered with WCDMA Base Station (BS), WMAN Base Station and Access Point (AP) of WLAN. A multi-homed terminal is able to connect to these wireless access networks simultaneously.

In the heterogeneous wireless network environment, we assume that there are  $I$  users ( $I=\{1,2,\dots,I\}$ ) using multi-homed terminals in a certain geographic region which is covered by  $N$  heterogeneous wireless access networks ( $\mathcal{N}=\{1,2,\dots,N\}$ ). Each user transmits a video stream to the server. Since the wireless network bandwidth is restricted, a single wireless network may not guarantee the user's QoS requirements. In addition, the full

utilization of network resources may be not achieved. Therefore,  $I$  users should split their video streams among all of the  $N$  wireless access networks and transmit simultaneously.

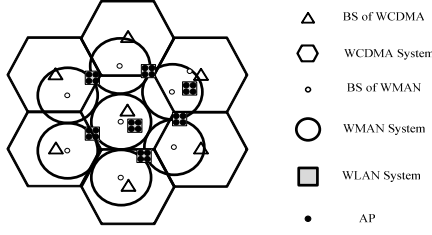


Fig. 1. Heterogeneous wireless network environment

Let  $C_n$  be the available bandwidth of the network  $n$ ,  $n \in \mathcal{N}$ .  $R_{i,n}$  is the allocated rate of user  $i$  over the network  $n$ .  $R_i$  is the total allocated rate of user  $i$ ,  $i \in I$ . Then

$$R_i = \sum_{n=1}^N R_{i,n}, \quad i \in I. \quad (1)$$

From the perspective of the user  $i$ , the available bandwidth of the network  $n$  is

$$C_{i,n} = C_n - \sum_{\substack{i'=1 \\ i' \neq i}}^I R_{i',n}. \quad (2)$$

In this paper, we use the Distortion-Rate (DR) model as below [11]

$$D = \omega e^{-\mu R}, \quad R_{\min} \leq R \leq R_{\max}, \quad \omega > 0, \mu > 0, \quad (3)$$

where  $D$  is the distortion of the video measured as the mean square error (MSE), and  $R$  is the rate for the video sequence,  $R_{\min}$  and  $R_{\max}$  are the minimum rate and maximum rate, respectively;  $\omega$  and  $\mu$  are the parameters which are dependent on video content.

Then, we define the utility of the user  $i$  with  $D$  and  $R$  as

$$U_i(D_i, R_i) = G(D_i) - aE(R_i), \quad (4)$$

where  $U_i$  is the utility of the user  $i$ ,  $G(D_i)$  is the gain function which should be a monotonically decreasing function of  $D_i$ ,  $E(R_i)$  is the cost function that should be a monotonically increasing function of  $R_i$ , and  $a$  is a tradeoff parameter.

The Peak Signal to Noise Ratio (PSNR) is a more objective measure of the video quality than the MSE. Given the distortion  $D$ ,  $PSNR = 10 \lg(255^2 / D)$ . And the second derivative of the gain function about PSNR should be negative. Hence,

$$G(D_i) = \ln(PSNR_i) = \ln(10 \lg \frac{255^2}{D_i}). \quad (5)$$

In this paper, we assume that the cost of the unit of bandwidth offered by each wireless network is the same. Therefore, we define the cost for the user  $i$  over the network  $n$  is  $pR_{i,n}$  where  $p$  is a positive constant. The user cost function is the sum of the cost that user pay for each network, that is,

$$E(R_{i,n}) = p \sum_{n=1}^N R_{i,n}. \quad (6)$$

Combining (3), (4), (5) and (6) and ignoring the constant term, we have the utility function of the user  $i$  as follows

$$U_i(R_i, R_{i,n}) = \ln(2 \ln 255 - \ln \omega_i + \mu_i R_i) - \kappa \sum_{n=1}^N R_{i,n}, \quad (7)$$

where  $\kappa = a \cdot p$ .

### III. EFFICIENT AND FAIR ALLOCATION IN RATE

In this section, we address the problem of the rate allocation among multi-user with multiple video sequences over multiple wireless networks. Each video sequence is assumed to be elastic with a maximum rate  $R_i^{\max}$  and a minimum rate  $R_i^{\min}$ . The problem to solve is that how to allocate the  $N$  networks' resources to  $I$  users fairly and efficiently. Since each user is selfish, from the user's point of view, the user  $i$  chooses its own rate over each network  $\mathbf{R}_i = \{R_{i,1}, R_{i,2}, \dots, R_{i,N}\}$  under certain constraint conditions, and tries to maximize its own utility function. The multi-user multi-network rate allocation problem is formulated as follows

$$\max_{\mathbf{R}_i} U_i(R_i, R_{i,n}), \quad (8)$$

$$s.t. R_i = \sum_{n=1}^N R_{i,n}, \quad \forall i \in I, \quad (9)$$

$$R_i^{\min} \leq R_i \leq R_i^{\max}, \quad \forall i \in I, \quad (10)$$

$$R_n = \sum_{i=1}^I R_{i,n} \leq C_n, \quad \forall n \in \mathcal{N}, \quad (11)$$

$$R_{i,n} = \rho_{i,n} R_i, \quad \forall n \in \mathcal{N}. \quad (12)$$

Eq. (11) indicates the total rate of the network  $n$  should not exceed its available bandwidth  $C_n$ . In (12),  $\rho_{i,n}$  denotes the proportion of the total rate of the user  $i$  allocated over the network  $n$ .

From above, we observe that the gain and the cost of each user only depend on  $R_i$ ,  $i \in I$ . Hence, we first consider the following model to determine  $R_i$

$$\max_{R_i} U_i(R_i) = \ln(2 \ln 255 - \ln \omega_i + \mu_i R_i) - \kappa R_i, \quad (13)$$

$$s.t. R_i^{\min} \leq R_i \leq R_i^{\max}, \quad \forall i \in I.$$

Note that the constraints are linear and the second derivative of  $U_i$  is negative that means  $U_i$  is convex, which implies that the K-K-T conditions are necessary and sufficient for optimality. If  $R_i^*$  is the optimal solution of (13), then,

$$R_i^* = \max \left[ R_i^{\min}, \min \left( \frac{1}{\kappa} - \frac{2 \ln 255 - \ln \omega_i}{\mu_i}, R_i^{\max} \right) \right], \quad \forall i \in I. \quad (14)$$

And according to (12), the optimal rate of the user  $i$  over the network  $n$  is

$$R_{i,n}^* = \rho_{i,n} R_i^*, \quad \forall i \in I, \forall n \in \mathcal{N}. \quad (15)$$

Therefore, the corresponding optimal strategy of the user  $i$  is  $\mathbf{R}_i^* = \{R_{i,1}^*, R_{i,2}^*, \dots, R_{i,N}^*\}$ .

#### A. Selecting the parameter $\rho_{i,n}$

As mentioned above,  $\rho_{i,n}$  denotes the allocated proportion of the total rate of the user  $i$  over the network  $n$ , which guarantees the fairness of network utilization. Here, fairness means that  $R_{i,n}$  is in direct proportion to its available bandwidth observed over the network  $n$ . Therefore,  $R_{i,n} = \rho_{i,n} R_i^*$  satisfies the weighted proportionally fair[1]

$$\begin{aligned} & \max_{\rho_{i,n}} \sum_{n=1}^N C_{i,n} \ln(\rho_{i,n} R_i^*), \\ & \text{s.t. } \sum_{n=1}^N \rho_{i,n} = 1. \end{aligned} \quad (16)$$

From (16), we have

$$\rho_{i,n} = C_{i,n} / \sum_{n=1}^N C_{i,n}, \forall n \in \mathcal{N}. \quad (17)$$

*Lemma 1*  $\rho_{i,n}$  is only dependent on the available bandwidth of the network  $n$  and independent of other users' rates over the network  $n$ . That is,

$$\rho_{i,n} = C_{i,n} / \sum_{n=1}^N C_{i,n} = C_n / \sum_{n=1}^N C_n = \rho_n. \quad (18)$$

Without loss of generality, we assume that there are  $i-1$  users connect to the network  $n$  before user  $i$  starts to access to the network  $n$ . We can use the mathematical induction to prove that the existence of  $i-1$  users dose not affect  $\rho_{i,n}$ .

Eq. (18) guarantees the fairness of the network utilization, which is a good solution for the load balance problem.

Combining (14), (15) and (18), the optimal rate of the user  $i$  over the network  $n$  is

$$R_{i,n}^* = \frac{C_{i,n}}{\sum_{n=1}^N C_{i,n}} \max \left[ R_i^{\min}, \min \left( \frac{1}{\kappa} - \frac{2 \ln 255 - \ln \omega_i}{\mu_i}, R_i^{\max} \right) \right]. \quad (19)$$

$R_{i,n}^*$  is determined by  $C_{i,n}$ ,  $R_i^{\min}$ ,  $R_i^{\max}$ ,  $\mu_i$ ,  $\omega_i$  and  $\kappa$ ,  $C_{i,n}$  can be obtained by observing each network independently,  $R_i^{\min}$ ,  $R_i^{\max}$ ,  $\mu_i$  and  $\omega_i$  depend on the source coding scheme and the video content. Therefore, we have to select an appropriate  $\kappa$  to make the bandwidth allocation more efficient.

#### B. Selecting the parameter $\kappa$

The network utilization is defined as  $W_n = (\sum_{i=1}^I R_{i,n}^*) / C_n$ .

Lemma 2 will show that available network bandwidth can be fully utilized if  $\kappa$  is appropriately selected.

*Lemma 2* For each network, if  $\kappa^*$  is the constant satisfying

$$\sum_{i=1}^I \left( \max \left[ R_i^{\min}, \min \left( \frac{1}{\kappa^*} - \frac{2 \ln 255 - \ln \omega_i}{\mu_i}, R_i^{\max} \right) \right] \right) = \sum_{n=1}^N C_n, \quad (20)$$

then  $W_n = (\sum_{i=1}^I R_{i,n}^*) / C_n = 1$ .

*Proof:* For user  $i$ ,

$$\begin{aligned} R_{i,n}^* &= \frac{C_{i,n}}{\sum_{n=1}^N C_{i,n}} \max \left[ R_i^{\min}, \min \left( \frac{1}{\kappa^*} - \frac{2 \ln 255 - \ln \omega_i}{\mu_i}, R_i^{\max} \right) \right] \\ &= \frac{C_{i,n}}{\sum_{n=1}^N C_{i,n}} R_i^* \text{ and } C_{i,n} = C_n - \sum_{i'=1, i' \neq i}^I R_{i',n}^*. \text{ Then,} \end{aligned}$$

$$\sum_{i=1}^I R_{i,n}^* = \sum_{i=1}^I \left( (C_{i,n} \cdot R_i^*) / \sum_{n=1}^N C_{i,n} \right) = \sum_{i=1}^I \left( (C_{i,n} \cdot R_i^*) / \left( \sum_{n=1}^N C_n - \sum_{i'=1, i' \neq i}^I R_{i',n}^* \right) \right)$$

Since  $\sum_{i=1}^I \left( \max \left[ R_i^{\min}, \min \left( \frac{1}{\kappa^*} - \frac{2 \ln 255 - \ln \omega_i}{\mu_i}, R_i^{\max} \right) \right] \right)$

$$= \sum_{i=1}^I R_i^* = \sum_{n=1}^N C_n, \text{ we have } \sum_{i=1}^I R_{i,n}^* = \sum_{n=1}^N C_n - R_i^*.$$

And  $\sum_{i=1}^I R_{i,n}^* = \sum_{i=1}^I \left( \frac{C_{i,n}}{R_i^*} R_i^* \right) = \sum_{i=1}^I C_{i,n} = C_n, \forall n \in \mathcal{N}.$

Finally,  $W_n = \frac{\sum_{i=1}^I R_{i,n}^*}{C_n} = \frac{C_n}{C_n} = 1, \forall n \in \mathcal{N}.$  ■

Therefore, selecting  $\kappa = \kappa^*$  that guarantees each network's available bandwidth is fully utilized, indicates that our proposed EFAR is an efficient bandwidth allocation scheme in terms of the network utilization.

It is obvious that  $\kappa$  is a global parameter calculated in the Joint Resource Management Module (JRMM) of the heterogeneous wireless networks, and the calculation needs the parameters of users and networks. In order to reduce the computational complexity of the JRMM and adopt a decentralized mechanism, we use bi-section method as shown in Algorithm 1 to obtain  $\kappa^*$ .

Algorithm 1(Bi-section Method):

(1) The JMMR collects the available bandwidth of all the networks and the parameters of each user, sets the lower bound as  $\kappa_l = 1 / (\sum_{n=1}^N C_n + \max_{i \in \mathcal{I}} \frac{2 \ln 255 - \ln \omega_i}{\mu_i})$ , and the upper bound

as  $\kappa_u = I / \sum_{n=1}^N C_n$ , initializes  $\kappa_0 = (\kappa_u + \kappa_l) / 2$ , and chooses the threshold  $\varepsilon > 0$ .

(2) The JMMR sends  $\kappa_0$  to all the users and each user calculates

$$R_i = \max \left[ R_i^{\min}, \min \left( \frac{1}{\kappa_0} - \frac{2 \ln 255 - \ln \omega_i}{\mu_i}, R_i^{\max} \right) \right], \forall i \in I \text{ and}$$

sends it back to the JMMR.

(3) The JMMR sums up all the  $R_i$ s, and compares it with  $\sum_{n=1}^N C_n$ .

$$\text{If } \sum_{i=1}^I R_i > \sum_{n=1}^N C_n, \kappa_l = \kappa_0; \text{ else } \kappa_u = \kappa_0.$$

(4) The JMMR checks the difference between  $\kappa_u$  and  $\kappa_l$ . If  $\kappa_u - \kappa_l > \varepsilon$ , set  $\kappa_0 = (\kappa_u + \kappa_l)/2$  and goes to (2); otherwise, the iteration terminates, and  $\kappa^* = \kappa_0$ .

### C. Proportionally Fair in Utility

As defined in [1], if the utility is proportionally fair, for each  $U_i \in \mathcal{U}$  ( $\mathcal{U} = \{U_1, U_2, \dots, U_I\}$ ), the aggregate of proportional changes is zero or negative, i.e.,

$$\sum_{i=1}^I \frac{U_i - U_i^*}{U_i^*} \leq 0, \quad (21)$$

where  $U_i^*$  is the utility at the optimal rate of the user  $i$ ,  $U_i$  is any other feasible utility of the user  $i$ . Since  $U_i^*$  is the maximal utility for the EFAR scheme, (21) holds obviously. Therefore, for the EFAR scheme, the utility of user is proportionally fair.

## IV. SIMULATION RESULTS AND ANALYSIS

In our simulations,  $I=7$ , each user transmits one video sequence, and the values of the parameters,  $R_i^{\min}$ ,  $R_i^{\max}$ ,  $\mu_i$  and  $\omega_i$ , are shown in Table I. The allocated rate of the user  $i$  over network  $n$  is not only dependent on the content of video stream, but also is associated with available network bandwidth. We set  $N=3$ , that is, there are 3 wireless networks, assumed to be WLAN, WCDMA, and WMAN whose available bandwidth is  $C_1$ ,  $C_2$  and  $C_3$ , respectively.

TABLE I. THE PARAMETER  $R_i^{\min}$ ,  $R_i^{\max}$ ,  $\mu_i$  AND  $\omega_i$  OF VIDEO SEQUENCE

Sequence	$\omega_i$	$\mu_i$	$R_i^{\min}$ (kbps)	$R_i^{\max}$ (kbps)
1	69.2433	0.0416	1.5119	84.5447
2	81.9924	0.0114	20.2554	322.0153
3	81.6896	0.0043	28.4987	878.8011
4	76.7633	0.0093	17.8168	388.7091
5	129.3377	0.0024	286.311	1720
6	113.9919	0.0025	255.0682	1610
7	71.4521	0.0074	12.7781	481.1014

In order to validate our proposed bandwidth allocation scheme, we compare it with the User-based Absolute Fairness scheme (UAF), the Network-based Absolute Fairness scheme (NAF) and the Media-Aware allocation scheme (Media-Aware) [10]. In the UAF scheme, each user's optimal total rate is equally divided to all the networks. In the NAF scheme, each network's available bandwidth is equally divided to all the users. In the Media-Aware scheme, the total distortion of all

the video sequences sharing multiple access networks is minimized.

Fig. 2 shows the comparison of the allocated rates of the UAF scheme, the NAF scheme, the Media-Aware scheme and the EFAR scheme. In Fig. 2 (a) and (b),  $\{C_1, C_2, C_3\} = \{800\text{kbps}, 700\text{kbps}, 1000\text{kbps}\}$  and  $\{C_1, C_2, C_3\} = \{1300\text{kbps}, 1200\text{kbps}, 1500\text{kbps}\}$ , respectively. From Fig. 2, we observe that the EFAR scheme allocates each network's bandwidth to users according to the characteristic of different video sequences, while the NAF scheme allocates to users equally. Moreover, the EFAR scheme keeps a certain balance between the videos with large  $\mu$  and small  $\mu$ , while the Media-Aware scheme keeps the video with smaller  $\mu$  at its minimal rate until other videos with larger  $\mu$  have achieved their maximal rates. For example, when the total available bandwidth increases from 2.5Mbps (as shown in Fig.2(a)) to 4Mbps (as shown in Fig.2(b)), the allocated rates of sequence 5 and sequence 6 increase over all 3 networks in the EFAR scheme, while only the allocated rate of sequence 6 increases over all 3 networks in the Media-Aware scheme. Compared with the UAF scheme, the EFAR scheme splits users' total rates among 3 networks based on their different available bandwidths and makes all 3 networks be fully utilized. Therefore, the proposed EFAR scheme performs more efficiently as well as higher fairly.

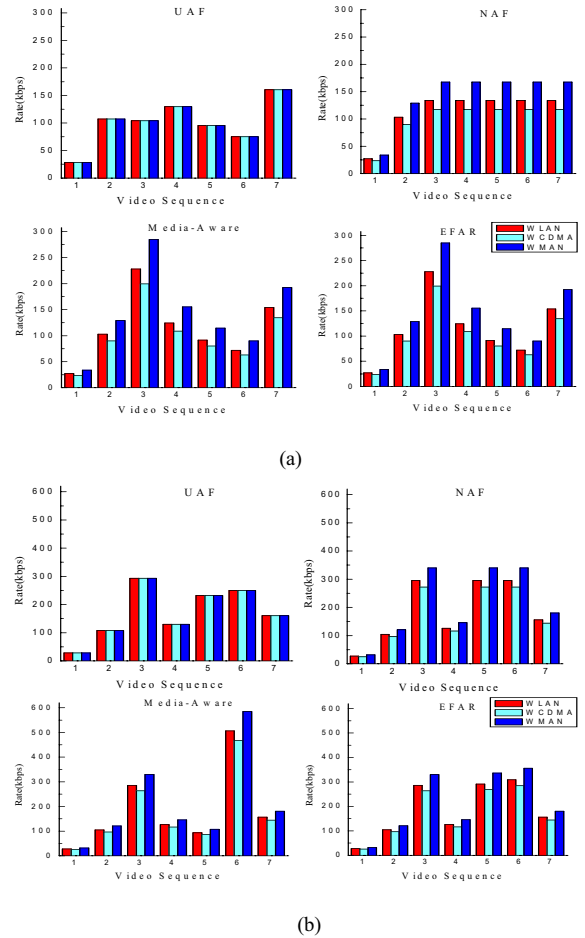


Fig. 2. The allocated rate for different video sequences.

(a)  $\{C_1, C_2, C_3\} = \{800\text{kbps}, 700\text{kbps}, 1000\text{kbps}\}$

(b)  $\{C_1, C_2, C_3\} = \{1300\text{kbps}, 1200\text{kbps}, 1500\text{kbps}\}$

Fig. 3 shows the utility of each user for 9 cases shown in table II. Since the UAF and NAF schemes ignore the differences among networks or users, the performance of the utility of mostly users is worse than that of the EFAR scheme.

TABLE II. 9 CASES OF THE AVAILABLE NETWORK BANDWIDTH

case	$C_1(kbps)$	$C_2(kbps)$	$C_3(kbps)$
1	300	200	500
2	500	300	700
3	650	450	900
4	800	700	1000
5	900	900	1200
6	1250	1000	1250
7	1300	1200	1500
8	1500	1500	1500
9	1700	1500	1800

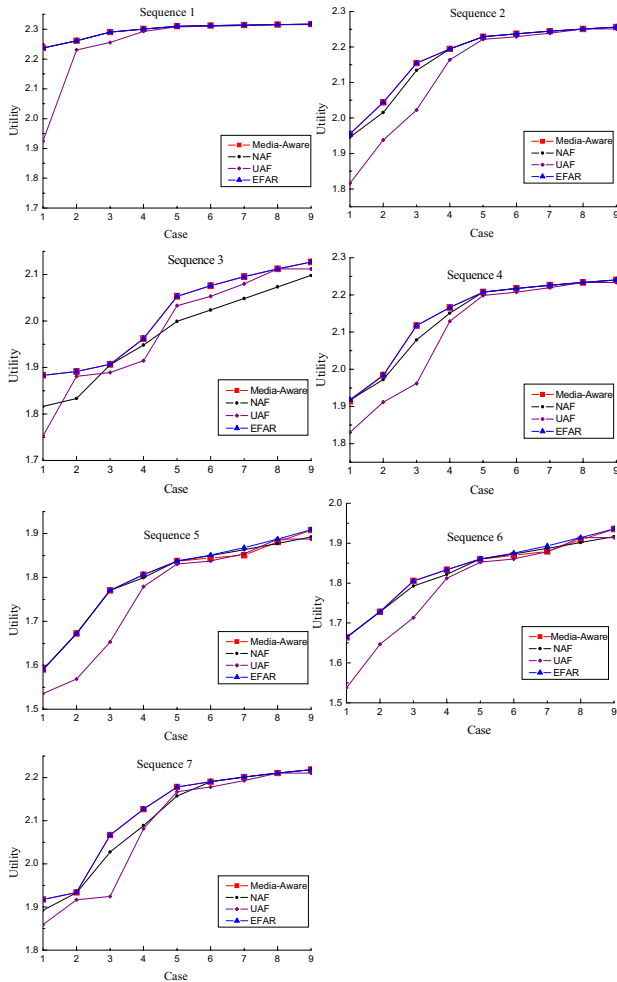


Fig. 3. The utility of each video sequence for 9 cases

On the other hand, for the video sequences with large  $\mu$ , the performance of the EFAR scheme in terms of the utility of each

user is almost as same as that of the Media-Aware scheme. However, for the video sequences with small  $\mu$ , the EFAR scheme has greater performance in some cases than the Media-Aware scheme. The reason for this phenomenon is that the EFAR scheme allocates rate more fairly among video sequences with small  $\mu$ , while the Media-Aware scheme does not allocate more rate to a sequence than its minimal rate until other sequences with large  $\mu$  achieves the maximal rate. Therefore, from the perspective of the utility of each user, the EFAR scheme performs better than other schemes.

## V. CONCLUSIONS

In the future heterogeneous wireless networks, the bandwidth allocation among multiple users sharing multiple heterogeneous access networks is an important and challenging problem. In this paper, an EFAR scheme for multi-user over heterogeneous wireless access networks was proposed. To maximize the utility of each user and split the total rate among the multiple access networks effectively, the proposed EFAR scheme can be formulated as a convex optimization problem. Simulation results show that the EFAR scheme allocates rate among the users more efficiently and fairly compared with other scheme, such as the UAF scheme, the NAF scheme and the Media-Aware scheme. Moreover, from the perspective of the utility of each user, the EFAR scheme also outperforms other three schemes.

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