Queuing Analysis of Multi-Hop CSMA/CA Wireless Networks Handling Many Traffic Flows

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ABSTRACT

In this paper a semi-analytical model for the evaluation of the queuing performance in CSMA-CA multi-hop wireless networks with arbitrary topology is proposed. The intrinsic fairness of single-hop wireless networks can be used to model the network as a processor sharing system with multiple queues, one for each node, and a fair scheduling regime. In the multi-hop scenario, the queuing performance depends on the access protocol, the network topology and the traffic profiles on all links. In a previous paper a model is introduced which allows the calculation of the saturation throughput of nodes in multi-hop CSMA-CA networks with RTS/CTS enabled access. A simplified version of the method is used to estimate the resources of the servers in a many-sources large deviations analysis of a queuing system with multiple coupled servers. The queuing performance of wireless networks can be evaluated by mapping the different servers to the nodes in the network. The cumulative complementary distribution function of the buffer occupation of a node for different topologies is calculated by a novel method and compared to the results of an event-driven simulation with the same settings. A good fit between the semi-analytical model and the simulation is obtained.

Categories and Subject Descriptors

C.2 [**Computer-Communication Networks**]: Wireless communication; G.3 [**Probability and Statistics**]: Queuing Theory

1. INTRODUCTION

Recently there has been a large interest in multi-hop wireless networks. The theoretical capacity of IEEE802.11n[1] makes this recent standard a viable wireless local loop technology. A semi-analytical approach for the evaluation of the queuing performance can be an important tool for the initial planning of a network or for the troubleshooting in the case of QOS problems.

The analysis of the queuing behavior in a wireless network is a rather complex problem. In a multi-hop scenario the throughput depends not only on the maximum over-the-air data-rate but also on the topology, the exact sender-receiver pair and the workload of the other nodes in the network. Semi-analytical throughput models [4] for multi-hop CSMA-CA packet radio networks existed but only recently the same theory is applied to IEEE802.11 [1,2]. In a previous paper [10] the throughput in a IEEE802.11 network with RTS/CTS enabled access is calculated as a function of the topology and the mean link occupation of a sender-receiver pair. The interaction of the varying capacity as seen by a node and the queuing process has never been addressed properly for a CSMA-CA network with an arbitrary topology. The authors of [3,11] do consider only a single hop approach and in [7,12] the performance is evaluated by calculating the throughput without considering the queuing process.

An exact solution for the multi-hop queuing problem is not possible if the traffic distribution is not trivial or the traffic is characterized by a measured trace but the analysis can be simplified if the traffic is supposed to be composed of many flows. A many sources large deviations estimate for the buffer occupation of a node can be calculated which takes into account the capacity of the node as the saturation throughput of the node considering the capacity seen by the other nodes. The basic assumptions for the application of a many sources large deviations theory in a practical network scenario can be summarized as follows:

- the queuing process has a steady-state solution;
- the input to the queuing process is generated by many independent sources and
- the event of interest is a “rare event”.

The first hypothesis can be reformulated: the capacity of a node has to be higher than the mean rate, otherwise the queue is not stable and less than the peak rate, otherwise no buffer overflow can occur. The second assumption can be relaxed [16] to include the case of long range dependent and fractal WAN traffic respecting some technical conditions. In practice all the experiments with real traffic trace [17,18] have given satisfying results. The buffer in a well designed

9[1]248 Mbps for 2 streams.

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queuing system empties regularly. The probability that the buffer level exceeds a certain level is low and the event of interest, the overflow of the buffer, can be labeled as a "rare event".

In [14,15] the queuing analysis of respectively a FIFO and a priority queuing system accessed by a large number of independent stationary sources is detailed. The same approach is used here to make a large deviations model of the multi-hop CSMA/CA wireless network. The main differences with other approaches are the following:

- real traffic traces or arbitrary traffic distributions can be used as input;
- the multiplexing of sources is well defined in the large deviations limit;
- the evaluation can be carried out for an arbitrary topology and
- the cumulative complementary distribution function of the buffer occupation is calculated.

These features allow the evaluation of several realistic QOS parameters, as packet loss, delay and jitter both on a node level and a flow level.

To evaluate the proposed solution for the calculation of the cumulative complementary distribution function of the buffer occupation of a node, a simplified MAC protocol is used. The intricacies of IEEE802.11 with RTS/CTS enabled access make the queuing analysis not trivial due to starvation effects [7]. To study the interaction of the varying capacity with the queuing process an ideal RTS/CTS schema is used:

- the sending and receiving of a RTS/CTS message is instantaneous and does not interfere with other transmissions (side channel) so that collisions are prohibited and impossible;
- on reception of a RTS message, a node returns a CTS message if the channel as seen by the node is empty;
- if a node is not blocked by physical of virtual carrier sensing and the buffer is not empty, a packet will be served without back-off and
- if several nodes can serve a packet at the same time, the oldest packet to be served has priority.

The method in [5] is applied to the simplified MAC protocol and a product form evaluation of the air-time of the nodes is used to evaluate the constitutive equations which restricts the space of possible throughput tuples for all nodes. These equations turn out to be linear in this case and the rule "always send if not blocked" states that the capacities as seen by the nodes, are always situated on the bounds of the constitutive equations.

The main contribution in this paper is a novel method which models a multi-hop CSMA-CA network as a many sources large deviations queuing problem. An ideal RTS/CTS access schema is used to study the interaction between the MAC protocol and the queuing process. This is a first step to construct a semi-analytical framework which models the queuing performance of real CSMA-CA networks as IEEE802.11. The more complicated capacity evaluation described in [10] can easily be integrated in the many sources large deviations approach to obtain a semi-analytical framework for the performance evaluation of an IEEE802.11 network with RTS/CTS enabled access.

This paper is organized as follows. In section 2 some background information and preliminaries are given for the many sources large deviations analysis. Section 3 details the novel method for the calculation of the cumulative complementary distribution function of the buffer occupation of a node in the multi-hop CSMA/CA network. Whereas section 4 introduces the simplified RTS/CTS access protocol. In section 5 a case-study is presented which compares the semi-analytical calculations with event-driven simulations for a same setting.

2. BACKGROUND AND PRELIMINARIES

A discrete-time fluid queuing system with M servers each one serving a buffer with size $b^{(i)}$ and accessed by $n^{(i)}$sources of class i represents a multi-hop CSMA-CA network with M nodes and associated traffic. The instantaneous capacities of the servers are related by the topology of the multi-hop network, the behavior of the CSMA/CA protocol and the load due to the different sources.

At any discrete time instant, denoted by n, a finite number of traffic units $0 \leq \lambda_n^{(i)} \leq K^{(i)}$ is transmitted from a class i source into the buffer i, where $K^{(i)}$ is referred to as the peak rate of the class i. If there is insufficient space in the buffer then the traffic units which can not be admitted are lost. It is assumed that the sequence $\left(\lambda_n^{(i)}, n \in \mathbb{Z}\right)$ forms a stationary and ergodic process for each source of class i and that the resulting sequences are independent among the classes and within them.

The many sources regime studies the particular case when each buffer i is fed by the superposition of a large number of class i sources, noted $N^{(i)} = Nn^{(i)}$. The instantaneous service rates $C_n^{(i)} = Nc_n^{(i)}$and the buffer sizes $B^{(i)} = Nb^{(i)}$ are also scaled with a factor N. The total arrival process for a source of class i is defined by

$$\lambda_n^{N^{(i)}} = \sum_{j=1}^{N^{(i)}} \lambda_n^{(i)} \ . \tag{1}$$

The workload present in the buffer i at the discrete moment n can be defined by Lindley's recursion formula [13]

$$X_n^{N^{(i)}} = \max\left(0, X_{n-1}^{N^{(i)}} + \lambda_n^{N^{(i)}} - C_n^{(i)}\right) \ . \tag{2}$$

The following stability condition guarantees the existence of a stationary queue length process

$$n^{(i)}\mathbb{E}\lambda_n^{(i)} = n^{(i)}\mu^{(i)} < \mathbb{E}c_n^{(i)} \tag{3}$$

and the relation

$$n^{(i)}K^{(i)} > \mathbb{E}c_n^{(i)} \tag{4}$$

has to be satisfied otherwise losses cannot occur.

The cumulative arrival process $\Lambda_t^{(i)}$, i.e. the total number of traffic units arrived into the buffer $B^{(i)}$ in the interval $[n, m)$, is expressed by

$$\Lambda_t^{(i)} = \sum_{l=n}^{m-1} \lambda_l^{(i)} \tag{5}$$

with $t=(m-n)\,\Delta t$, a multiple of the elementary time-step $\Delta t = t_0$.

The tail distribution of $\Lambda_t^{N^{(i)}} = \sum_{j=1}^{N^{(i)}} \Lambda_t^{(i)}$ determines the buffer asymptotics by Loynes's theorem [14]. This theorem specifies the stationary, ergodic queue length process if the stability criterion (3) holds

$$X^{N^{(i)}} = \sup_{t\geq 0}\left(\Lambda_t^{N^{(i)}} - C_t^{(i)}\right) \tag{6}$$

where $C_t^{(i)} = \sum_{l=n}^{m-1} C_l^{(i)}$.

It is assumed that the moment generating function of $\lambda_m^{(i)}$ exists

$$\phi_t^{(i)}(s) = \mathbb{E}\mathrm{e}^{s\Lambda_t^{(i)}} \tag{7}$$

for all finite t.

The density $f_{\Lambda_t^{N^{(i)}}}(Nu)$ and the tail probability $\Pr\left[\Lambda_t^{N^{(i)}} \geq Nu\right]$ are respectively stated by the local limit theorem due to Petrov and the Bahadur-Rao theorem [14]. As $N\to\infty$ uniformly for $0\leq u\leq n^{(i)}K^{(i)}t$ then

$$f_{\Lambda_t^{N^{(i)}}}(Nu) = \frac{\mathrm{e}^{-NJ_t^{(i)}(u)}}{\sqrt{2\pi N\left(\sigma_t^{(i)}(u)\right)^2}}\left(1+O(\frac{1}{N}\right) \tag{8}$$

and as $N\to\infty$ uniformly for $n^{(i)}\mu^{(i)}t < u < n^{(i)}K^{(i)}t$ then

$$\Pr\left[\Lambda_t^{N^{(i)}} \geq Nu\right] = \frac{\mathrm{e}^{-NJ_t^{(i)}(u)}}{s_t^{(i)}(u)\sqrt{2\pi N\left(\sigma_t^{(i)}(u)\right)^2}}\left(1+O(\frac{1}{N}\right) \tag{9}$$

where

$$J_t^{(i)}(u) = s_t^{(i)}(u)u - n^{(i)}\ln\phi_t^{(i)}\left(s_t^{(i)}(u)\right) \tag{10}$$

with $s_t^{(i)}(u)$ the unique solution to

$$\frac{\phi_t^{(i)}\left(s_t^{(i)}(u)\right)'}{\phi_t^{(i)}\left(s_t^{(i)}(u)\right)} = \frac{u}{n^{(i)}} \tag{11}$$

and

$$\sigma_t^{(i)}(u) = n^{(i)}\left(\frac{\phi_t^{(i)}\left(s_t^{(i)}(u)\right)''}{\phi_t^{(i)}\left(s_t^{(i)}(u)\right)} - \left(\frac{u}{n^{(i)}}\right)^2\right). \tag{12}$$

These equations are well known in the theory of Large Deviations where $J_t^{(i)}(u)$ denotes the rate function. $s_t^{(i)}(u)$ is positive if $u > n^{(i)}\mu^{(i)}t$ and it is zero if $u = n^{(i)}\mu^{(i)}t$. If $u < n^{(i)}\mu^{(i)}t$ then it is assumed that $\Pr\left[\Lambda_t^{N^{(i)}} \geq Nu\right] = 1$ and $J_t^{(i)}(u) = 0$. If $u > n^{(i)}K^{(i)}t$ then packet loss can not occur and $\Pr\left[\Lambda_t^{N^{(i)}} \geq Nu\right] = 0$.

3. MULTI-HOP QUEUING

3.1 Formulation

The Large Deviations probability that the steady-state workload present in the buffer M is larger than the value $B^{(M)}$ given $C_t^{(M)}$ can be expressed in the terminology of the previous subsection

$$\Pr\left[X^{N^{(M)}} \geq B^{(M)}\right]\left(C_t^{(M)}\right) = \Pr\left[\sup_{t\geq 0}\left(\Lambda_t^{N^{(M)}} - C_t^{(M)}\right) \geq B^{(M)}\right].$$

By symmetry this can be done for each node in the network.

The principle of the largest term states that rare events occur in the most likely way [13]. The overflow of a buffer can be considered a rare event so that the supremum can be moved out the probability

$$\Pr\left[X^{N^{(M)}} \geq B^{(M)}\right]\left(C_t^{(M)}\right) = \sup_{t\geq 0}\Pr\left[\Lambda_t^{N^{(M)}} \geq C_t^{(M)} + B^{(M)}\right].$$

In a multi-hop CSMA/CA network, the capacities of the nodes are related to satisfy the following constraints:

- the stability criterion for each node has to be respected;
- no backlogging of traffic in a node, except the node M is authorized;
- the intrinsic capacity specified by the access protocol has to be optimally used and
- the topology restricts the M-dimensional space of available capacities.

The next section gives a detailed overview of the constitutive equations linking the capacities of nodes in a multi-hop network with an ideal RTS-CTS access schema and evaluated for different topologies.

Due to the dependency of $C_t^{(M)}$ on the capacities of the other nodes the following M-multiple integral has to be evaluated for the calculation of $\Pr\left[X^N \geq B\right]$ for all authorized capacities $C_t^{(M)}$

$$\Pr\left[X^N \geq B\right] = \int_{\mathcal{D}} \sup_{t\geq 0}\Pr\left[\left(\Lambda_t^N\right) \geq C_t^{(M)} + B^{(M)}\right]\mathrm{d}C_t^{(M)} \prod_{i=1}^{M-1}\sup_{t\geq 0} f_{\Lambda_t^{N^{(i)}}}(C_t^{(i)})\mathrm{d}C_t^{(i)} \tag{13}$$

where the domain $\mathcal{D}$ is limited by $\left(N\mu^{(i)}t^{(i)}, NK^{(i)}t^{(i)}\right)$ and the constitutive equations due to the access protocol and the topology. Substituting the tail probability and the densities expressed in the previous section gives

$$\Pr\left[X^N \geq B\right] = \int_{\mathcal{D}} \frac{\mathrm{e}^{-NJ_t^{(M)}\left(u^{(M)}\right)}}{s_t^{(M)}\left(u^{(M)}\right)\sqrt{2\pi N\left[\sigma_t^{(M)}\left(u^{(M)}\right)\right]^2}}\mathrm{d}u^{(M)} \prod_{i=1}^{M-1}\frac{\mathrm{e}^{-NJ_{t_0}^{(i)}\left(u^{(i)}\right)}}{\sqrt{2\pi N\left[\sigma_{t_0}^{(i)}\left(u^{(i)}\right)\right]^2}}\mathrm{d}u^{(i)} \tag{14}$$

where $t = \arg\sup_{t\geq 0} J_t^{(M)}\left(u^{(M)}\right)$ and $Nu^{(M)} = C_t^{(M)} + B^{(M)}$. For $i < M$ no backlogging of traffic is authorized, e.i. the buffer capacity of node i equals 0, so that $t^{(i)} = t_0$ and $Nu^{(i)} = C_t^{(i)}$.

3.2 Evaluation

Laplace's method [19] gives an analytical approximation for the following integral as $N \to \infty$

$$\int_{\mathcal{D}} g(\boldsymbol{x})\mathrm{e}^{-Nf(\boldsymbol{x})}\mathrm{d}\boldsymbol{x} = \frac{(2\pi)^{\frac{M-1}{2}} g(\boldsymbol{x_0})\mathrm{e}^{-Nf(\boldsymbol{x_0})}}{N^{\frac{M+1}{2}}\sqrt{|J|}}$$

with $\arg\min f(\boldsymbol{x}) = \boldsymbol{x}_0 \in \mathcal{D}$ and

$$J = \frac{\partial f(\boldsymbol{x_0})}{\partial x_p}\frac{\partial f(\boldsymbol{x_0})}{\partial x_q}\mathrm{cof}\left[\frac{\partial^2 f(\boldsymbol{x_0})}{\partial x_p \partial x_q} - \sum_j k_j \frac{\partial^2 h_j(\boldsymbol{x_0})}{\partial x_p \partial x_q}\right]$$

with j active constraints $h_j(\boldsymbol{x_0}) = 0$ and k_j the corresponding Lagrangian multipliers. For non-active constraints, the Lagrangian multipliers equal 0. $\mathrm{cof}[A_{pq}]$ is used to denote the co-factor of the element A_{pq} of the matrix $[A_{pq}]$.

For $\boldsymbol{u} = (u^{(1)}, \ldots, u^{(M)})$ the elements of the integral 14 can be reformulated in terms of Laplace's method

$$f(\boldsymbol{u}) = J_t^{(M)}\left(u^{(M)}\right) + \sum_{i=1}^{M-1} J_{t_0}^{(i)}\left(u^{(i)}\right) \tag{15}$$

and

$$g(\boldsymbol{u}) = \left(s_t^{(M)}\left(u^{(M)}\right)\sqrt{2\pi N\left[\sigma_t^{(M)}\left(u^{(M)}\right)\right]^2} \prod_{i=1}^{M-1}\sqrt{2\pi N\left[\sigma_{t_0}^{(i)}\left(u^{(i)}\right)\right]^2}\right)^{-1}.$$

Both can analytically be calculated for a given traffic distribution, e.g. periodic ON/OFF sources, or the functions can numerically be evaluated for a file containing a traffic trace consisting of a time-stamp and a packet size.

The key point of the evaluation of the integral is the optimization of the function $f(\boldsymbol{u})$ with a search domain limited by the constraint functions $h_j(\boldsymbol{u})$. The gradient of the function $f(\boldsymbol{u})$ can easily be obtained by combining

$$\frac{\partial f(\boldsymbol{u})}{\partial u^{(i)}} = \frac{\partial J_t^{(i)}\left(u^{(i)}\right)}{\partial u^{(i)}}$$

and

$$\begin{aligned}\frac{\partial J_t\left(u^{(i)}\right)}{\partial u^{(i)}} &= s_t^{(i)}\left(u^{(i)}\right) + \frac{\partial s_t^{(i)}\left(u^{(i)}\right)}{\partial u^{(i)}}u^{(i)} \\ &- \frac{n}{\phi_t^{(i)}\left(s_t^{(i)}\left(u^{(i)}\right)\right)}\frac{\partial \phi_t\left(s_t^{(i)}\left(u^{(i)}\right)\right)}{\partial s_t^{(i)}\left(u^{(i)}\right)}\frac{\partial s_t^{(i)}\left(u^{(i)}\right)}{\partial u^{(i)}} \\ &= s_t^{(i)}\left(u^{(i)}\right).\end{aligned}$$

The last equation uses equation 11 to give the following straightforward result

$$\nabla f(\boldsymbol{u}) = \left[\begin{array}{cccc} s_t^{(1)}\left(u^{(1)}\right) & s_t^{(2)}\left(u^{(2)}\right) & \cdots & s_t^{(M)}\left(u^{(M)}\right)\end{array}\right].$$

Similarly the Hessian of the function $f(\boldsymbol{u})$ can be calculated

$$\begin{aligned}\frac{\partial^2 f(\boldsymbol{u})}{\partial\left(u^{(i)}\right)^2} &= \frac{\partial^2 J_t\left(u^{(i)}\right)}{\partial\left(u^{(i)}\right)^2} = \frac{\partial s_t^{(i)}\left(u^{(i)}\right)}{\partial u^{(i)}} \\ &= 2\frac{\partial s_t^{(i)}\left(u^{(i)}\right)}{\partial u^{(i)}} - \sigma_t^{(i)}\left(u^{(i)}\right)\left(\frac{\partial s_t^{(i)}\left(u^{(i)}\right)}{\partial u^{(i)}}\right)^2 \\ &= \frac{1}{\sigma_t^{(i)}}\end{aligned}$$

and

$$\frac{\partial^2 f(\boldsymbol{u})}{\partial u^{(i)}\partial u^{(j)}} = 0$$

resulting in a diagonal Hessian matrix

$$\mathcal{H}f(\boldsymbol{u}) = \left[\begin{array}{cccc} \frac{1}{\sigma_t^{(1)}\left(u^{(1)}\right)} & 0 & & 0 \\ 0 & \frac{1}{\sigma_t^{(2)}\left(u^{(2)}\right)} & & 0 \\ & & \ddots & \\ 0 & 0 & 0 & \frac{1}{\sigma_t^{(M)}\left(u^{(M)}\right)}\end{array}\right].$$

The optimization can in the general case be carried out by a numerical algorithm. DONLP2 [20] by Peter Spellucci is a FORTRAN subroutine which uses a sequential equality constrained quadratic programming method with an active set technique to solve the constraint minimization problem. Depending on the constitutive equations more optimized procedures can be chosen.

4. CONSTITUTIVE EQUATIONS

To evaluate of the model of previous section a simple MAC protocol is used. It is based on the RTS/CTS enabled access of IEEE802.11 with the following modifications:

- the sending and receiving of a RTS/CTS message is instantaneous and does not interfere with other transmission (side channel) so that collisions are prohibited and impossible;
- on reception of a RTS message, a node returns a CTS message if the channel as seen by the node is empty;
- if a node is not blocked by physical of virtual carrier sensing and the buffer is not empty, a packet will be served without back-off and
- if several nodes can serve a packet at the same time, the oldest packet to be served has priority.

This CDMA-CA protocol can be called ideal because the scheduling is completely fair and collisions are completely avoided. It is however completely unrealistic. RTS/CTS messages have a certain cost, e.g. capacity, and collisions will occur in a real wireless network. In [7] a complete analyzes of both effects is detailed. For the evaluation of the queuing model, this ideal MAC protocol allows a better

Table 1: Channel states and characteristics

State	Time Interval	Occurrence Probability
Successful	T_s	$P_s = (1-e)(1-b)$
Idle	T_e	$P_e = e(1-b)$
Busy	T_b	$P_b = b$

understanding between the varying capacities, as seen by a node, and the queuing process. Starvation effects due to collisions of hidden terminals and nodes with an information asymmetry relationship are not possible, so that the constitutive equations limiting the domain of possible capacity tuples are easier to evaluate.

4.1 Throughput and Channel State

Generally, a channel as seen by a node in a multi-hop CSMA/CA network can have 4 states [7]:

- idle channel;
- channel occupied by a successful transmission of the node;
- channel occupied by a collision of the node and
- channel busy due to activities of other nodes, detected by means of physical or virtual carrier sensing (NAV).

These states are characterized by a time interval during which a node remains in each of the states and the corresponding occurrence probability. The simple access protocol prohibits collisions so the collision channel state is not allowed. The probabilities can be defined as a function of 2 auxiliary variables:

b the probability that the channel is busy due to activities of other nodes and

e the conditional empty queue probability.

Table 1 summarizes the different channel states and associated parameters. Using renewal theory, the steady-state throughput, s, of a node can be expressed by

$$s = \frac{P_s}{P_s T_s + P_e T_e + P_b T_b} \ . \tag{16}$$

$T_s$and T_b are known a priori

$$T_s = \frac{\mu}{C}$$
$$T_b = \frac{\mu^*}{C}$$

with μ the mean length of a packet leaving the node, $\mu^*$the mean length of a packet blocking the node and C the over-the-air data-rate. In this paper it is assumed that $\mu = \mu^*$ so that several equations are simplified. For sake of completeness, the equations affected by this assumption will be commented in a footnote. b, e and T_e are specific for each node and depend on the topology and the throughput of the other nodes. The next section introduces the concept of air-time which is the basis for the calculation of all remaining unknowns.

4.2 Air-time

In [10] the air-time of a link, $a^{(l)}$, is defined by the fraction of time during which the data over a link l can start to be transmitted. In the setting of this paper it is no longer necessary to distinct the different links of a node. This was needed to have a reversible Markov-chain so that the steady-state probability can be expressed by a product form. A product form evaluation based on nodes is possible for the simple MAC and $A^{(i)}$ is defined as the air-time of a node. When all nodes are in sensing range of each other, the air-time is common to all of them. The throughput of a node is directly related to its air-time. The larger $A^{(i)}$, the more the node can start to transmit.

A technique originally developed in [4] can be used to compute $A^{(i)}$. The channel activity of each node i is characterized by an ON/OFF process where the ON period has a duration[2] of T_s and where the corresponding link is active after an exponentially distributed amount of time with mean $\frac{1}{g^{(i)}}$. The steady-state probability of a set of all nodes which can be simultaneously active, noted by D, can be expressed in product form[3] by

$$Q(D) = \left(\prod_{i \in D} g^{(i)} T_s\right) Q(\varnothing) = \left(\prod_{i \in D} G^{(i)}\right) Q(\varnothing) \tag{17}$$

where $Q(\varnothing)$ is the probability that no node is active and $G^{(i)}$ the normalized scheduling rate. In [5] the authors give the necessary condition for a product form probability: the corresponding Markov-chain has to be reversible, i.e. if a node blocks another node, the latter blocks also the former.

The air-time of a link l can be computed by considering all independent sets in which no link in conflict with link l is active

$$A^{(i)} = \frac{\sum_{D \in \overline{C^{(i)}}} \prod_{j \in D} G^{(j)}}{\sum_{D \in \mathcal{N}} \prod_{k \in D} G^{(k)}} \tag{18}$$

where $\overline{C^{(i)}}$ is the complement of the set of nodes including node i or a node in conflict with node i, given $\mathcal{N}$ the set of all independent active node combinations. The rates $g^{(i)}$ need to be computed iteratively by imposing that

$$A^{(i)} g^{(i)} = s^{(i)} \tag{19}$$

where $s^{(i)}$ is the throughput of the node i. This equations can also be normalized

$$A^{(i)} G^{(i)} = S^{(i)} \tag{20}$$

with $S^{(i)} = s^{(i)} T_s$ the normalized throughput of the node i.

The air-time of a node $A^{(i)}$ also corresponds to the fraction of time during which the node is idle[4]

$$A^{(i)} = \frac{P_e^{(i)} T_e}{P_s^{(i)} T_s + P_e^{(i)} T_e + P_b^{(i)} T_s} \tag{21}$$

and $P_e^{(i)}$ can be calculated as the probability that no scheduling event from all the states in which node i can transmit

9[2]Generally speaking $T_s^{(i)} = \frac{\mu^{(i)}}{C}$ depends on the mean length of a packet leaving a node.

9[3]$Q(D) = \left(\prod_{i \in D} g^{(i)} T_s^{(i)}\right) Q(\varnothing)$

9[4]$A^{(i)} = \frac{P_e^{(i)} T_e}{P_s^{(i)} T_s^{(i)} + P_e^{(i)} T_e + P_b^{(i)} T_b^{(i)}}$

during an interval T_e

$$P_e^{(i)} = e^{(i)}(1 - b^{(i)}) = \mathrm{e}^{-G_{agg}^{(i)} T_e^{(i)}} \tag{22}$$

where $G_{agg}^{(i)}$ is the normalized aggregated exponential scheduling rate of all nodes in conflict with node i conditioned by the air-time of node i

$$G_{agg}^{(i)} = G^{(i)} + \sum_{i' \in C^{(i)}} G^{(i')} A^{(i'|i)} \tag{23}$$

and the conditional air-time of node i', given node i, is defined by

$$A^{(i'|i)} = \frac{A^{(i' \cap i)}}{A^{(i)}} = \frac{\sum_{D \in \overline{C^{(i)} \cap C^{(i')}}} \prod_{j \in D} G^{(j)}}{\sum_{D \in \overline{C^{(i)}}} \prod_{k \in D} G^{(k)}} \,. \tag{24}$$

Equations 18, 20, 21 and 22 combined with the normalized throughput equation[5]

$$S^{(i)} = \frac{P_s^{(i)}}{P_s^{(i)} + P_e^{(i)} \frac{T_e^{(i)}}{T_s} + P_b^{(i)}} \tag{25}$$

can be iteratively solved for a given tuple of throughputs. If the system of equations converges the tuple is a possible capacity distribution for the wireless multi-hop network.

4.3 Constitutive Equations

For the evaluation of the queuing performance, the bounds for the space of capacity tuples is needed. These bounds are reached when every node in the network is sending or is blocked by a transmission of another node. This saturation state can be expressed formally

$$\forall i : e^{(i)} = 0 \,. \tag{26}$$

On the boundary, the air-time of a node becomes 0 and $T_e^{(i)}$ can no longer be evaluated. The normalized throughput equation[6] reduces to

$$S^{(i)} = 1 - b^{(i)} \,. \tag{27}$$

The blocking probability of a saturated node can be calculated

$$b^{(i)} = \sum_{j \in F_1^{(i)}} S^{(j)} - \sum_{(k,l) \in F_2^{(i)}} S^{(k,l)} + \sum_{(m,n,o) \in F_3^{(i)}} S^{(m,n,o)} - \ldots \tag{28}$$

where $F_1^{(i)}$ is the set of all nodes which block node i, $F_2^{(k,l)}$ is the set of all pair of nodes which can simultaneous block node i and $S^{(k,l)}$ is the corresponding probability.

A first example is a linear topology of 4 nodes

$$n^1 \quad n^2 \quad n^3 \quad n^4$$

with an equal distance between each two nodes. The nodes can only transmit to their closest neighbor(s). The constitutive equations can be written down

$$\begin{aligned} 1 - S^1 &= S^2 + S^3 \\ 1 - S^2 &= S^1 + S^3 + S^4 - S^{14} \\ 1 - S^3 &= S^1 + S^2 + S^4 - S^{14} \\ 1 - S^4 &= S^2 + S^3 \end{aligned}$$

where the second and the third equation are identical. S^{14} can be eliminated and the bounds can be expressed by the matrix equation

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} S^1 \\ S^2 \\ S^3 \\ S^4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} .$$

The second topology is a square of 4 nodes

$$\begin{matrix} n^1 & n^2 \\ n^3 & n^4 \end{matrix}$$

with an equal distance between neighboring nodes. The constitutive equation is easily found

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} S^1 \\ S^2 \\ S^3 \\ S^4 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} .$$

A circular topology of 6 nodes

$$\begin{matrix} & n^1 & n^2 & \\ n^6 & & & n^3 \\ & n^5 & n^4 & \end{matrix}$$

can also be evaluated

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} S^1 \\ S^2 \\ S^3 \\ S^4 \\ S^5 \\ S^6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} .$$

A rectangle of 6 nodes

$$\begin{matrix} n^1 & n^2 & n^3 \\ n^4 & n^5 & n^6 \end{matrix}$$

is the last example. The following constitutive equations are obtained

$$\begin{aligned} 1 - S^1 &= S^2 + S^3 + S^4 + S^5 - S^{34} \\ 1 - S^2 &= S^1 + S^3 + S^4 + S^5 + S^6 - S^{16} - S^{34} \\ 1 - S^3 &= S^1 + S^3 + S^5 + S^6 - S^{16} \\ 1 - S^4 &= S^1 + S^2 + S^5 + S^6 - S^{16} \\ 1 - S^5 &= S^1 + S^2 + S^3 + S^4 + S^6 - S^{16} - S^{34} \\ 1 - S^6 &= S^2 + S^3 + S^4 + S^5 - S^{34} \end{aligned}$$

and after elimination of S^{16} and S^{34} the following matrix equation describes the boundary of the domain

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} S^1 \\ S^2 \\ S^3 \\ S^4 \\ S^5 \\ S^6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} .$$

9[5] $S^{(i)} = \frac{P_s^{(i)}}{P_s^{(i)} + P_e^{(i)} \frac{T_e^{(i)}}{T_s^{(i)}} + P_b^{(i)} \frac{T_b^{(i)}}{T_s^{(i)}}}$

9[6] $S^{(i)} = \frac{1 - b^{(i)}}{1 - b^{(i)} + b^{(i)} \frac{T_b^{(i)}}{T_s^{(i)}}}$

5. CASE STUDY

The semi-analytical method is illustrated by a rather simple example illustrating the different steps in the evaluation of the queuing performance. A linear chain of 5 nodes forms a CDMA/CA network with the simplified RTS/CTS access. The nodes serve traffic composed of a superposition of packets from multiple periodic ON/OFF packet generators. Although more complicated scenarios have been analyzed (tens of nodes fed by real traffic traces and networks with an arbitrary topology) with success, it is more instructive and comprehensible to give the detailed results for a small scenario.

5.1 Periodic ON/OFF Source

All nodes have a FIFO queue which is fed by 2 different periodic ON/OFF packet generators. The period for the first $p_1 = 40$ traffic units and $p_2 = 5$ for the second. The peak rates of these sources in traffic units per time slot are $a_1 = 4$ and $a_2 = 1$. The durations of the ON states are $on_1 = 11$ time units and $on_2 = 4$ time units. The basic stream consist of $n_1 = 20$ flows of the first source and $n_2 = 10$ flows of the second. The mean rate of the basic stream equals 30 traffic units per time slot. For the experiment a variable number N of basic streams are multiplexed.

The calculation for $\phi_t(s) = \mathbb{E}e^{s\Lambda_t}$ and derived quantities is straightforward for a periodic ON/OFF source. Let a be the peak number of traffic units per time unit, on the number of time units during which the source is generating traffic units and p the period of the source then

$$\phi_t(s) \quad = \quad \mathrm{e}^{on.s.m.a}\phi_r(s)$$

with $\phi_r(s)$ the moment generating function for a time-step r smaller than the period of the source p, $r = (t-1)\mathrm{mod}p + 1$ and $m = \lfloor (t-1)/d \rfloor$. Once the values for one period are calculated the others can be directly obtained.

5.2 Constitutive Equations

The networks is composed of 5 nodes in a linear chain topology

$$n^1 \quad n^2 \quad n^3 \quad n^4 \quad n^5$$

and the constitutive equations are

$$\begin{aligned}
1 - S^1 &= S^2 + S^3 \\
1 - S^2 &= S^1 + S^3 + S^4 - S^{14} \\
1 - S^3 &= S^1 + S^2 + S^4 + S^5 - S^{14} - S^{15} - S^{25} \\
1 - S^4 &= S^2 + S^3 + S^5 - S^{25} \\
1 - S^5 &= S^3 + S^4
\end{aligned}$$

where S^{14}, S^{15} and S^{25} can be eliminated to give the following matrix equation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} S^1 \\ S^2 \\ S^3 \\ S^4 \\ S^5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

5.3 Numerical Evaluation

The constraints for the multi-dimensional minimization problem are all linear so that the TOLMIN procedure [21] of Mike Powell can be applied to solve the system. it is a sequential equality constrained quadratic programming method with an active set technique and the Hessian matrix is approximated by a BFGS formula.

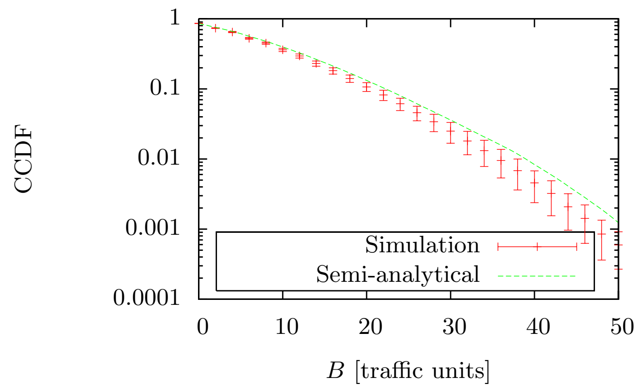

Figure 1: Semi-analytical model compared to a event-driven simulation for the second node in the chain topology with $N = 20$.

For each feasible tuple of throughputs the values of $t^{(i)}$ and $s_t^{(i)}$ have to be evaluated. $s_t^{(i)}$ is calculated by a fail-save root finding routine which utilizes a combination of bisection and Newton-Rhapson [6]. A linear search is performed to obtain the optimal value for $t^{(i)}$.

In one second one global optimization can be carried out for the settings in this case study. 25 runs of the event-driven simulation with random seeds are processed to get a 95% confidence interval. The run time for one simulation is 25 times longer than the semi-analytical method. Both the semi-analytical calculation as the event-driven simulation are not optimized.

The over-the-air data rate is $100N$ traffic units per time unit.

The first figure shows the complementary cumulative distribution function of the second node calculated by the semi-analytical calculation and compared to an event-driven simulation for $N = 20$. The second figure shows also the CCDF of the second node for $N = 80$. Both figures[7] show a good qualitative fit of the novel method to the event-simulation. It is clear however without any quantification that with a larger N the accuracy of the procedure becomes higher.

6. CONCLUSION

In this paper a model for the evaluation of the queuing performance in a multi-hop CSMA-CA network is proposed based on the many sources large deviations theory and the concept of the air-time of a node to calculate the varying capacity as seen by a node. The method is implemented for a network with a simplified RTS/CTS enabled access schema. The evaluation of the queuing performance for a 5 node linear topology is detailed and simulations have shown that the results of the semi-analytical approach fits closely the event-driven simulations.

9[7]Due to paper length restrictions only these figures can be presented. A complete set of scenarios is analyzed and the overall results of the semi-analytical approach fit very well the simulations.

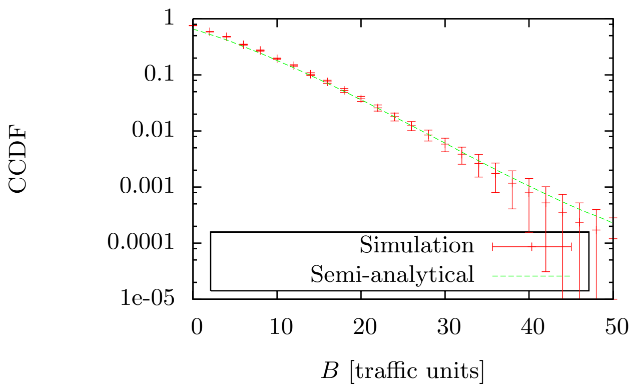

Figure 2: Semi-analytical model compared to a event-driven simulation for the second node in the chain topology with $N = 80$.

In a future paper an extensive set of scenarios will be evaluated and discussed. The framework for an efficient evaluation of the queuing performance of an IEEE802.11 network with RTS/CTS access will be presented. The constraints are no longer linear and starvation effects due to collisions can dominate the queuing process. Topics for further research also include:

- a mathematical analysis of the stability of the proposed methods and a quantitative evaluation of the goodness of fit between the model and the event-driven simulation;
- other wireless MAC protocols, e.g. IEEE802.15.4, can be analyzed to see if a product form evaluation of the air-time is possible and the framework can be adapted to include these access protocols and
- a hybrid wireless simulator can be developed combining the accuracy of event-driven simulations with the speed of semi-analytical models. A similar methodology is already applied to wired networks in [8,9].

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