Distributed Beam-Forming and Power Control in Multi-Relay Underlay Cognitive Radio Networks: A Game-Theoretical Approach

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***Abstract*—In this paper, we propose a power control algorithm incorporating distributed beam-forming via multi-relay structure with underlay cognitive radio architecture. This problem is modeled as a non-cooperative game and a novel distributed algorithm is designed to achieve Nash equilibrium (NE). At each iteration, power and beam forming weights are determined while quality of service (QoS) of primary and secondary users are guaranteed in terms of interference threshold and signal to interference and noise (SINR) threshold levels, respectively. Convergence of the proposed algorithm to a unique and sub-optimal fixed point for any given initial resource allocation has been proved. Numerical simulations demonstrate significant improvement in average rate of secondary users in comparison with single-relay algorithms.**

I. Introduction

The growth in demand for wireless services coupled with underutilization of radio spectrum is the main motivation behind deployment of cognitive radios and improving spectrum efficiency. In traditional "overlay" cognitive radio networks, the secondary users transmit their signals when primary users have no activity [1], [2]. However, in the more general "underlay" approach, primary and secondary users could simultaneously transmit information and achieve improved spectrum efficiency at the expense of more control messages and thereby more signaling overhead. On the other hand, the most important issue in this approach is that quality of service (QoS) of primary users should be guaranteed by ensuring that interference level at primary receiver is below a certain threshold level [3], [4].

Spectrum efficiency improvement as a result of vertical spectrum sharing between primary and secondary users may be damaged due to multi-user interference and fading. Spatial diversity schemes are among the main approaches proposed to resist against fading effects and improve spectrum efficiency. Moreover, such approach provides the possibility of using beam-forming to control multi-user interference. In [5] and [6], the problem of beam-forming design in multi-antenna underlay cognitive radio networks has been investigated. For example, authors in [6] consider an underlay multi-input single-output (MISO) cognitive radio network, in which zero interference from secondary users is achieved by imposing null-shaping constraints. Also, optimality and convergence is modeled through game theory. Nevertheless, using multi-antenna structure at each node is not practical due to complexity, size, and power consumption constraints. In this regard, cooperative communication is another area that has attracted a lot of attention for improving system performance in recent years [7], [8]. In this paper, our goal is to propose a cooperative relay-based structure that uses distributed beam-forming to improve the performance of an underlay cognitive radio network.

Earlier research on resource allocation in wireless networks can be divided into three general categories. Centralized approach is the first option in which a base station calculates transmission parameters and transmits such information to secondary users. As an example of a centralized approach, authors in [9] proposed an algorithm which is not suitable for cognitive radio networks due to the fact that a base station with complete information about the entire network is required. The second option alleviates the need for a central station but relies on full cooperation between secondary users. However, such approach is also not desired due to large amount of signaling and control messages passed between nodes. As an example, resource allocation algorithm in [10] is based on the network utility maximization (NUM), which results in an optimal solution but suffers from high volume of control messages and inefficient use of the available bandwidth. The third category is based on distributed schemes which rely on self or at most local information. Such schemes are, therefore, more suited to cognitive radio network scenarios and generally adopt non-cooperative game theory architectures. In [11], an overlay relay-based cognitive radio network is considered and the NE of power control game for interference relay channel is computed. One approach to achieve an optimum resource allocation in a distributed manner is to use pricing

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mechanisms. For example, in [12] the pricing function based on SINR level of each user is computed at each step. The power control problem in a multi-relay cellular network is also investigated in [13] where the selfish behavior of relays is modeled by non-cooperative game theory and the corresponding convergence criteria are calculated. The disadvantage of the scheme proposed in [13] is that for any destination, a separate relay is considered and consequently, large number of relays may be required in general.

In this work, a network of peer-to-peer secondary users with multi intermediary relays is assumed. Secondary users are selfish and maximize their utility functions. Subsequently, a novel power allocation problem is solved in which QoS of primary and secondary users are guaranteed simultaneously in a distributed manner.

Throughout this paper, matrices and vectors are denoted by upper-case boldface letters and lower-case boldface letters, respectively. The operators $(.)^T$ and $(.)^H$ denote transpose and conjugate transpose, respectively. Also, the vector contains diagonal elements of matrix $\mathbf{X}$ is represented by diag($\mathbf{X}$).

This paper is organized as follows. In Section II, the system model and game problem formulation are introduced. The distributed algorithm to reach NE is proposed in Section III. Numerical results are presented in Section IV. Finally, Section V concludes the paper.

II. System Model and Game Problem Formulation

A. System Model

Our network model, as shown in Fig. 1, consists of N secondary transmitter-receiver pairs. Moreover, one primary link operating at the same channel is assumed. Naturally, the model can be extended to more than one primary links, nevertheless for the sake of simplicity of presentation, the case of one primary link is considered in this paper. The direct link between secondary transmitters and receivers are assumed to be weak and is further neglected. In this way, R relays is assumed to facilitate communication between secondary sources and destinations and also to provide distributed beam-forming. As secondary users are used in an underlay scenario, the

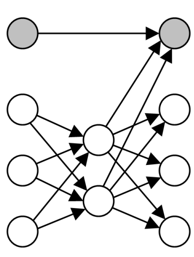

Fig. 1. Network model for $N = 3$ and $R = 2$

quality of service (QoS) of primary users must be guaranteed by ensuring that the interference inflicted from secondary users is less than a certain threshold level. The intermediary relays forward the sources' information in half duplex mode, in other words sources send their information at the first time slot and at the second time slot the relays retransmit an amplified and phase adjusted version of received power in the Amplify-and-Forward (AF) mode, simultaneously. The channels between transmitters and receivers are assumed to be frequency flat fading.

Let $f_{i,1}$, $g_{1,i}$ be channel coefficients from the i-th secondary source to the first relay and from the first relay to the i-th secondary receiver, respectively. The received signal at the first relay is given by

$$x_r^1 = \sum_{i=1}^{N} f_{i,1} x_i + n_r^1 \tag{1}$$

where x_i is the transmitted signal from the i-th secondary source and n_r^1 is the additive white Gaussian noise at the first relay. Therefore, the received signal at all the relays will be

$$\mathbf{x}_r = v \sum_{i=1}^{N} \mathbf{f}_{i,r} x_i + \mathbf{n}_r \tag{2}$$

where $\mathbf{x}_r = [x_r^1, x_r^2, ..., x_r^R]^T$, $\mathbf{f}_{i,r} = [f_{i,1}, f_{i,2}, ..., f_{i,R}]^T$, and $\mathbf{n}_r = [n_r^1, n_r^2, ..., n_r^R]^T$.

Each relay multiplies the received signal by a complex weight parameter $w^{(i)}$ (for the i-th relay) and transmits it to destination nodes. The output signal of the first relay is

$$y^{(1)} = w^{(1)} (\sum_{i=1}^{N} f_{i,1} x_i + n_r^1) \tag{3}$$

and the received signal at the i-th secondary receiver will be

$$z_i = \mathbf{g}_i^T(\mathbf{y}) + n_d^i = \mathbf{g}_i^T \mathbf{W}^H (\mathbf{f}_{i,r} x_i) + \mathbf{g}_i^T \mathbf{W}^H (\sum_{j=1, j\neq i}^{N} \mathbf{f}_{j,r} x_j) + \mathbf{g}_i^T \mathbf{W}^H \mathbf{n}_r + n_d^i \tag{4}$$

where $\mathbf{g}_i = [g_{1,i}, g_{2,i}, ..., g_{R,i}]^T$, $\mathbf{y} = [y^{(1)}, y^{(2)}, ..., y^{(R)}]^T$, $\mathbf{W}$ is a diagonal matrix with elements $\{w^{(i)}\}_{i=1}^R$ and n_d^i denotes the additive white Gaussian noise at the i-th secondary receiver.

The first term in the right-hand side of (4) denotes the desired signal, the second term is the multi-user interference, and remaining terms constitute the noise. In the following calculations, we assume that all channel coefficients, source signals, receiver noises, and the relay noise are jointly independent.

We denote the power of the desired signal, the interference power, and power of noise terms by p_{des}, p_{int}, and p_{noise}, respectively; then $p_{\text{des}}^i = E\{\mathbf{g}_i^T \mathbf{W}^H \mathbf{f}_{i,r} x_i \mathbf{f}_{i,r}^H \mathbf{W} \mathbf{g}_i^* x_i^*\} = E\{x_i x_i^*\} \mathbf{w}^H E\{h_i h_i^*\} \mathbf{W} = p_i \mathbf{w}^H \mathbf{R}_D^i \mathbf{w}$, where $\mathbf{w} = \text{diag}(\mathbf{W})$, $\mathbf{R}_D^i = E\{\mathbf{h}_i \mathbf{h}_i^H\}$, and $\mathbf{h}_i = [f_{i,1} g_{1,i}, f_{i,2} g_{2,i}, ..., f_{i,R} g_{R,i}]^T$ is the effective channel coefficient between the i-th secondary transmitter and its correspondent destination via R relays. Also, $P_i \triangleq E\{x_i x_i^*\}$ is transmitted power of i-th secondary transmitter. The interference power level is consequently given by $p_{\text{int}}^i = E\{\mathbf{g}_i^T \mathbf{W}^H (\sum_{j=1, j\neq i}^N \mathbf{f}_{i,j} x_j)(\sum_{k=1, k\neq i}^N \mathbf{f}_{i,k}^* x_k^*) \mathbf{W} \mathbf{g}_i^*\} = \mathbf{w}^H \mathbf{R}_{\text{int}}^i \mathbf{w}$, where $\mathbf{h}_j^i = [f_{j,1} g_{1,i}, f_{j,2} g_{2,i}, ..., f_{j,R} g_{R,i}]^T$, and $\mathbf{R}_{\text{int}}^i = E\{\sum_{j\neq i} p_j \mathbf{h}_j^i (\mathbf{h}_j^i)^H\}$; and the power of noise is $p^i_{\text{noise}} = E\{n^i_d n^{i^*}_d\} + E\{\mathbf{g}^T_i \mathbf{W}^H \mathbf{n}_r \mathbf{n}^*_r \mathbf{W}\mathbf{g}^*_i\} = \mathbf{W}^H \mathbf{R}^i_{\text{noise}} \mathbf{W} + \delta^2_{n_d}$, where $\mathbf{R}^i_{\text{noise}}$ is diagonal matrix contains the diagonal elements of $\delta^2_{N_r} E\{\mathbf{g}_i \mathbf{g}^H_i\}$. It should be noted that in general, it is possible to use interference cancelation techniques to reduce interference levels. However, such solutions rely on some level of coordination between secondary users and may result in high amount of control messaging between nodes [14]. Therefore, in this paper, we assume no interference cancelation is applied. Consequently, the multiuser interference acts as additive noise and the SINR level at the i-th secondary receiver will be

$$\gamma_i = \frac{p_i \mathbf{w}^H \mathbf{R}^i_D \mathbf{w}}{\mathbf{w}^H \mathbf{R}^i_{\text{int}} \mathbf{w} + \mathbf{w}^H \mathbf{R}^i_{\text{noise}} \mathbf{w} + \delta^2_{N_d} + I^{\text{primary}}_i} \tag{5}$$

where I^{primary}_i is the inflicted interference at secondary receivers by primary transmitter. Finally, the received interference over the primary receiver will be $x_{\text{int}} = \mathbf{l}^T \mathbf{y}$, where $\mathbf{l} = [l_1, l_2, ..., l_R]$ denotes the channel coefficients vector from the relays to the primary receiver. Therefore,

$$x_{\text{int}} = \mathbf{l}^T (\mathbf{W}^H \mathbf{x}_r) = \mathbf{l}^T \mathbf{W}^H (\sum_{i=1}^{N} \mathbf{f}_{i,r} x_i + \mathbf{n}_r) \tag{6}$$

and the interference power will be $I = \mathbf{w}^H \mathbf{R}^{\text{primary}}_i \mathbf{w} + \mathbf{w}^H \mathbf{R}^{\text{primary}}_n \mathbf{w}$, where $\mathbf{R}^{\text{primary}}_i = E\{\sum_i p_i \mathbf{h}^p_i (\mathbf{h}^p_i)^H\}$, $\mathbf{h}^p_i = [f_{i,1} l_1, f_{i,2} l_2, ..., f_{i,R} l_R]^T$ and $\mathbf{R}^{\text{primary}}_n$ is diagonal matrix contains the diagonal elements of $\delta^2_{N_r} E\{\mathbf{l}\,\mathbf{l}^H\}$.

B. Game Problem Formulation

In our studied network, no central node or coordination between secondary users is assumed. Coordination-based algorithms generally require large amount of control message exchange. As a result, in addition to high power consumption and increased complexity, inefficient bandwidth usage is required for signaling which is not proper for cognitive radio networks. Another approach is to assume selfish secondary users that compete to improve their utilities and use resources offered by relays in a competitive fashion. Selfish behavior of secondary users is modeled through non-cooperative game theory. We consider a game in which each secondary user adjusts its power iteratively under QoS constraint of the primary user. The proposed game is represented by $G = \langle \mathrm{N}, \{A_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$, where $\mathrm{N} = \{1, 2, ..., N\}$ is the set of players (secondary users), $A = A_1 \times A_2 \times ... \times A_N$ is the action space of players reflecting their transmitted power, and u_i is the utility function defined as the Shannon rate minus pricing function defined in our model. Distributed beam-forming leads to improved resource allocation due to control of interference. In addition, we use a linear pricing scheme which is a simple, low complexity approach. The game formulation will then be given by

$$\max_{p_i} \; u_i(p_i, p_{-i}) = K_1 \sqrt{\gamma_i - \gamma^{\text{th}}_i} - K_2 (I - I_{\text{th}}) - K_3 p_i \tag{7}$$

$$A_i = \{p_i \in \mathbb{R}^N : 0 \le p_i \le p_{\max}\}, \tag{8}$$

where I_{th} is the interference threshold level at the primary receiver reflecting its target QoS level and subsequently, the second term is added to control the amount of inflicted interference on primary receiver. The aforementioned three terms are weighted using the non-negative weighting factors K_1 to K_3. The value of I_{th} is computed based on the minimum required rate ($R_{\min}$) and the outage probability at this rate (P_{outage}), as follows.

$$\Pr\{\log_2(1 + \frac{P_{\text{primary}} h_{p,p}}{I + N_0 W}) \le R_{\min}\} \le P_{\text{outage}} \tag{9}$$

$$E\{1 - 2^{\frac{1 - e^{R_{\min}}}{P_{\text{primary}}}(I + N_0)}\} \le P_{\text{outage}},$$

where P_{primary} is transmitted power of primary user and assumed fixed during the game convergence interval. $h_{p,p}$ is channel coefficient between primary transmitter and receiver and assuming to have a Rayleigh distribution. Since $E\{I\} \le I_{\text{th}}$, the interference threshold level will be equal to

$$I_{\text{th}} = P_{\text{primary}} \frac{\log_2(1 - P_{\text{outage}})}{(1 - 2^{R_{\min}})} - N_0 W. \tag{10}$$

As a result, as P_{outage} is reduced, I_{th} decreases rapidly.

As we conclude this section, we note that according to the utilization defined in (14), the required information for each player of our game (G) are the second order statistic of channel coefficients between the i-th secondary transmitter and the relays, the relays and the i-th secondary receiver, between the relays and the primary receiver, and finally the beamforming weights which are calculated by the relays and are sent over a common control channel. The second order statistic could be obtained by assuming a certain channel model as we assumed in this paper.

III. Distributed Algorithm Design

In this section, we propose an iterative algorithm to reach NE of the optimum power control game. The NE is defined as follows.

Definition 1: Nash equilibrium (NE) is a point that no user has incentive to improve its utility unilaterally [15]. In other words, a set of actions $P^+_i \in A_i$ will be NE, if and only if

$$u_i(p^*_i, p^*_{-i}) \ge u_i(p^+_i, p^*_{-i}), \;\; \forall i \in \mathrm{N}, \forall p^+_i \in A_i. \tag{11}$$

Each user at each iteration updates its action such that the utility function in (7) is maximized. As shown in Table I, the distributed algorithm is based on two update processes. First, the action space updates which is based on our strategy of best response to action of other players. In this stage, beam-forming weights are set and do not change. In the second step, beam-forming weights are updated.

TABLE I
Synchronous Power Allocation Algorithm

1 : Initialize $P_i = 0$, $\forall i \in \mathrm{N}$.
2 : Set $t = 0$.
3 : Until the convergence criterion is satisfied, Repeat step 4 and 5.
4 : Update $\mathbf{W}$
5 : $P^{(t)}_i = BR_i(P^{(t-1)}_{-i})$.

The algorithm is executed in the synchronous timing mode where all the secondary users update their action simultaneously. The algorithm continues until the convergence criterion is satisfied. This criterion is met when the deviation of the transmitted power for all secondary users is less than a little value. Currently, we want to obtain these update processes.

The beam-forming weights are calculated by minimizing the total transmitted power of the relays while QoS of secondary users in terms of their SINR levels are guaranteed. The problem of finding the beam-forming weights can be summarized as

$$\begin{array}{c}\min(P_T)\\ \gamma_i \geq \gamma_{\text{th}}\end{array} \quad \forall i \in N \tag{12}$$

where P_{T} is total transmitted power of the relays, given by $P_{\text{T}} = E\{\mathbf{y}\mathbf{y}^H\} = E\{\mathbf{W}^H \mathbf{x}_r \mathbf{x}_r^H \mathbf{W}\} = \mathbf{w}^H \mathbf{D} \mathbf{w}$, where $\mathbf{D}$ is a diagonal matrix with elements $\{E\{(x_r^i)(x_r^i)^H\}\}_{i=1}^R$.

It is clear that the optimization problem in (12) is non-convex. In [16] and [17], the semi-definite relaxation scheme is used to obtain a new semi-definite programming optimization problem. So, the relaxed optimization problem is given by

$$\begin{aligned} &\min\{tr(\mathbf{DX})\}\\ &tr(\mathbf{T}_i \mathbf{X}) \geq \gamma_{\text{th}}(\delta_{n_d}^2 + I_i^{\text{primary}})\\ &\mathbf{X} = \mathbf{w}^H \mathbf{w} \end{aligned} \tag{13}$$

where $\mathbf{T}_i = \mathbf{R}_D^i - \gamma_{\text{th}}(\mathbf{R}_{\text{int}}^i + \mathbf{R}_{\text{noise}}^i)$. Therefore, the new optimization problem is convex and using the Lagrangian method, the optimum beam-forming weights for each iteration are obtained.

The beam-forming weights change over time; hence, we cannot prove convergence or obtain criteria for convergence analytically. The memory-based update used is

$$\mathbf{w}^{t+1} = \omega \mathbf{w}^t + (1-\omega)\mathbf{w}_{\text{opt}}^{t+1}, \tag{14}$$

where ω is memory factor and $\mathbf{w}_{\text{opt}}^{t+1}$ is optimum solution which is obtained via solving (13) at $t{+}1$ iteration. As proved in [18], this algorithm converges for memory factors between 0, 1 and results in a more robust solution as well. Therefore, using such approach will lead to a practical solution for cognitive radio networks which are generally sensitive to channel variations and estimation errors. It should be noted that by using larger values of ω , more robustness is achieved at the expense of slower convergence rate.

Before analysis of NE of our game (G), we note that updating BF weights only requires second order statistic of all communicating channels and by assuming a certain channel model (such as Rayleigh model) the signaling load will not be significant.

Theorem 1: For fixed p_{-i}, the best response update of i-th secondary use will be

$$\begin{aligned} BR_i(p_i, p_{-i}) = [(&\frac{\gamma_{\text{th}}}{\mathbf{w}^H \mathbf{R}_D^i \mathbf{w}})(I_i)\\ &+ (\frac{K_2(\mathbf{w}^H E\{\mathbf{h}_i^p (\mathbf{h}_i^p)^H\}\mathbf{w}) + K_3}{2K_1})(\mathbf{w}^H \mathbf{R}_D^i \mathbf{w})(\frac{1}{I_i})]_0^{p_{\max}} \end{aligned} \tag{15}$$

where I_i is total interference plus noise over the i-th secondary receiver.

Proof: $\frac{\partial U_i}{\partial p_i} = \frac{1}{2}K_1(\gamma_i - \gamma_{\text{th}})^{-\frac{1}{2}}\frac{\partial \gamma_i}{\partial p_i} - K_2(\frac{\partial I}{\partial p_i}) - K_3$; then solving $\frac{\partial U_i}{\partial p_i} = 0$ completes the proof. □

Theorem 2: At least one NE exits for the proposed game G in (7).

Proof: The proof is based on the sufficient conditions for the existence of NE provided in [19]. For our game, the conditions are met as (i) the action space is compact and convex, (ii) the utility function is continuous in its action space, (iii) the utility function is concave and also quasi-concave in its action space. □

Also, we will use Yates result in [20] to investigate the uniqueness of NE. In [20], it is shown that if the best response is standard function, then the algorithm converges to a unique fixed point. As the Nash equilibrium is also a fixed point of the best response function (Eq. 23), uniqueness of such fixed point will also prove that the NE is also unique.

Theorem 3: The Nash equilibrium of game G is unique.

Proof: The key aspect of Nash equilibrium's uniqueness is to show that the best response function is a standard function. In order to prove that the Nash equilibrium is unique, the best response function, $BR_i(.)$ should be a standard function and satisfy the positivity, monotonicity, and scalability properties. (i) The positivity property holds when for all $i \in N$, we have $BR_i(p_i, p_{-i}) \geq 0$. This property follows from (15). (ii) To prove the monotonicity property we need to show that if $p^1 \geq p^2$, then $BR_i(p_i^1, p_{-i}^1) \geq BR_i(p_i^2, p_{-i}^2)$. For $p^1 \geq p^2$, $BR_i(p_i^1, p_{-i}^1) - BR_i(p_i^2, p_{-i}^2) = (\frac{\gamma_{\text{th}}}{\mathbf{w}^H \mathbf{R}_D^i \mathbf{w}})(I_i^1 - I_i^2) + (\frac{K_2(\mathbf{w}^H E\{\mathbf{h}_i^p(\mathbf{h}_i^p)^H\}\mathbf{w}) + K_3}{2K_1})(\mathbf{w}^H \mathbf{R}_D^i \mathbf{w})(\frac{1}{I_i^1} - \frac{1}{I_i^2})$. For $I_i > (\frac{K_2(\mathbf{w}^H E\{\mathbf{h}_i^p(\mathbf{h}_i^p)^H\}\mathbf{w}) + K_3}{2K_1\sqrt{\gamma_{\text{th}}}})(\mathbf{w}^H \mathbf{R}_D^i \mathbf{w})$, monotonicity property is proved. (iii) The scalability property holds when for all $\alpha > 1$, we have $\alpha BR_i(p_i^1, p_{-i}^1) \geq BR_i(\alpha p_i^2, \alpha p_{-i}^2)$, where p^1 and p^2 are two different set of the secondary users powers. For all $\alpha > 1$, with the same condition for monotonicity, the scalability property is proved. Consequently, our algorithm fits in the standard framework and its convergence to a unique NE is guaranteed [20]. □

IV. Simulation Setup and Numerical Results

In this section, we present the numerical results for the performance analysis of the proposed distributed algorithm. First, a network of two peer-to-peer secondary pairs and one peer-to-peer primary pair and also three relays is considered. The noise power at the destinations and the relay are equal. The channel coefficients are modeled based on path loss and Rayleigh fading. The path loss exponent is set to be four. Also, $\frac{K_2}{K_1} = 4$ and $\frac{K_1}{K_3} = 2$ are set in (7). In addition, the minimum required rate for the secondary users is assumed to be 1.5. The convergence rate of the proposed algorithm is presented in Fig. 2. As expected, the algorithm leads to a monotonic increasing transmit power level. It should also be noted that the proposed algorithm converges to a interference value that is lower than the threshold level. This is due to the fact that a large cost is associated for interference levels exceeding the threshold. Fig. 3 demonstrates the improvement in terms of average rate as the number of relays is increased from 1 to 3. As shown in this figure, as the minimum required rate for primary user is increased, performance of the single relay setups decreases significantly[1]. However, with the proposed beamforming approach, performance degradation is much smaller. To summarize, as shown in Fig.2 and Fig.3, use of proper pricing in combination with distributed beamforming, it is possible to meet target QoS levels of primary and secondary users in the defined competitive game scenario.

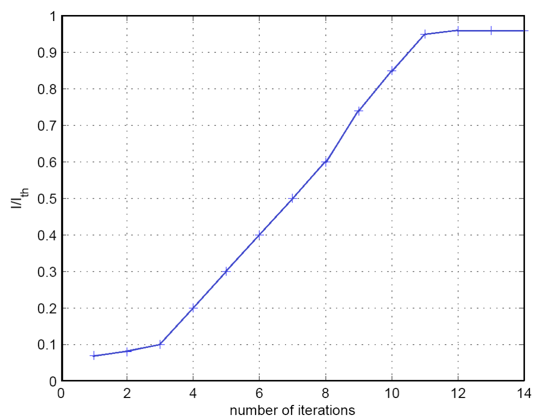

Fig. 2. Convergence of the proposed algorithm

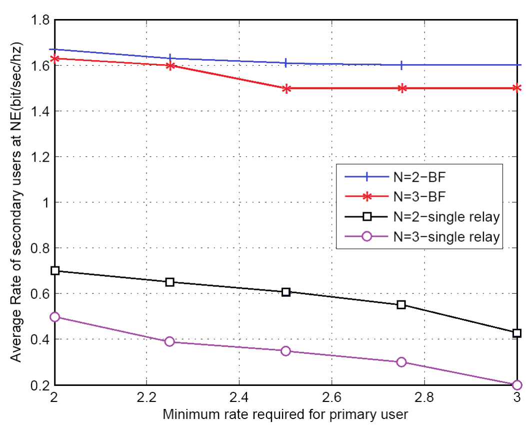

Fig. 3. Average rate of secondary users at NE versus minimum required rate of primary user

V. Conclusion

In this paper, a multi-relay cognitive radio network consisting of a number of peer-to-peer secondary users and one primary link is considered. In this model, secondary users compete selfishly to maximize their utility function. We defined a proper utility function and distributed beam-forming update process to ensure QoS level of primary and secondary users (in terms of average supported rate) are satisfied simultaneously. Game theory is applied to analyze the convergence and uniqueness of the proposed algorithm. Finally, overall system performance in terms of QoS of primary and secondary users is validated through simulations.

[1]The single relay setup is based on amplify-and-forward scheme and the amplification factor is set according to the available power constraint (See, e.g., [11]).

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