

An Improved Fischer-Huber Loading Algorithm for Reliable Applications on Access Power Line Communications

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Abstract—The original Fischer-Huber loading algorithm attempts to minimize the probability of bit error by maximizing the signal to noise ratio. Bits are allocated to achieve the same error probability in each of the used subcarriers. However, in channels with a high difference in attenuation like access Power Line Communications (PLC) channels, this could result in over-allocation for parts of the subcarriers, which is unreasonable in practice but is neglected during the bit allocation in the algorithm. Furthermore, after defining the upper boundary of constellation, the bit allocation is changed and then the method of flat power allocation generates a bad error probability. In this paper, we present an improved Fischer-Huber loading algorithm for access PLC channels with high attenuation variability. By adding constellation limitations and adjusting the power allocation policy, the improved algorithm appears more flexible. Simulation results show the improved performance.

Keyword- Loading algorithm; high attenuation variance; access PLC; constellation limitation; reliability

I. INTRODUCTION

Using Power line communications (PLC) for reliable data transmission, especially power grid related data transmission, is full of contradictions: on one hand, the power supply grids are originally designed for energy delivery rather than high speed communication, the power line network turns out to be rather hostile. The line impedance variation, high attenuation and phase shift may vary with frequency, time, location, and distance [1-4]. Reliable communication in this hostile environment requires appropriate signaling schemes. On the other hand, the PLC technique is so attractive for at least the following reason. No new wires are needed, resulting in low cabling cost. Power line is owned by the distribution utility, for power grid related data transmission, it ensures a certain level of security. The obvious advantages propel researchers to find solutions to overcome the drawbacks of PLC.

Orthogonal Frequency Division Multiplexing (OFDM) modulation has been proved to be effective for PLC. In an OFDM system, additional significant gains can be achieved by allocating more bits to subcarriers with larger margins and less or even no bits to seriously faded carriers using a bit loading algorithm. The power allocated to each subcarrier also could be adjusted to obtain better performance. Over the past years, different power and bit allocation schemes with diverse

optimization objectives have been studied [5-9] and can roughly be classified to three categories, namely Margin Adaptive (MA), Rate Adaptive (RA) and Power Adaptive (PA). The Margin Adaptive algorithms minimize the probability of error for a given bit-rate and power, the Rate Adaptive algorithms maximize the bit-rate when assuming a given power and error boundary, the Power Adaptive algorithms minimize the total energy transmission under the restriction of a predetermined error probability and total bit rate. While MA algorithms are suitable for applications that require high reliability or systems with fixed data-rate, RA algorithms are popular in variable data-rate applications without strict requirements of reliability. Finally, PA algorithms suit the cases where the energy consumption is of importance.

There are three algorithms that are of practical interest. The Fischer-Huber algorithm [5], The Chow-Leke-Cioffi algorithm [6], and The Hughes-Hartogs algorithm [7] which belong to the three categories, respectively. From a earlier study from Ref.[9], the Hughes-Hartogs algorithm generates the lowest bit error rate when Chow-Leke-Cioffi gives the worst performance for a given signal to noise ratio (SNR), this occurs due to the bit and power round off that the Fischer-Huber algorithm and Chow algorithm have to do, which will introduce a “quantization error”. However, as in the PLC environments, when the number of subcarriers and the number of bits per symbol are large, the computational complexity of H-H algorithm becomes the weak point and makes it impractical. The Fischer-Huber algorithm is then the best choice of the three for reliable data transmission on power lines.

Nevertheless, the access PLC brings new challenges for the loading algorithm. The attenuation variability is considerable in power lines. This is even getting worse when the cable length increases to several hundreds meter, which is a normal length in the access network. The assumption of infinite granularity in constellation sizes becomes unpractical. It is then necessary in this situation to add a limitation of the constellation size, e.g. 10 bits/symbol at most. When the upper boundary of constellation size is defined, the flat energy allocation needs to be modified since it does not generate the same SNR for all the subcarriers any more. In this paper we propose an improved Fischer-Huber algorithm for access PLC with two changes: adding a limitation of the constellation size before bit

allocation makes it more flexible; the new power allocation policy improves the performance sharply.

This paper is organized in the following manner. An overview of the Fischer-Huber algorithm is presented in section II; the possible problems in access PLC and the corresponding solutions are shown in section III; simulation results are reported in section IV; finally, the conclusions are given in section V.

II. REVIEW OF FISCHER-HUBER ALGORITHM

Fischer and Huber proposed the first minimum BER bit loading algorithm in Ref. [5]. In this paper, the minimization problem of BER is solved for QAM constellations with the restrictions that both the total energy and the total bit rate are constant. It takes advantage of the fact that the symbol error probability of subcarriers that use QAM modulation is given by a uniform formula and the assumption that in the optimum system all the subcarriers perform with the same error rate, otherwise the highest error rate would dominate. Thus, the demand is

$$P_e^i = K \cdot Q\left(\sqrt{\frac{d_i^2/4}{N_i/2}}\right) = \text{const.}, \quad \forall i \quad (1)$$

where i represents the i^{th} subcarrier, d_i is the minimum Euclidean distance between signals point, N_i is the noise energy in that subcarrier band and $Q(x)$ is the complementary Gaussian integral function. K is constant and has been assumed to be equal for all subcarriers. The minimizing P_e problem could be replaced by maximizing SNR of each subcarrier, which is:

$$\begin{aligned} \frac{d_i^2/4}{N_i/2} &= \text{SNR} \rightarrow \max, \\ \text{s.t.} \quad \begin{cases} R_T = \sum_{i=1}^N R_i = \text{const.} \\ P_T = \sum_{i=1}^N P_i = \text{const.} \end{cases} \end{aligned} \quad (2)$$

where R_T is the total bits to be transmitted, R_i is the allocated number of bits to the i^{th} subcarrier, P_T denotes the total power and P_i denotes the transmit power of the i^{th} subcarrier. N is the number of subcarriers. Finally, the optimization problem has solution as shown in Eq.3:

$$R_i = R_T / N + 1 / N \cdot \log\left(\prod_{k=1}^N \frac{N_k}{N_i}\right) \quad (3)$$

For negative R_i the corresponding subcarrier is turned off and this equation can be applied once again, the operation is done iteratively until all R_i of the remaining subcarriers are positive.

The algorithm needs to perform bit round off since the R_i obtained until now is non-integer number; the power then is distributed evenly among the remaining subcarriers.

III. IMPROVEMENTS OF THE ALGORITHM

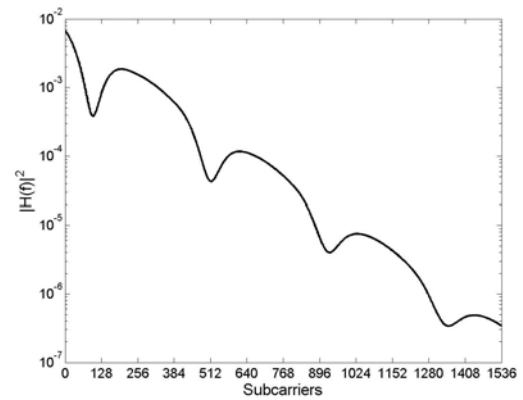
As in the OPERA specification [10] and the Homeplug proposal [11], for the allowed PLC bandwidth from 2 to 30 MHz, there are 1536 parallel subcarriers. First we define the dynamic range D of the subcarrier transmission coefficients H_i :

$$D = 20 \log\left(\frac{|H_{\max}|}{|H_{\min}|}\right) \quad (4)$$

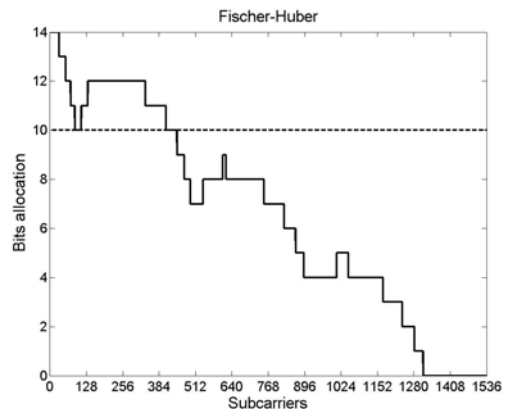
where $H_{\max} = \max(H_i)$ and $H_{\min} = \min(H_i)$ for $i=1 \dots N$ [8]. From the definition, it is clear that D represents the variance of the attenuation among all the subcarriers.

A. Limitations of original algorithm

Fig. 1 a) is an attenuation sample for access PLC when the cable length is 200 meters. The D reaches 45 dB. Fig. 1 b) shows the bit allocation result when using the original Fischer-Huber algorithm.



(a) Attenuation of the subcarriers



(b) Allocation result

Figure 1: A Fischer-Huber bit allocation result without considering the modulation limitation

From the allocation result, it is obvious that a number of subcarriers have been allocated more bits than the constellation limitation, which of course should be avoided in practice but was unfortunately not solved well in [5]. In the original algorithm, there is no restriction in the constellation size during

the bit allocation, the R_{max} is mentioned when doing round off only and the over-allocated bits are removed roughly without further treatment.

In addition, from [5], it is worthwhile to notice that the allocated power P_i can be written as:

$$P_i = k \cdot SNR \frac{N_i}{2} 2^{R_i} = \text{cost}. \quad (5)$$

the R_i here is the non-integer value before being rounded off, which means the allocated power for the remaining subcarriers does not generate the same SNR any more, that is, the round off operation brings quantization errors. In fact, by multiplying adaptively with a coefficient related to the difference between R_i and $\text{round}(R_i)$, the quantization error could be eliminated perfectly.

B. Possible solutions for improvement

The improved algorithm solves the problem with one more restriction, which is

$$\begin{cases} R_i = R_T / N + 1 / N \cdot \log\left(\prod_{k=1}^N \frac{N_k}{N_i^N}\right) & 1) \\ R_{\max} = 10 & 2) \end{cases} \quad (6)$$

Part 1 of the formula could be rewritten as:

$$N_i \cdot e^{NR_i} = e^{R_T} \cdot \prod_{k=1}^N N_k = N_{const1}. \quad (7)$$

Suppose there are R_o ($R_o < R_T$) bits need to be reallocated due to constellation size limitation, the problem is then the same as the original problem with two changed pre-conditions: first, the subcarriers with fully allocated bits (M) should be removed from the set P , which is the set of used subcarriers before reallocation. Second, for the i^{th} subcarrier with $R_i > 0$, the new N_i , here we represented as N_{i_new} , should be updated to $N_{i_new} = N_i \cdot e^{NR_i}$. For the subcarriers without bits being allocated, the N_i remains the same. The solutions of the original and improved algorithm can be expressed as (8) and (9), respectively.

$$R_{j_ori} = R_o / (P - M) + 1 / (P - M) \cdot \log\left(\prod_{k=1}^{P-M} \frac{N_k}{N_{j_new}^{P-M}}\right) \quad (8)$$

$$R_{j_imp} = R_o / (N - M) + 1 / (N - M) \cdot \log\left(\prod_{k=1}^{N-M} \frac{N_k}{N_{j_new}^{N-M}}\right) \quad (9)$$

After reallocation, the optimal case is that all the subcarriers generate the same BER, which means $N_i \cdot \exp(N \cdot (R_i + R_{j_ori})) = N_{const2}$, when power is flatly allocated. It is clear that N_{const1} is the smallest from the set of N_i , otherwise the R_i of the subcarrier with smallest N_i should be positive to make sure the $N_i \cdot \exp(N \cdot R_i) = N_{const1}$. When the threshold becomes N_{const2} , which is higher than N_{const1} , parts of the subcarriers out of set P have positive R_i for $N_i \cdot \exp(N \cdot R_i) = N_{const2}$. ($i \in N, i \notin P$), since $N_{const1} < N_{const2}$. The original algorithm does not take this

into account and it is not the optimal solution when considering constellation size limitation. This means that the original method excludes the influence of over-allocated bits from the decision of whether or not the subcarrier should be turned off, which leads to a possible result that some subcarriers are turned off but in fact should remain open. This could be critical when D is high or the number of bits to be allocated is large. The reasonable flow should consider the constellation size first, and close the subcarriers with negative R_i subsequently. The modified algorithm uses an extra vector to record the subcarriers that have reached the maximum bits and remove these subcarriers from the subcarrier set for the next operation. When there is no fully occupied subcarrier any more, the algorithm goes as the original one for the left bits and subcarriers; finally the used subcarriers are the one with positive R_i . The main part of the improved algorithms is shown in Table I. the added part from the original algorithm is bold marked.

TABLE I. PROCEDURE OF THE MODIFIED FISCHER-HUBER ALGORITHM

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1 Initiate N, RT, PT, H, sub_gain.
2 Set bit_alloc=zeros(1,N), Γ={ N_use }={1...N}, S={0};
   Ri=(RT+ Σi∈Γlog(Ni))/N_use-log(Ni), i∈Γ.
   flag=1. flag2=1.
3 While flag2=1 do
   a Find Ri >= 10, S=S+{i | Ri >= 10}, Γ=Γ-{i | Ri >= 10}. Set
     Ri=10. Rt=Rt-10*length(S).
   b Recalculate Ri for new set Γ, N_use=length(Γ).
   c if for i belongs to Γ, Ri < 10
     Break,
     else
     Repeat step 3
   End
4 While flag=1 do
   a For new RT and N_use, calculate Ri=(RT+ Σi∈Γlog(Ni))/N_use-
log(Ni), i∈Γ.
   Find Ri < 0, remove the corresponding subcarriers from set Γ.
   b If all the Ri in set Γ are positive
     break;
   end
   c Recalculate Ri for new set Γ, the N_use is the number of
elements in set Γ.
   d flag=1.
   end
5 bit_alloc(i)=Ri for i∈Γ, bit_alloc(i)=10 for i∈S, bit_alloc(i)=0
for else.
6 Bit round off
   Rfinal =  $\begin{cases} R_{\max} & R_i \geq R_{\max}^{-0.5} \\ INT(R_i) & 0.5 \leq R_i < R_{\max}^{-0.5} \\ 0 & R_i < 0.5 \end{cases}$ 
7 End

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Fig.2 shows the bit allocation results for both algorithms. The main difference of the two algorithms is the sequence of solving the two problems, as marked with 1 and 2 in the figure.

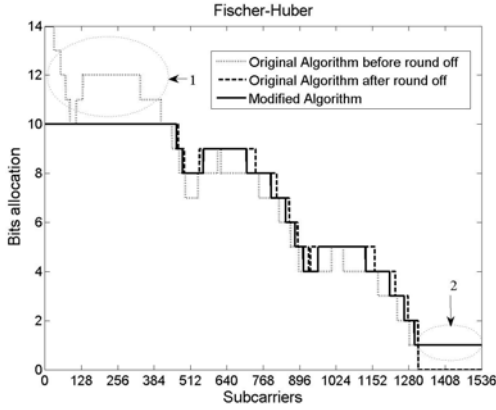


Figure 2: Comparison of bit allocation results for the original algorithm and the modified algorithm

Even for the bit allocation result of the original algorithm before round off, the flat power allocation policy can not suppress the quantization error, when introducing the constellation upper boundary; the flat power allocation generates worse performance since the bit allocation result has changed sharply.

Eq.5 reveals the relation of P_i with R_i for optimal power allocation, however, the R_i here is non-integer. After bit round off, the R_i has changed, to get same BER, the P_i could be modified correspondingly: A new vector is introduced to hold the difference between original R_i and $\text{round}(R_i)$, by multiplying with a coefficient of $2^{\text{round}(R_i) - R_i}$ for P_i , the power could be modified to generate the same BER for each subcarrier. The sum of allocated energy after modification is not the same as the initial energy, this could be corrected by dividing $\text{sum}(P_i)/P_T$ for every subcarrier in use. The power allocation results are plotted in Fig.3.

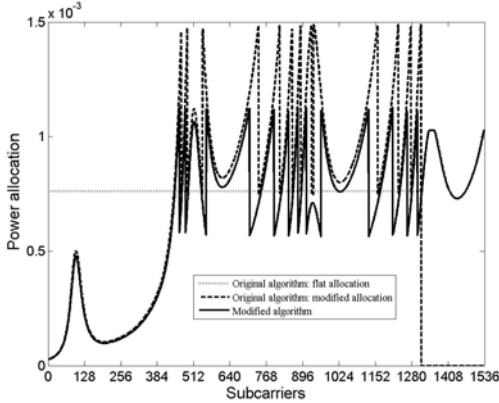


Figure 3: Comparison of power allocation results for the original algorithm and the modified algorithm

IV. RESULTS ANALYSIS

The performance of the loading algorithms is evaluated by computer simulations. The noise power is characterized by σ^2 , which is composed of additive white Gaussian noise (AWGN) and impulsive noise; the model is the same as in [13-15]. The

average SNR is defined as the average ratio of the received signal power and the noise power:

$$SNR_{av} = \sum_{i=1}^N SNR_i = \sum_{i=1}^N \frac{P_i |H_i|^2}{\sigma_i^2} = \sum_{i=1}^N \frac{P_i |H_i|^2}{(\sigma_n^2 + p\sigma_{im}^2)} \quad (10)$$

the channel's transfer function H_i used in the simulations are obtained from the measurement of Ref.[12].

Fig.4 and Fig.5 present the BER vs. the Average SNR performance for two different scenarios. In Fig.4, the R_T is set to be 10000 bits, D is 40 dB and N is 1536. It was found that the original algorithm with flat power allocation has the worst output, the proposed algorithm in this paper is better than all the others. Specifically it presents 0.5 dB better performance compared to the original algorithm with modified power allocation for almost every SNR. This occurs because of the reuse of several subcarriers that are turned off in the original algorithm. The difference is getting larger when D is higher, i.e. when the length of cable is larger. Another result in Fig.5 shows the comparison of the algorithms for R_T equal to 4000 bits, $D=40$ dB and $N=1536$. In this case, when use the same power allocation policy, the proposed algorithm and the original algorithm generates the same BER; this is due to the same bit allocation result since no subcarrier has been allocated more than 10 bits at the beginning.

Last but not least, not shown in these figures, because the higher D induces more bit reallocation due to the limitation of the constellation size, the higher the D is, the better the proposed algorithm will be.

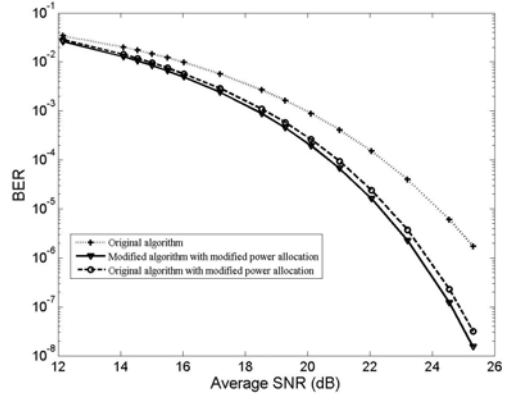


Figure 4: Performance Comparison (BER versus Average SNR) between the Algorithms for: $R_t = 10000$ bits, $D = 40$ dB, $N = 1536$.

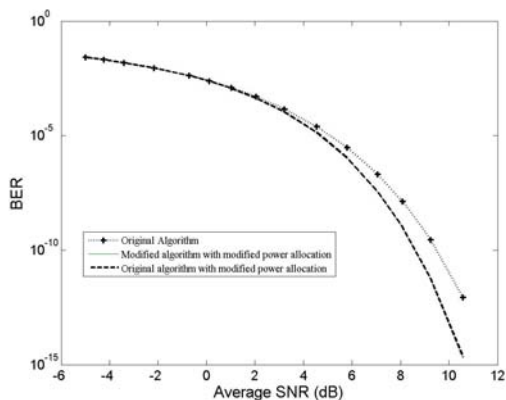


Figure 5: Performance Comparison (BER versus Average SNR) between the Algorithms for: $R_t = 4000$ bits, $D = 40$ dB, $N = 1536$.

V. CONCLUSION

A modified Fischer-Huber algorithm is presented in this paper. From the simulation results, it is clear that the proposed algorithm over performs the original one, especially when the upper boundary is reached for some of the used subcarriers. The new power allocation policy improves the performance with only a slight increase in complexity. For reliable applications, the tradeoff is definitely worth the extra effort. In addition, the modified algorithm considers the effect of constellation size first, and subsequently excludes the subcarriers with negative R_i . This sequence avoids the situation that some subcarriers are turned off but in fact should remain used.

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