Low Complexity Cooperative Multicast Beamforming in Multiuser Multicell Downlink Networks

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***Abstract*—This paper presents a low complexity sub-optimal algorithm for the max-min beamforming problem in multicell multicast transmission. The beam pattern is jointly optimized for all users inside a cell in a multicell scenario such that the signal to interference plus noise ratio (SINR) of the worst users is maximized subject to a limited sum power. The presented algorithm is based on the Lagrangian dual problem of the downlink max-min beamforming problem. The SINR of the presented algorithm is very close to the SINR of the known solution based on the more complex semidefinite programming.**

I. Introduction

In the next generation mobile communication standards like Long Term Evolution (LTE) or Worldwide Interoperability for Microwave Access (WiMAX) multiple narrow band orthogonal subcarriers help to overcome inter- symbol and inter-carrier interference in an efficient manner, thus, eliminating the intracell interference. However, limited number of the available bandwidth means reuse of these subcarriers in neighboring cells and as a result intercell interference (ICI) emerges as a significant performance limiting factor in multicell networks.

Scenario: In this work, a multicell network with multiple base stations (BSs) with the capability of beamforming is considered. Each BS serves multiple users, each equipped with one antenna. At a time step, a BS transmits the same content to all users inside the cell which corresponds to the multicast scenario. If only one user per cell is scheduled, a unicast transmission is possible as well.

Related work: Closed-loop beamforming is a useful technique to jointly improve the received signal at the regarded user and reduce the interference to other users. This technique makes use of the channel state information (CSI) at the base station (BS) to optimize the beamforming weights [1]. Complete and accurate instantaneous CSI availability is too idealistic for practical implementations but statistical (second order) CSI can be assumed. Using long-term (statistical) CSI, the multicell multicast max-min beamforming is a very practical method to maximize the signal to interference plus noise ratio (SINR) of the cell edge users. This technique results to an adaptation of the sector pattern, such that no user is close to the cell border any more [2]. Beside the availability of accurate CSI, a fast execution of the max-min beamforming algorithm is important for practical systems. A centralized unit has to optimize the beamforming weights for a large network with multiple users. The multicast max-min beamforming problem (MMP) is NP-hard in general [3]. Thus, only sub-optimal solutions can be achieved. In [2], a near-optimal solution based on a semidefinite relaxation is presented. The resulting algorithm is a bisection algorithm to find the optimal beamforming weights for a maximization of the SINR of the worst users with a semidefinite program (SDP). The solution is quite close to the optimum, because a lot of matrices already fulfill the rank-1 constraint [3]. On the other hand, the worst case complexity of a SDP is quite large, which makes it impractical for large networks. A low complexity algorithm for the multicast MMP in single BS multiuser scenario is proposed in [4].

Contribution: The solution in [4] for the multicast MMP is heuristically derived based on a solution for the unicast MMP, which mainly is based on the algorithm of [5]. In contrast to [4], in this paper it is proven that a solution can be directly derived based on the Lagrangian dual problem of the primal multicast MMP. Furthermore the problem is extended to the multicell case. Due to the complexity of the original problem only weak duality is given but the resulting low complexity algorithm finds solutions which are close to the performance of the bisection algorithm based on the SDP [2]. Finally, the presented solution outperforms the solution of [4] for the strongest users.

Notation: Lower case and upper case boldface symbols denote vectors and matrices, respectively and the transpose conjugate of a matrix $\mathbf{A}$ is denoted by $\mathbf{A}^H$. The matrix element of $\mathbf{A}$ with index i, j is given by $[\mathbf{A}]_{i,j}$.

II. System setup and data model

A multiuser multicell network is considered with N_C cooperating BSs equipped with N_t antennas each, serving M users given by the set $\mathcal{S}$, each equipped with a single antenna. A group of users is served by one BS c_i, i.e., $N_C \leq M$. The signal r_i received at a time instant by a user i is given by

$$r_i = \mathbf{h}_{i,c_i}^H \mathbf{w}_{c_i} s_i + \sum_{c \neq c_i} \mathbf{h}_{i,c}^H \mathbf{w}_c s_c + n_i, \tag{1}$$

where $\mathbf{h}_{i,c} \in \mathbb{C}^{N_t \times 1}$ is the channel vector from the c-th BS to the i-th user. The transmit beamforming vector at BS c is $\mathbf{w}_c \in \mathbb{C}^{N_t \times 1}$, s_i is the information signal for user i with $\mathcal{E}\{|s_i|^2\} = 1$ and $n_i \sim \mathcal{CN}(0, \sigma_i^2)$ is the complex additive Gaussian noise with zero mean and variance σ_i^2. Let $\mathbf{R}_{i,c} = \mathcal{E}\{\mathbf{h}_{i,c}\mathbf{h}_{i,c}^H\}$ and assuming long-term channel information is perfectly known then the downlink (DL) SINR of user i is

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$$\gamma_i^D = \frac{\mathbf{w}_{c_i}^H \mathbf{R}_{i,c_i} \mathbf{w}_{c_i}}{\sum_{c \neq c_i} \mathbf{w}_c^H \mathbf{R}_{i,c} \mathbf{w}_c + \sigma_i^2}. \quad (2)$$

In the following the DL SINR is the performance metric for the presented max-min beamforming algorithm.

III. Optimization Problem and the Uplink Downlink Duality

The fairness among users is achieved by a max-min optimization approach of the DL SINR of all users in the network. Defining $\hat{\mathbf{R}}_{i,c} = \mathbf{R}_{i,c}/\sigma_i^2$, the DL SINR is given by:

$$\gamma_i^D = \frac{\mathbf{w}_{c_i}^H \hat{\mathbf{R}}_{i,c_i} \mathbf{w}_{c_i}}{\sum_{c \neq c_i} \mathbf{w}_c^H \hat{\mathbf{R}}_{i,c} \mathbf{w}_c + 1}. \quad (3)$$

It is desired to improve the worst SINR of the users with the power $\sum_{c \in \mathcal{C}} \mathbf{w}_c^H \mathbf{w}_c$ constrained by P, $\mathcal{C}$ being the set of the currently active BSs. This optimization problem is called max-min beamforming problem (MMP) in the following and can be stated as

$$\max_{\mathbf{W}} \quad \min_{i \in \mathcal{S}} \ \gamma_i^D \quad (4)$$
$$\text{s.t.} \quad \sum_{c \in \mathcal{C}} \mathbf{w}_c^H \mathbf{w}_c \leq P. \quad (5)$$

Here $\mathcal{S}$ is the set of currently active users, and $\mathbf{W} = [\mathbf{w}_1, \ldots, \mathbf{w}_{N_C}]$. It has been shown that the problem (4) is non-convex. In [2], this problem was turned to a convex problem with a semidefinite relaxation. The resulting SDP was iteratively solved with a bisection algorithm. In this paper, another low complexity approach is used. For this new algorithm, the MMP will be at first reformulated to the standard form of a optimization problem. Introducing the auxiliary variable γ in (4) the MMP can be converted to

$$\begin{aligned} \max_{\mathbf{W}} \quad & \gamma \quad (6) \\ \text{s.t.} \quad & \gamma_i^D \geq \gamma, \ \ \forall i \in \mathcal{S} \\ & \sum_{c \in \mathcal{C}} \mathbf{w}_c^H \mathbf{w}_c \leq P. \end{aligned}$$

The second constraint of (6) can be further reformulated to:

$$-\frac{\mathbf{w}_{c_i}^H \hat{\mathbf{R}}_{i,c_i} \mathbf{w}_{c_i}}{\gamma} + \sum_{c \neq c_i} \mathbf{w}_c^H \hat{\mathbf{R}}_{i,c} \mathbf{w}_c + 1 \leq 0. \quad (7)$$

With the new constraint (7) the optimization problem (6) in its standard form is given by:

$$\begin{aligned} \max_{\mathbf{W}} \quad & \gamma \quad (8) \\ \text{s.t.} \ & -\frac{\mathbf{w}_{c_i}^H \hat{\mathbf{R}}_{i,c_i} \mathbf{w}_{c_i}}{\gamma} + \sum_{c \neq c_i} \mathbf{w}_c^H \hat{\mathbf{R}}_{i,c} \mathbf{w}_c + 1 \leq 0, \ \forall i \in \mathcal{S} \\ & \sum_{c \in \mathcal{C}} \mathbf{w}_c^H \mathbf{w}_c - P \leq 0. \end{aligned}$$

Instead of the SDP based on a semidefinite relaxation [2], in this paper, a low complexity approach based on the Lagrangian duality theory is presented. The original problem is non-convex, therefore, the dual problem will deliver only an upper bound. The power minimization problem (PMP) [3], which is equivalent to the MMP if the SINR constraint of the PMP is equal to the balanced SINR of the MMP. In [3] it was proven that the duality gap of the dual problem of the PMP is as tight as the gap of the SDP to the optimal problem. The optimization of the dual variables of the dual problem results to further SDP [3].

In this paper, the uplink (UL) downlink relation is derived based on the Lagrangian dual problem of the original problem. This relation is used to derive a low complexity solution of the original problem (4). The Lagrangian of the primal problem (8) is formed with the knowledge that the DL SINR γ is positive and with the non-negative Lagrange multipliers $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_M]$ and λ_P:

$$\begin{aligned} L(\gamma, \mathbf{W}, \boldsymbol{\lambda}, \lambda_P) &= \gamma - \lambda_P \Big(\sum_{c \in \mathcal{C}} \mathbf{w}_c^H \mathbf{w}_c - P\Big) \quad (9) \\ &- \sum_{i \in \mathcal{S}} \lambda_i \Big(-\frac{\mathbf{w}_{c_i}^H \hat{\mathbf{R}}_{i,c_i} \mathbf{w}_{c_i}}{\gamma} + \sum_{c \neq c_i} \mathbf{w}_c^H \hat{\mathbf{R}}_{i,c} \mathbf{w}_c + 1\Big). \end{aligned}$$

This Lagrangian can be converted to:

$$\begin{aligned} L(\gamma, \mathbf{W}, \boldsymbol{\lambda}, \lambda_P) &= \gamma - \lambda_P \Big(\sum_{c \in \mathcal{C}} \mathbf{w}_c^H \mathbf{w}_c - P\Big) \quad (10) \\ &+ \underbrace{\sum_{i \in \mathcal{S}} \frac{\lambda_i \mathbf{w}_{c_i}^H \hat{\mathbf{R}}_{i,c_i} \mathbf{w}_{c_i}}{\gamma} - \sum_{c \neq c_i} \lambda_i \mathbf{w}_c^H \hat{\mathbf{R}}_{i,c} \mathbf{w}_c}_{\Psi} - \sum_{i \in \mathcal{S}} \lambda_i. \end{aligned}$$

With the definition of the set $\mathcal{U}_c$ of user served by BS c, the function Ψ can be rearranged to:

$$\Psi = \sum_{c \in \mathcal{C}} \sum_{i \in \mathcal{U}_c} \frac{\lambda_i \mathbf{w}_c^H \hat{\mathbf{R}}_{i,c} \mathbf{w}_c}{\gamma} - \sum_{j \notin \mathcal{U}_c} \lambda_j \mathbf{w}_c^H \hat{\mathbf{R}}_{j,c} \mathbf{w}_c. \quad (11)$$

With the rearranged Ψ the Lagrangian is given by:

$$\begin{aligned} L(\gamma, \mathbf{W}, \boldsymbol{\lambda}, \lambda_P) &= \gamma + \lambda_P P - \sum_{i \in \mathcal{S}} \lambda_i - \lambda_P \sum_{c \in \mathcal{C}} \mathbf{w}_c^H \mathbf{w}_c \quad (12) \\ &+ \sum_{c \in \mathcal{C}} \sum_{i \in \mathcal{U}_c} \frac{\lambda_i \mathbf{w}_c^H \hat{\mathbf{R}}_{i,c} \mathbf{w}_c}{\gamma} - \sum_{j \notin \mathcal{U}_c} \lambda_j \mathbf{w}_c^H \hat{\mathbf{R}}_{j,c} \mathbf{w}_c. \end{aligned}$$

Now, there is a summation over c and a summation over fixed $\mathbf{w}_c$ and the beamforming vectors can be removed from the inner sums as follows:

$$\begin{aligned} L(\gamma, \mathbf{W}, \boldsymbol{\lambda}, \lambda_P) &= \gamma + \lambda_P P - \sum_{i \in \mathcal{S}} \lambda_i \quad (13) \\ &+ \sum_{c \in \mathcal{C}} \mathbf{w}_c^H \Big(-\lambda_P \mathbf{I} + \sum_{i \in \mathcal{U}_c} \frac{\lambda_i \hat{\mathbf{R}}_{i,c}}{\gamma} - \sum_{j \notin \mathcal{U}_c} \lambda_j \hat{\mathbf{R}}_{j,c}\Big) \mathbf{w}_c. \end{aligned}$$

The dual function of this problem is:

$$g(\boldsymbol{\lambda}, \lambda_P) = \sup_{\gamma, \mathbf{W}} L(\gamma, \mathbf{W}, \boldsymbol{\lambda}, \lambda_P). \quad (14)$$

By observation it can be inferred that $g(\boldsymbol{\lambda}, \lambda_P) \leq \infty$ (non-trivial solution) only if

$$-\lambda_P \mathbf{I} + \sum_{i \in \mathcal{U}_c} \frac{\lambda_i \hat{\mathbf{R}}_{i,c}}{\gamma} - \sum_{j \notin \mathcal{U}_c} \lambda_j \hat{\mathbf{R}}_{j,c} \preccurlyeq 0, \quad (15)$$

which implies the constraint of the dual function. Hence, the Lagrangian dual problem can be stated as:

$$\min_{\boldsymbol{\lambda},\lambda_P} \max_{\gamma} \; \gamma + \lambda_P P - \sum_{i\in\mathcal{S}} \lambda_i \tag{16}$$
$$\text{s.t.} \quad \lambda_P \mathbf{I} + \sum_{j\notin\mathcal{U}_c} \lambda_j \hat{\mathbf{R}}_{j,c} \succcurlyeq \sum_{i\in\mathcal{U}_c} \frac{\lambda_i \hat{\mathbf{R}}_{i,c}}{\gamma},$$
$$\lambda_P \geq 0,\; \lambda_i \geq 0,\; \forall\, i \in \mathcal{S}.$$

With the definition of the optimization variable $\chi = \sum_{i\in\mathcal{S}} \lambda_i$ the additional constraint $\chi \geq \sum_{i\in\mathcal{S}} \lambda_i$, the problem can be reformulated to

$$\max_{\chi} \min_{\boldsymbol{\lambda},\lambda_P} \max_{\gamma} \; \gamma + \lambda_P P - \chi \tag{17}$$
$$\text{s.t.} \quad \lambda_P \mathbf{I} + \sum_{j\notin\mathcal{U}_c} \lambda_j \hat{\mathbf{R}}_{j,c} \succcurlyeq \sum_{i\in\mathcal{U}_c} \frac{\lambda_i \hat{\mathbf{R}}_{i,c}}{\gamma},$$
$$\lambda_P \geq 0,\; \lambda_i \geq 0,\; \forall\, i \in \mathcal{S},\; \chi \geq \sum_{i\in\mathcal{S}} \lambda_i.$$

Using the additional variable substitutions $\chi = \chi^{'} P, \lambda_P = \lambda_P^{'} \chi^{'}$ and $\lambda_i = \lambda_i^{'} \chi^{'}$, the following simplification of the problem (17) is given by:

$$\max_{\chi^{'}} \min_{\boldsymbol{\lambda}^{'},\lambda_P^{'}} \max_{\gamma} \; \gamma + \lambda_P^{'} \chi^{'} P - \chi^{'} P$$
$$\text{s.t.} \quad \lambda_P^{'} \mathbf{I} + \sum_{j\notin\mathcal{U}_c} \lambda_j^{'} \hat{\mathbf{R}}_{j,c} \succcurlyeq \sum_{i\in\mathcal{U}_c} \frac{\lambda_i^{'} \hat{\mathbf{R}}_{i,c}}{\gamma},$$
$$\lambda_P^{'} \geq 0,\; \lambda_i^{'} \geq 0,\; \forall\, i \in \mathcal{S},\; \chi^{'} P \geq \sum_{i\in\mathcal{S}} \lambda_i^{'} \chi^{'}.$$

This problem results to a simple formulation:

$$\max_{\chi^{'}} \min_{\boldsymbol{\lambda}^{'},\lambda_P^{'}} \max_{\gamma} \; \gamma + \chi^{'} P (\lambda_P^{'} - 1) \tag{18}$$
$$\text{s.t.} \quad \lambda_P^{'} \mathbf{I} + \sum_{j\notin\mathcal{U}_c} \lambda_j^{'} \hat{\mathbf{R}}_{j,c} \succcurlyeq \sum_{i\in\mathcal{U}_c} \frac{\lambda_i^{'} \hat{\mathbf{R}}_{i,c}}{\gamma},$$
$$\lambda_P^{'} \geq 0,\; \lambda_i^{'} \geq 0,\; \forall\, i \in \mathcal{S},\; P \geq \sum_{i\in\mathcal{S}} \lambda_i^{'}.$$

Consider the following problem:

$$\min \;\; f(\mathbf{x}) \;\; \text{s.t.}\; \mathbf{g}(\mathbf{x}) \leq 0, \tag{19}$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}$ and $\mathbf{g} = [g_1, \dots, g_m]^T$ and each $g_i : \mathbb{R}^n \rightarrow \mathbb{R}$ and $\mathbf{x} \in \mathbb{R}^n$. The dual problem is obtained by first relaxing the constraints of the dual problem for the problem (19), which yields the dual function $q : \mathbb{R}^m \rightarrow \mathbb{R}$.

$$q(\mu) = \inf_{\mathbf{x}} (f(\mathbf{x}) + \mu \mathbf{g}(\mathbf{x})). \tag{20}$$

The dual problem is then given by:

$$\max \;\; q(\mu) \;\; \text{s.t.}\; \mu \geq 0, \mu \in \mathbb{R}^m. \tag{21}$$

Applied to (18) the variable $\chi^{'}$ is the dual variable for the minimization over $\lambda_P^{'}$ with $P(\lambda_P - 1) \leq 0 \Leftrightarrow \lambda_P \leq 1$. The objective function and constraints are linear functions over $\lambda_P^{'}$ with other variables fixed, therefore, the strong duality holds for the primal and dual problem over $\lambda_P^{'}$ and the primal problem of (18) is given by:

$$\min_{\boldsymbol{\lambda}^{'},\lambda_P^{'}} \max_{\gamma^{'}} \; \gamma \tag{22}$$
$$\text{s.t.} \quad \lambda_P^{'} \mathbf{I} + \sum_{j\notin\mathcal{U}_c} \lambda_j^{'} \hat{\mathbf{R}}_{j,c} \succcurlyeq \sum_{i\in\mathcal{U}_c} \frac{\lambda_i^{'} \hat{\mathbf{R}}_{i,c}}{\gamma},$$
$$\lambda_i^{'} \geq 0,\; \forall\, i \in \mathcal{S},$$
$$\sum_{i\in\mathcal{S}} \lambda_i^{'} \leq P,\; 0 \leq \lambda_P^{'} \leq 1.$$

Multiplying both sides of the first constraint in (24) by $\mathbf{v}_c^H$ from the left and $\mathbf{v}_c$ from the right, it can be rewritten as

$$\gamma \geq \frac{\mathbf{v}_c^H (\sum_{i\in\mathcal{U}_c} \lambda_i^{'} \hat{\mathbf{R}}_{i,c}) \mathbf{v}_c}{\mathbf{v}_c^H (\lambda_P^{'} \mathbf{I} + \sum_{j\notin\mathcal{U}_c} \lambda_j^{'} \hat{\mathbf{R}}_{j,c}) \mathbf{v}_c}. \tag{23}$$

With $\mathbf{V} = [\mathbf{v}_1, \dots, \mathbf{v}_{N_C}]$ the optimization problem is now given by:

$$\min_{\boldsymbol{\lambda}^{'},\lambda_P^{'}} \max_{\gamma,\mathbf{V}} \; \gamma \tag{24}$$
$$\text{s.t.} \quad \gamma \geq \frac{\mathbf{v}_c^H (\sum_{i\in\mathcal{U}_c} \lambda_i^{'} \hat{\mathbf{R}}_{i,c}) \mathbf{v}_c}{\mathbf{v}_c^H (\lambda_P^{'} \mathbf{I} + \sum_{j\notin\mathcal{U}_c} \lambda_j^{'} \hat{\mathbf{R}}_{j,c}) \mathbf{v}_c},$$
$$\lambda_i^{'} \geq 0,\; \forall\, i \in \mathcal{S},$$
$$\sum_{i\in\mathcal{S}} \lambda_i^{'} \leq P,\; 0 \leq \lambda_P^{'} \leq 1.$$

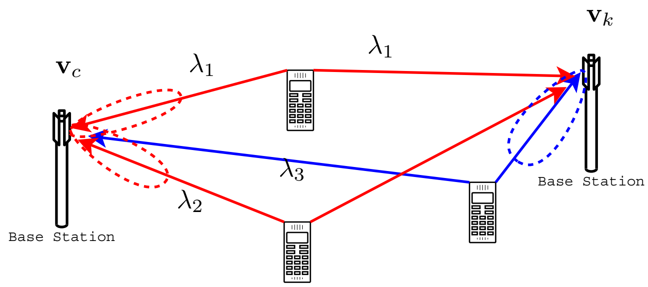
Similar to [6], the optimal value of the dual variable $\lambda_P^{'}$ can be obtained by investigating the objective function of (24). The objective over $\lambda_P^{'}$ is the minimization over γ with regard to $\lambda_P^{'}$. Therefore the optimal $\lambda_P^{'}$ should reach its upper bound, i.e., $\lambda_P^{'} = 1$. With the substitutions $\lambda_i = \lambda_i^{'}$ and $\lambda_P = \lambda_P^{'}$ the final Lagrangian dual problem is then:

$$\min_{\boldsymbol{\lambda}} \max_{\gamma,\mathbf{V}} \; \gamma \tag{25}$$
$$\text{s.t.} \quad \gamma \geq \frac{\mathbf{v}_c^H (\sum_{i\in\mathcal{U}_c} \lambda_i \hat{\mathbf{R}}_{i,c}) \mathbf{v}_c}{\mathbf{v}_c^H (\mathbf{I} + \sum_{j\notin\mathcal{U}_c} \lambda_j \hat{\mathbf{R}}_{j,c}) \mathbf{v}_c}, \tag{26}$$
$$\sum_{i\in\mathcal{S}} \lambda_i \leq P \;\; \lambda_i \geq 0,\; \forall\, i \in \mathcal{S}.$$

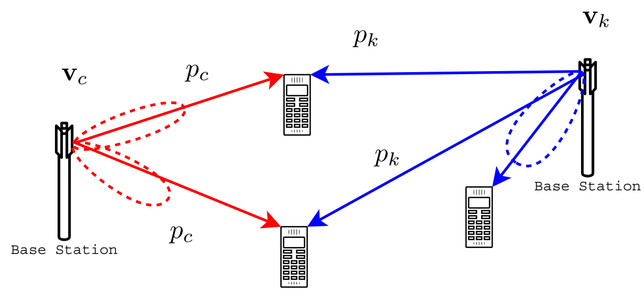
The constraint (26) is an upper bound of the optimal balanced SINR γ. In the case of $|\, \mathcal{U}_c \,| = 1 \; \forall c$, e.g., [6], strong duality is fulfilled. In this case, the reversal of the SINR constraints and the reversal of the minimization as a maximization over λ_is do not affect the optimal solution. But this is not the general case, in the case of $|\, \mathcal{U}_c \,| > 1 \; \forall c$, a balanced SINR can not be achieved. Then (26) will be an upper bound and the strong duality is not fulfilled.

IV. Algorithm

The optimal solution of the Lagrangian dual problem (25) delivers an upper bound of the primal problem (8). Based on the dual problem, a low complexity but sub-optimal method is presented in this section. In Fig. 1, the UL-DL downlink relation, based on the previous derivation of the Lagrangian duality, is illustrated. In the uplink case (a), a BS c using the UL beam pattern $\mathbf{v}_c$, receives UL signals (red) from its users with the UL power λ_i, with $i \in \mathcal{U}_c$, but the BS c also receives interference (blue) from users of other cells, e.g. cell k. In



(a) Uplink



(b) Downlink

Fig. 1: UL-DL relation

the downlink case (b), a user receives the useful signal (red) transmitted from the BS c over the beam pattern $\mathbf{w}_c = p_c \mathbf{v}_c$, where p_c denotes the DL power transmitted by BS c. On the other hand, the user receives interference (blue) from adjacent BSs, e.g., BS k. Based on this investigation the following computation of the beam pattern is made. Regarding the upper bound (26) of the UL SINR

$$\gamma_c^{UL} = \sum_{i \in \mathcal{U}_c} \frac{\mathbf{v}_c^H \lambda_i \hat{\mathbf{R}}_{i,c} \mathbf{v}_c}{\mathbf{v}_c^H (\mathbf{I} + \sum_{j \notin \mathcal{U}_c} \lambda_j \hat{\mathbf{R}}_{j,c}) \mathbf{v}_c}, \tag{27}$$

the UL signal received at a BS consists of $\mathcal{U}_c$ links of different MSs served by this BS c. With (27) only the strongest link is optimized. But the maximization of UL SINR has to be done by an independend optimization of the beamformer for each user, otherwise only the strongest users will get high downlink SINR. Thus, a smarter computation of the UL beamforming vectors $\mathbf{v}_c$ is needed. In contrast to [7], the optimal beamforming vector is given by its linear hull $\text{span}(\mathbf{v}_{c,1}, \mathbf{v}_{c,2}, \ldots, \mathbf{v}_{c,|\mathcal{U}_c|}) = \{\lambda_1 \mathbf{v}_{c,1} + \lambda_2 \mathbf{v}_{c,2} + \ldots + \lambda_{|\mathcal{U}_c|} \mathbf{v}_{c,|\mathcal{U}_c|} \mid \lambda_1, \lambda_2, \ldots, \lambda_{|\mathcal{U}_c|} \in \mathbf{K}\}$, spanned by the $\mathcal{U}_c$ UL beamforming vectors of the $\mathcal{U}_c$ users over the field $\mathbf{K}$ of the UL powers. A vector of this vector space is then given by:

$$\mathbf{v}_c = \sum_{i \in \mathcal{U}_c} \lambda_i \mathbf{v}_{c,i}, \tag{28}$$

where in each link the optimal beamforming vector $\mathbf{v}_{c,i}$ is computed by:

$$\mathbf{v}_{c,i} = \arg\max_{\mathbf{v}_{c,i}} \frac{\mathbf{v}_{c,i}^H \lambda_i \hat{\mathbf{R}}_{i,c} \mathbf{v}_{c,i}}{\mathbf{v}_{c,i}^H (\mathbf{I} + \sum_{j \notin \mathcal{U}_c} \lambda_j \hat{\mathbf{R}}_{j,c}) \mathbf{v}_{c,i}}. \tag{29}$$

For fixed beamforming vectors $\mathbf{v}_c$, the UL powers λ_i are then given by:

$$\lambda_i = \frac{\mathbf{v}_c^H (\mathbf{I} + \sum_{j \notin \mathcal{U}_c} \lambda_j \hat{\mathbf{R}}_{j,c}) \mathbf{v}_c}{\mathbf{v}_c^H \hat{\mathbf{R}}_{i,c} \mathbf{v}_c}. \tag{30}$$

Like in [6], if the total transmit power $P_\Sigma = \sum_{i \in \mathcal{S}} \lambda_i$ has not reached the power budget P, every UL power can be scaled by P/P_Σ . In Algorithm 1, the complete outline for the UL beam patter optimization is listed. The UL powers are initialized with $\lambda_i = 1 \ \forall i$ and the initial beamforming vectors are given by $\mathbf{v}_c = [1, 0, 0, 0]^T$, $\forall c$. After Algorithm 1 converged, the computation of the UL power and the beam pattern is done. But the DL power is not determined. In contrast to [6], the DL power can not be computed by solving a linear system of equations by using the UL SINR, because it is an upper bound. Therefore the correct UL SINR has to be correctly determined afterwards. The DL beamforming weights are given by $\mathbf{w}_c = p_c \mathbf{v}_c$. In contrast to [4], where a solution based on a generalized eigenvalue decomposition is used, in this paper a linear program finds the optimal power allocation. It is desired to improve the worst DL SINR of all users with the total power constraint given by P. This can be defined as:

Algorithm 1 UL vector iteration

repeat
  for $c = 1$ to N_c **do**
    for $i = 1$ to M **do**
      $\tilde{\lambda}_i = \frac{\mathbf{v}_c^H (\mathbf{I} + \sum_{j \notin \mathcal{U}_c} \lambda_j \hat{\mathbf{R}}_{j,c}) \mathbf{v}_c}{\mathbf{v}_c^H \hat{\mathbf{R}}_{i,c} \mathbf{v}_c}$
      $\mathbf{v}_{c,i} = \arg\max_{\mathbf{v}_{c,i}} \frac{\mathbf{v}_{c,i}^H \lambda_i \hat{\mathbf{R}}_{i,c} \mathbf{v}_{c,i}}{\mathbf{v}_{c,i}^H (\mathbf{I} + \sum_{j \notin \mathcal{U}_c} \lambda_j \hat{\mathbf{R}}_{j,c}) \mathbf{v}_{c,i}}$
    end for
    $\mathbf{v}_c = \sum_{i \in \mathcal{U}_c} \lambda_i \mathbf{v}_{c,i}$
    $\lambda_i = \beta \tilde{\lambda}_i, \forall i \in \mathcal{S}$, with $\beta = P / \sum_{i \in \mathcal{S}} \tilde{\lambda}_i$
  end for
until convergence
return $\mathbf{V}$

$$\begin{aligned} \max_{\mathbf{p}} \quad & \min_{i \in \mathcal{S}} \ \frac{p_c h_{i,c_i}}{\sum_{c \neq c_i} p_c h_{i,c} + 1} \\ \text{s.t.} \quad & \sum_{c \in \mathcal{C}} p_c \leq P, \end{aligned} \tag{31}$$

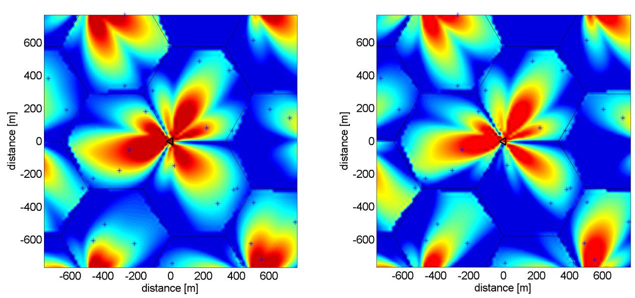
with $\mathbf{p} = [p_1, \ldots, p_{N_c}]$ and $h_{i,c} = \mathbf{v}_c^H \hat{\mathbf{R}}_{i,c} \mathbf{v}_c$. The problem (31) is the well known optimal transmit power allocation. This problem can be transformed to a linear problem.

V. Simulation Results

In Fig. 2, the SINR as a function of the location is depicted. An example of a user drop and the adaptation of the sector pattern by the presented algorithm and the SDP is shown. For this plot the spatial correlation matrices are created based on a simple Laplacian power angular distribution [8] and each BSs has $N_a = 4$ antenna elements and each user has one antenna element. It can be observed that the sector pattern is adapted: Regions with high SINR are only around user positions. Regions without users have a very low SINR, thus no energy is wasted and, therefore, also no unnecessary interference is generated to users in adjacent cells. Comparing the sector pattern of the SDP and the new low complexity algorithm, both plots show a similar SINR distribution.

The numerical results presented in Fig. 4 and Fig. 5 are based on multiuser multicell system level simulations with $N_C = 21$ cells with BSs with the capability of beamforming equipped with $N_a = 4$ antenna elements. Each BS uses a 120° antenna pattern to avoid interference to adjacent sectors.

In each user drop $2, 3$, or 4 users are equally distributed inside each cell. The channels between each user and each BS are created based on the known SCME channel model. The island (see Fig. 3) consisting of cooperating cells and users is surrounded by a ring of omnidirectionally transmitting BSs to simulate the incoming intercell interference of adjacent networks. In total 400 user drops are generated for the simulation results. The presented algorithm is compared with the SDP of [2]. The new algorithm achieves results approximately 0.5 dB close to the SDP for the weakest 10% of the users and it outperforms the SDP for the strongest 40% of the users for the case of two users per cell. Increasing the number of users per cell to four, the distance increased to less than 1dB for the weakest 10% of the users (see Fig. 5). Comparing the distance to the quasi optimal solution, the presented solution outperforms the solution of [4] for the strongest users and achieves similar results for the weakest users [7]. The UL beamformer calculation of [4] is more complex, because it requires an additional generalized eigenvalue decompositon for the computation of the UL powers. In all cases the new algorithm uses less than the half of the computation time compared to the SDP of [2] (see Fig. 4).



(a) low complexity alg.

(b) SDP

Fig. 2: Example of a solution for the beam pattern optimization with the presented algorithm. The asteriks denote users distributed in the world. Regions with red color experience high SINR and regions with blue color have a low SINR.

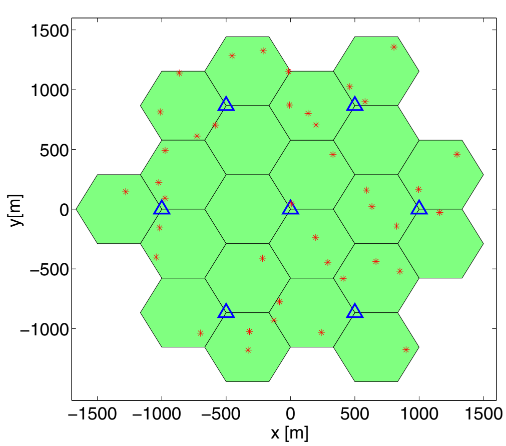


Fig. 3: Simulation scenario, the asteriks denote users distributed in the world.

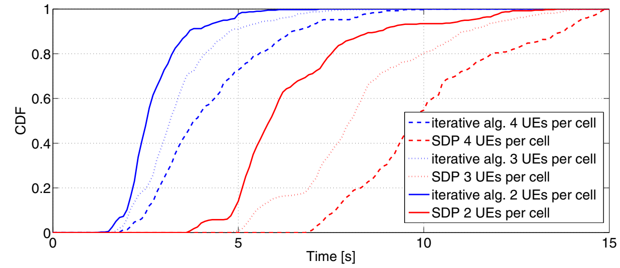


Fig. 4: Convergence of new Alg. and the Alg. of [2].

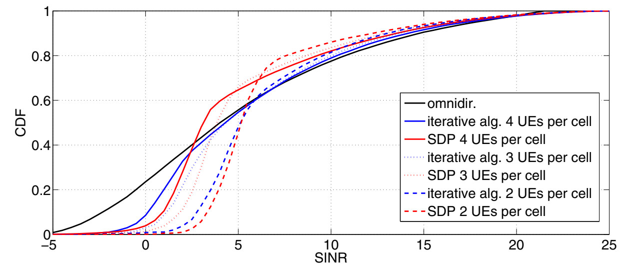


Fig. 5: Cumulative distribution function (CDF) of the SINR.

VI. Conclusions

In this paper, the beam pattern is adapted for multiple users in a cell to mitigate the intercell interference. The resulting problem is the multicast max-min beamforming problem which is known to be NP-hard. Based on the Lagrangian duality, a low complexity algorithm is derived. For the same accuracy, the new algorithm achieves solutions close to the solutions of the SDP, with approximately the half computation time. In this paper, the sum power of the network is constrained. A reasonable extension of this is a per-base station power constraint. This can be simply fulfilled by the final power control problem (31), but because of the complexity it needs to be integrated in the beam patter computation as well and is topic for future research.

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