The Auxiliary Parametric Sensitivity Method as a Means of Improving Project Management Analysis and Synthesis of Executive Elements

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Abstract. To write differential state equations of investigated actuating devices in the needed Cauchy's normal form, we must abandon the traditional theory of electrical circuits for the theory of electromagnetic circuits. In this work, the known and developed new mathematical models for the analysis of special states have been involved. For solving this problem, it is necessary, at first, to construct a mathematical model of the actuating device. This model is based on the construction of a monodromy matrix, and simulation of transient and steady-state processes at the same time. To make the numerical analysis more convenient, differential equations for models of electromechanical state are written down in Cauchy's normal form. The algorithm involves the study of transient and stationary processes by decomposing them into constituent parts using the mathematical apparatus of the classical theory of nonlinear differential equations, which are calculated in a relatively simple way. The transitional process is obtained as a result of the differentiation of state equations for given initial conditions. We obtain a steady-state process by the initial conditions that exclude transient response. Such conditions we receive by the iterative Newton method. The proposed method of auxiliary variation equations allowed bypass procedure of differentiation of matrix coefficients over the argument that ensured the possibility of the algorithm application of the method of parametric sensitivity. The method of analysis can be spread to more complex nonlinear systems, such as electric motors.

Keywords: Asynchronous motor · Auxiliary parametric sensitivity · Project management · Transition process

1 Introduction

The tasks that solve the problem of improving the management of projects of analysis and synthesis of technical computerized systems are characterized by the use of gradient methods, which leads to the problem of constructing matrices of parametric sensitivity [1, 2]. When the gradients (rows of such a matrix) of certain variables in the field that characterizes the constant values of the parameters allow accelerating the procedure of

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designing variational differential equations to the optimal level [3, 4]. Since the main current engineering problem is to develop a mechanism for creating differential equations that on the one hand determine the physical state of a particular electrodynamic system, and on the other hand they are variational. Therefore, a method for constructing auxiliary variational equations of parametric sensitivity is proposed [5, 6].

The auxiliary model of parametric sensitivity is a modern apparatus of classical nonlinear differential equations theory, which creates preconditions for facilitating the calculation and modeling of electromagnetic processes. This makes it possible to determine all stages of the analysis of electromagnetic systems: the calculation of transient and stable processes of a real physical system, the determination of static stability of processes and the parametric sensitivity of this system. Thus, in this case, the auxiliary model of parametric sensitivity is a kind of generalized auxiliary model of sensitivity to the initial conditions of the physical system.

This proposed method has been carefully tested in the most complex practical problems of project management analysis and synthesis of electromagnetic nonlinear technical systems [1, 7].

2 Literature Review

The system of differential equations of the physical system in vector form has the form [8, 9]

$$\frac{d\mathbf{x}}{dt} = f_1(\mathbf{x}, \boldsymbol{\lambda}, t); \quad 0 \le t \le \infty, \tag{1}$$

where $f_1(\mathbf{x}, \boldsymbol{\lambda}, t)$: t – certain periodic function, $\mathbf{x} = (x_1, x_2, \ldots, x_n)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ – vectors that characterize the constant parameters of the system.

It must be assumed that there is a periodic solution of the system of differential Eqs. (1) in the form $x(t) = x(t + T)$. Taking into account the initial conditions, $\mathbf{x}(0)$ becomes possible to enter directly into the batch solution, without taking into account the transient reaction of the system. In the periodicity equation, they will be an argument [9]

$$f(\mathbf{x}(0)) = \mathbf{x}(0) - \mathbf{x}(\mathbf{x}(0), T) = 0 \tag{2}$$

If we analyze Eqs. (1), (2), it is clear that they constitute a two-point T-periodic boundary value problem for nonlinear differential equations for determining the state of a physical system.

The obtained nonlinear transcendental Eq. (2) must be solved using Newton's iteration method.

$$\mathbf{x}(0)^{s+1} = \mathbf{x}(0)^s - f'\left(\mathbf{x}(0)^s\right)^{-1} f\left(\mathbf{x}(0)^s\right) \tag{3}$$

Equations in the form of Jacobi matrices $f'(\mathbf{x}(0))$ is obtained by differentiation $x(0)$ of the objective function (see Eqs. 2) [10]

$$f'(\mathbf{x}(0)) = \mathbf{E} - \boldsymbol{\Phi}(T) \tag{4}$$

where

$$\mathbf{\Phi}(T) = \left.\frac{\partial \mathbf{x}(\mathbf{x}(0), t)}{\partial \mathbf{x}(0)}\right|_{t=T} \tag{5}$$

Expression (5) can be called a matrix of monodromy and use it in the future. This matrix is obtained from the calculations of the equation of the first variation by differentiating expression (1) on $\mathbf{x}(0)$:

$$\frac{d\mathbf{\Phi}}{dt} = \frac{\partial f(\mathbf{x}, t)}{\partial \mathbf{x}}\mathbf{\Phi}. \tag{6}$$

Together with expression (6) it is necessary to integrate expression (1) on the s-th iteration of Newton's formula using a regulated time interval. Completion of the iteration process is confirmed by the given accuracy of the periodic solution of these equations [11, 12].

Parameter Φ (5) is a matrix of sensitivity to the initial conditions of the system. Thus, the rows of the obtained matrix determine the gradient of a certain one variable in the space of initial conditions, and the columns of the matrix - the sensitivity of the total set of obtained variables to one initial condition. Therefore, the study of the above differential equation is a prototype of the model of sensitivity to the initial conditions of real electromagnetic physical systems [13, 14].

The obtained eigenvalues of the monodromy matrix (5) characterize the static stability of the studied physical process (under regulated conditions). The calculation of parametric sensitivity is usually recommended by variational methods (based on Newtonian iterations (3)) [15].

Differentiation by $\boldsymbol{\lambda}$ we obtain a matrix of parametric sensitivities

$$\mathbf{S} = \frac{\partial \mathbf{x}}{\partial \boldsymbol{\lambda}} \tag{7}$$

The element of matrix $\boldsymbol{\lambda}$ can be any constant parameter of the system under study. Differentiating (1) by $\boldsymbol{\lambda}$ we obtain a linear parametric equation [16]

$$\frac{d\mathbf{S}}{dt} = \frac{\partial f_1(\mathbf{x}, \boldsymbol{\lambda}, t)}{\partial \mathbf{x}}S + \frac{\partial f_1(\mathbf{x}, \boldsymbol{\lambda}, t)}{\partial \boldsymbol{\lambda}} \tag{8}$$

In a stable state, $\mathbf{x}(0) = \mathbf{x}(T)$, therefore, Eq. (8) also has a $\mathbf{S}(t)$ periodic solution.

The complexity, in this case, is caused by partial derivatives of $\mathbf{x}$ and $\boldsymbol{\lambda}$ in the right part (6), (8). In order to implement the selected strategy, it is proposed to use a matrix of auxiliary parametric sensitivities of parameter $\boldsymbol{\chi}$ for some vector $\mathbf{y}$:

$$\boldsymbol{\chi} = \frac{d\mathbf{y}}{d\boldsymbol{\lambda}}; \quad \mathbf{S} = \mathbf{A}\chi, \tag{9}$$

where $\mathbf{A}$ is the matrix that determines the coefficients for Eq. (1) [1].

According to new vector $\mathbf{y}$ we form the equation of state [9]:

$$d\mathbf{y}/dt = f_2(\mathbf{x}, \boldsymbol{\lambda}, t) = f_3(\mathbf{y}, \boldsymbol{\lambda}, t) \tag{10}$$

f_2 – the periodic function of the parameter t.

After differentiating expression (10) by $\boldsymbol{\lambda}$ and taking into account the results of expressions (5), (6), we obtain the following [2]

$$\frac{d\boldsymbol{\chi}}{dt} = \frac{\partial f_3(\mathbf{y}, \boldsymbol{\lambda}, t)}{\partial \mathbf{x}}\boldsymbol{\chi} + \frac{\partial f_3(\mathbf{y}, \boldsymbol{\lambda}, t)}{\partial \boldsymbol{\lambda}} \tag{11}$$

This equation has a periodic solution $\boldsymbol{\chi}(t)$.

Replacing $\mathbf{x}$ with $\mathbf{y}$ should be done so that Eq. (10) is simpler than Eq. (1). Such substitution will be valid in the case when the relationship between the values of $\mathbf{x}$ and $\mathbf{y}$ is established [17].

Therefore, if we assume that $\boldsymbol{\lambda} = \boldsymbol{\chi}(0)$, then (11) turns into a homogeneous expression

$$\frac{d\boldsymbol{\chi}}{dt} = \frac{\partial f_2(\boldsymbol{\chi}, \boldsymbol{\lambda}, t)}{\partial \mathbf{x}}\mathrm{A}\chi \tag{12}$$

from will describe the sensitivity models of the system to its initial conditions (6).

The result of solving Eq. (3) will be the periodic solution of Eq. (11). In the process of solving Eqs. (1), (3), and (11), a periodic solution of the equations of state of the system in the form of a matrix (5) was obtained.

In order to continue the distribution on the column and implement the record in the form of a vector matrix (9) it is necessary to take into account

$$\boldsymbol{\chi} = \left(\boldsymbol{\chi}_1, \boldsymbol{\chi}_2, \ldots, \boldsymbol{\chi}_m\right) \tag{13}$$

where m is the number of elements that make up the vector of constant parameters of the expression $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_m)$, 2 $\boldsymbol{\lambda}$ = const. Moreover

$$\boldsymbol{\chi}_i = d\mathbf{y}/\partial\lambda_i, \quad i = 1, 2, \ldots, m \tag{14}$$

are vectors of parametric sensitivities of vector elements $\mathbf{y}$ to individual constant parameters.

The condition of the periodic solution of the differential Eq. (11) is written analogously to (2)

$$F\left(\boldsymbol{\chi}_i(0)\right) = \boldsymbol{\chi}_i(0) - \boldsymbol{\chi}_i\left(\boldsymbol{\chi}_i(0), T\right) = 0, \quad i = 1, 2, \ldots, n \tag{15}$$

Equation (15) can be solved using Newton's iteration method, but because expression (11) is a linear equation, the solution is greater than one iteration. In the case of zero approximation (3) will look like

$$\boldsymbol{\chi}_i(0) = F^{'}\left(\boldsymbol{\chi}_i(0)\right)^{-1}\boldsymbol{\chi}_i(T), \quad i = 1, \ldots, n. \tag{16}$$

The resulting Jacobi matrix should be expressed in terms of the known matrix, and in our case $\boldsymbol{\Phi}(T)$, which is obtained by calculating the periodic equation $x(t) = x(t + T)$

$$F^{'}\left(\boldsymbol{\chi}_i(0)\right) = \mathbf{E} - \boldsymbol{\Phi}(T) \tag{17}$$

Using the initial conditions found in (16) and integrating (11), and making the transition (9) from $\mathbf{S}_i(0)$ to $\boldsymbol{\chi}_i(0)$, we find the periodic solution $\boldsymbol{\chi}(t) + \boldsymbol{\chi}(t+T) = 0$. In accordance with expression (9), we find the sensitivity **S**.

The transformation of functions of continuous variables into functions of discrete variables is carried out by explicit or implicit methods. When using implicit methods of the Jacobi matrix, the basic equation and the goal equation coincide.

For a practical demonstration, let's take the example of an executive three-phase asynchronous motor of a computerized control system.

3 Researches Methodology

Equation (1) of the electromagnetic state of the electric motor is written as [1, 2]

$$\frac{d\mathbf{i}}{dt} = \mathbf{A}(\mathbf{u} - \Omega'\Psi - \mathbf{Ri}), \tag{18}$$

where

$$\begin{array}{c} \begin{bmatrix} \boldsymbol{\lambda}_S \\ \boldsymbol{\lambda}_R \end{bmatrix}, \ \boldsymbol{\lambda} = \mathbf{u}, \boldsymbol{\Psi}, \mathbf{i}; \quad \mathbf{A} = \begin{bmatrix} \mathbf{A}_S & \mathbf{A}_{SR} \\ \mathbf{A}_{RS} & \mathbf{A}_R \end{bmatrix}; \\ \boldsymbol{\Omega}' = \begin{bmatrix} & \\ & \boldsymbol{\Omega} \end{bmatrix}; \quad \mathbf{R} = \begin{bmatrix} \mathbf{R}_S & \\ & \mathbf{R}_R \end{bmatrix}. \end{array} \tag{19}$$

Here $\mathbf{i}_k = (i_{kA}, i_{kB})_t, \ k = S, R$ – columns of phase currents of a stator winding and the converted currents of a rotor winding; $\boldsymbol{\Psi}_k = (\mathbf{i}_{kA}, \mathbf{i}_{kB})_t, \ k = S, R$ – columns of the corresponding phase full flux couplings; $\mathbf{u}_k = (u_{kA}, u_{kB})_t, \ k = S, R$ – columns of phase voltages of the stator winding; $\mathbf{A}_S, \mathbf{A}_{SR}, \mathbf{A}_{RS}, \mathbf{A}_R$ – matrices

$$\begin{array}{c} \mathbf{A}_S = \alpha_S(1 - \alpha_S\mathbf{G}); \ \mathbf{A}_{SR} = \mathbf{A}_{RS} = -\alpha_S\alpha_R\mathbf{G}; \\ \mathbf{A}_R = \alpha_R(1 - \alpha_R\mathbf{G}), \end{array} \tag{20}$$

where **G**, **Ω** – matrices.

$$\mathbf{G} = \begin{bmatrix} T + b_A i_A & b_B i_A \\ b_A i_B & T + b_B i_B \end{bmatrix}, \quad \boldsymbol{\Omega} = \frac{\omega}{\sqrt{3}} \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}. \tag{21}$$

moreover

$$b_A = b(2i_A + i_B); \quad b_B = b(i_A + 2i_B); \quad b = \frac{2}{3}\frac{R - T}{i_m^2}; \tag{22}$$

$$R = \frac{1}{\alpha_S + \alpha_R + \rho}; \qquad T = \frac{1}{\alpha_S + \alpha_R + \tau}. \tag{23}$$

In expression (23) τ, ρ – inverse static and differential inductances, they are found by the characteristic of magnetization (idling) of the motor as:

$$\tau = \left[\frac{\psi_m(i_m)}{i_m}\right]^{-1}; \quad \rho = \left[\frac{d\psi_m(i_m)}{di_m}\right]^{-1} \tag{24}$$

where i_m is the modulus of the spatial vector of magnetizing currents

$$i_m = 2\sqrt{(i_A^2 + i_A i_B + i_B^2)/3}; \quad i_A = i_{SA} + i_{RA}; \ \ i_B = i_{SB} + i_{RB}. \tag{25}$$

In the absence of saturation, the magnetization characteristic degenerates into a straight line $i_m = \alpha_m \psi_m$, where α_m is the reverse main inductance, and matrix (21) according to (23) into a diagonal one, which greatly simplifies Eqs. (1).

$$\mathbf{G} = \frac{1}{\alpha_S + \alpha_R + \alpha_m} \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}, \tag{26}$$

In this case, we get the simplest of all known mathematical model of an induction motor; $\mathbf{R}_S$, $\mathbf{R}_R$ – resistance matrices and α_S, α_R, – reverse dissipation inductances of the stator and rotor windings; r_S – resistance of stator phases; r_R – reduced resistance of the rotor winding; $\mathbf{\Omega}$ – angular velocity matrix ω.

$$\mathbf{R}_s = \begin{bmatrix} r_S & \\ & r_S \end{bmatrix}; \ \ \mathbf{R}_R = \begin{bmatrix} r_R & \\ & r_R \end{bmatrix}, \tag{27}$$

The components of the actual electromagnetic quantities of the rotor and stator windings of an induction motor can be found by

$$\Psi_{kj} = \frac{1}{\tau}(i_{Sj} + i_{Rj}) + \frac{1}{\alpha_k} i_{kj}, \quad j = A,\ B; \quad k = S, R. \tag{28}$$

The equation of the mechanical state of an electric motor in a computerized control system has the form

$$\frac{d\omega}{dt} = \frac{p_0}{J}(M_E - M(\omega)), \ \ M_E = \sqrt{3} p_0 (\Psi_{SA} i_{SB} - \Psi_{SB} i_{SA}), \tag{29}$$

where $M(\omega)$ is the mechanical moment of an induction motor; p_0 – the number of pairs of magnetic poles of an induction motor; J – moment of inertia of the engine (on a shaft); M_E – the electromagnetic moment of the system as a whole.

The mathematical A-model of this device is described by the system of differential Eqs. (18), (29). With its help, we can calculate transient and stable processes. The practical use of this model requires knowledge of certain technical characteristics of the object under study.

4 Results

The formation of columns of unknowns is as follows

$$\mathbf{x} = (\mathbf{i},\ \omega)_t; \quad \mathbf{y} = (\mathbf{\Psi},\ \omega)_t. \tag{30}$$

The differential Eq. (1) corresponding to $\mathbf{y}$ has the form

$$\frac{d\mathbf{\Psi}}{dt} = \mathbf{u} - \Omega'\Psi - \mathbf{Ri}, \quad \mathbf{i} = \mathbf{L}^{-1}\mathbf{\Psi}, \tag{31}$$

where $\mathbf{L}^{-1}$ – the inverse matrix of static inductors.

$$\mathbf{L}^{-1} = T \begin{array}{|c|c|c|c|} \hline \alpha_S(\alpha_R+\tau) & & -\alpha_S\alpha_R & \\ \hline & \alpha_S(\alpha_R+\tau) & & -\alpha_S\alpha_R \\ \hline -\alpha_S\alpha_R & & \alpha_R(\alpha_S+\tau) & \\ \hline & -\alpha_S\alpha_R & & \alpha_R(\alpha_S+\tau) \\ \hline \end{array}. \tag{32}$$

We write the monodromy matrix similarly to expression (9)

$$\mathbf{\Phi} = (\mathrm{A}\mathbf{z}, \mathbf{w})_t, \tag{33}$$

where

$$\mathbf{z} = \frac{\partial \mathbf{\Psi}}{\partial \mathbf{x}(0)}; \quad \mathbf{w} = \frac{\partial \omega}{\partial \mathbf{x}(0)}. \tag{34}$$

Variation equations for calculating submatrices (34) are determined by differentiation by $\mathbf{x}(0)$ of the equations of the electromechanical state of an induction motor under the control of a computerized system (29), (31).

Differentiating expression (31), we obtain

$$\frac{d\mathbf{z}}{dt} = -(\Omega' + \mathbf{R}\mathrm{A})\mathbf{z} - \frac{\partial \Omega'}{\partial \omega}\mathbf{w}\Psi. \tag{35}$$

Performing differentiation operations on $\mathbf{x}(0)$ of expression (29), it was obtained

$$\begin{aligned} \tfrac{d\mathbf{w}}{dt} = \tfrac{p_0}{J}\Big(\sqrt{3}p_0(\tfrac{\partial \Psi_{SA}}{\partial \mathbf{x}(0)} i_{SB} + \Psi_{SA}\tfrac{\partial i_{SB}}{\partial \mathbf{x}(0)} - \\ -\tfrac{\partial \Psi_{SB}}{\partial \mathbf{x}(0)} i_{SA} - \Psi_{SB}\tfrac{\partial i_{SA}}{\partial \mathbf{x}(0)}) - \tfrac{\partial M(\omega)}{\partial \omega}\mathbf{w} \end{aligned} \tag{36}$$

Derivatives $\partial\Psi_{SA}/\partial x(0)$, $\partial\Psi_{SB}/\partial x(0)$, $\partial i_{SA}/\partial x(0)$, $\partial i_{SB}/\partial x(0)$ are elements of the matrices z, Az, so they are known.

Thus, the construction of the monodromy matrix of an induction motor requires, in conjunction with (18), (29), the integrations of the equations of the first variation (35), (36).

The equation of parametric sensitivities is obtained similarly to (35), (36) if we replace by $\boldsymbol{\lambda}$.

$$\begin{aligned} \frac{d\boldsymbol{\chi}}{dt} &= -(\Omega' + \mathbf{R}\mathrm{A})\boldsymbol{\chi} - \frac{\partial \Omega'}{\partial \omega}\mathbf{w}\Psi + \mathbf{F}; \quad \boldsymbol{\chi} = \frac{\partial \mathbf{\Psi}}{\partial \boldsymbol{\lambda}}; \\ \mathbf{F} &= \frac{\partial \mathbf{U}}{\partial \boldsymbol{\lambda}} + \mathbf{R}\mathbf{L}^{-1}\frac{\partial \mathbf{L}}{\partial \boldsymbol{\lambda}}\mathbf{i} - \frac{\partial \mathbf{\Omega}}{\partial \boldsymbol{\lambda}}\mathbf{\Psi} - \frac{\partial \mathbf{R}}{\partial \boldsymbol{\lambda}}\mathbf{i}. \end{aligned} \tag{37}$$

$$\begin{aligned} \frac{d\boldsymbol{\eta}}{dt} &= \frac{p_0}{J}\left(\frac{\partial M_E}{\partial \boldsymbol{\lambda}} - \frac{\partial M(\omega)}{\partial \omega}\boldsymbol{\eta}\right) + p_0(M_E - M(\omega))\frac{\partial (1/J)}{\partial \boldsymbol{\lambda}}; \\ \frac{\partial M_E}{\partial \boldsymbol{\lambda}} &= \sqrt{3}p_0(\boldsymbol{\xi}_{SA} i_{SB} + \Psi_{SA}\mathbf{S}_{SB} - \boldsymbol{\xi}_{SB} i_{SA} - \Psi_{SB}\mathbf{S}_{SA}). \end{aligned} \tag{38}$$

The matrix of static inductances of an induction motor L has the form [2, 9]

$$\mathbf{L} = \begin{array}{|c|c|c|c|} \hline l_S + l_\tau & & l_\tau & \\ \hline & l_S + l_\tau & & l_\tau \\ \hline l_\tau & & l_R + l_\tau & \\ \hline & l_\tau & & l_R + l_\tau \\ \hline \end{array}, \quad (39)$$

In this case $l_S = 1/\alpha_S$, $l_R = 1/\alpha_R$ – inductance of dissipation of windings of a stator and a rotor; $l_\tau = 1/\tau$ – basic static inductance.

The matrix of parametric sensitivities S in our case repeats the expression (33)

$$\mathbf{S} = (\mathrm{A}\chi, \boldsymbol{\eta})_t, \quad (40)$$

5 Conclusions

In order to formulate an application problem related to an induction motor, you need to write columns of unknown values of expression (30) in extended form, as follows

$$\mathbf{x} = (i_{SA}, i_{SB}, i_{RA}, i_{RB}, \omega)_t; \quad \mathbf{y} = (\Psi_{SA}, \Psi_{SB}, \Psi_{RA}, \Psi_{RB}, \omega)_t. \quad (41)$$

Derivatives $\boldsymbol{\lambda}$ from the elements of column $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ must be carried out according to the rules of differentiation of complex functions because they can be functions from the parameters of the system.

The use of formulas (1)–(17) allows for practical analysis. The analysis of the transient process is carried out by simultaneous integration of the system (1), (11) on a time interval $[0, \infty]$.

In practical implementation, this equation corresponds to the system of differential Eqs. (18), (29), (37), (38).

In the process of performing the iteration, the integration of differential Eqs. (1) and (12) was performed in parallel. The resulting system corresponds to a system of differential Eqs. (18), (29), (35), (36) by which it is possible to determine the parametric sensitivity of the physical system.

In the process of analysis of stationary parametric sensitivity, formula (16) is used in one iteration. On it, differential Eqs. (1), (11), which correspond to a specific system of differential Eqs. (18), (29), (37), (38), are subject to joint integration at time step $[0, T]$. As a result, we find periodic time functions of the set currents, angular velocity, and their parametric sensitivities to a set of constant parameters. The obtained results in the form of periodic solutions of equations are not convenient for practical use, so it is necessary to convert them to the form root mean square values:

$$\mathbf{S} = \sqrt{\frac{1}{T}\int_0^T \mathbf{S}(t)^2 dt}. \quad (42)$$

Table 1. Full RMS sensitivities

	r_S	r_R	U_m	J	l_R	l_S
ω	1.26	2.29	$1.22 \cdot 10^{-3}$	$3.29 \cdot 10^{-4}$	59.27	46.96
M_E	37.30	1.95	$5.18 \cdot 10^{-2}$	$1.44 \cdot 10^{-2}$	2085.60	2117.50

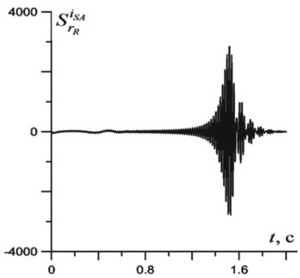

Fig. 1. The result of modeling the transient parametric current sensitivity on the stator of the motor to the resistance of the winding of its rotor

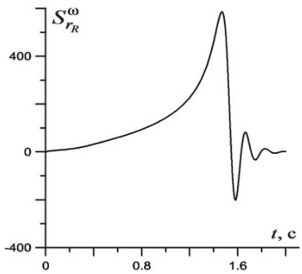

Fig. 2. The results of modeling the transient parametric sensitivity of the angular velocity of an induction motor to the resistance of its rotor winding

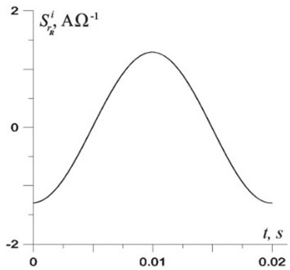

Fig. 3. The results of modeling the stable parametric sensitivity of the stator current of an induction motor to the resistance of its rotor winding

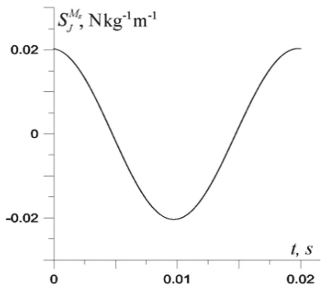

Fig. 4. Stable parametric sensitivity of the electromagnetic moment to the moment of inertia of

Figures 1–4 show the results of simulation of a model induction motor, a table of total RMS sensitivities (42) of angular velocity and electromagnetic moment in the corresponding constant parameters is developed (Table 1).

The method of auxiliary parametric sensitivity creates the preconditions for practical access to gradient methods to the problems of project management analysis and synthesis of nonlinear computerized systems with multipolar elements, as it removes the problem of constructing variational equations.

Thus, further research will focus on a more detailed description of the method of parametric sensitivity and the development of algorithms for its application. It is also planned to apply the developed provisions in practice in computerized control systems for electric motors. Control systems based on this method must have increased productivity and flexibility of setting the operating modes of electric motors.

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