Bandwidth Scanning when the Rivals Are Subjective

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Abstract. In this paper we consider how subjectivity affects the problem of scanning spectrum bands, and the impact on both the scanner and invader's strategy. To model such subjective behavior, we formulate a prospect theoretical (PT) extension of the Bayesian bandwidth scanning game where the Scanner knows only a priori probabilities about what type of intrusion (e.g. regular intensity or low intensity) occurs in the spectrum bands. Existence and uniqueness of the PT Bayesian equilibrium is proven. Moreover, these PT Bayesian equilibrium strategies are derived in closed form as functions of the detection probabilities associated with different invader types. Waterfilling equations are derived, which allows one to determine these detection probabilities. Bands where the Invader's strategies have band-sharing form are identified. The sensitivity of the strategies to the subjective factors and a priori probabilities are numerically illustrated.

Keywords: Bayesian equilibrium · Prospect theory · Bandwidth scanning

1 Introduction

Cognitive radio networks will support dynamic spectrum access (DSA). However, in spite of the potential benefits for DSA, the open nature of the wireless medium will make cognitive radios a powerful tool for conducting malicious activities or policy violations by secondary users. Therefore, detecting malicious users or unlicensed activities is a crucial problem facing DSA [19], and one of the challenges to enforcing the proper usage of spectrum is the development of an intrusion detection system (IDS) that will scan large amounts of spectrum and identify anomalous activities [4,6,19]. Since, in such security problems, there are two agents with different goals (the IDS aims to detect illegal spectrum usage, while the adversary intends to sneak into bands undetected for their illegal usage), *game theory* is an ideal tool to employ [13]. As examples of applying game theory to detect an adversary to prevent malicious attack on networks, we mention [2,3,5,8,9,11,12,14–16,20,23,25,27–30]. In all of these papers the rivals were *rational.*

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Prospect Theory (PT) [17] has been developed to model the *subjective* factor in an agent's behavior. In particular, PT models agents with subjective behavior by employing subjective probabilities, rather than the objective probabilities that might be used in rational behavior, to weight the values of possible outcomes. We note that, although PT originally was designed to take into account the possibility of the risk of irrational behavior by rivals in economic problems [17,18], it has been applied recently to different network and communication security problems. For example, in [24], for designing Trojan detection algorithm, in [31,32], for developing anti-jamming strategies in cognitive radio networks, in [26], for designing secure drone delivery systems, and in [33], for maintaining a cloud storage defense against advanced persistent threats.

In this paper, we consider how subjectivity affects the problem of scanning large amounts of bandwidth to detect illegal intrusion, where the scanner has incomplete information about the Invader's characteristics. To gain insight into this problem, we formulate a *PT extension* of the Bayesian scanning game between a *Scanner* and an *Invader* considered in [10]. We prove that this PT extension has a unique solution, and find the PT equilibrium strategies in closed form. To the best knowledge of the authors, this paper presents the first PT equilibrium strategies in closed form for an n-dimension payoff matrix and any subjective probabilities. The closed form solution allows us to reveal some interesting properties of the solution, such as the water-filling structure of the PT strategies as well as to identify the bands where Invader's strategies have band-sharing form. The solution also allows one to observe the sensitivity of the strategies on the subjective factors and on a priori information about the Invader's type.

The organization of this paper is as follows. In Sect. 2, the basic model for bandwidth scanning as a zero sum with a diagonal payoff matrix is formulated. In Sect. 3, the basics of prospect theory are presented as a basis for the rest of the paper. In Sect. 4, the PT extension of the basic bandwidth scanning game is described and solved. Next, in Sect. 5, the Bayesian extension of the basic bandwidth scanning game is formulated for the case where there is incomplete information about the mode of intrusion attack. In Sect. 7, auxiliary assumptions, notations and results are introduced, and they are employed in Sect. 8 to derive the equilibrium strategies for the PT Bayesian scanning game. Finally, in Sect. 9, conclusions are offered.

2 Model

In this section, we describe our basic problem model, which involves a scenario where a primary user (i.e. the Scanner) owns n frequency bands $1, 2, \dots, n$, which will be scanned by the Scanner. The Invader will attempt to "sneak" usage on *only* one of these bands, while the Scanner can only scan a single band at a time. We assume that the Invader will be detected with probability γ_i if he sneaks in band i and the Scanner scans that band. We note that the detection probability of the Invader depends on its SINR [21], and thus, in particular, on the power of the Invader's signal as well as on the distance between the Invader and the Scanner. If the Scanner does not scan the band that the Invader is using, then the Invader sneaks safely, i.e., its detection probability is zero. We assume that the payoff to the Scanner is one if the Invader is detected and it is zero otherwise. Let a (mixed) strategy for the Scanner be $\boldsymbol{x} = (x_1, \ldots, x_n)$, where x_i is the probability (reflecting the likelihood of revisiting that channel when the game is repeated) that he scans band i. So, $\sum_{i=1}^{n} x_i = 1$ and $x_i \geq 0$, $i = 1, \ldots, n$. Let a (mixed) strategy for the Invader be $\boldsymbol{y} = (y_1, \ldots, y_n)$, where y_i is the probability that he sneaks in band i. Thus, $\sum_{i=1}^{n} y_i = 1$ and $y_i \geq 0$, $i = 1, \ldots, n$. Denote by $\mathcal{P}$, the set of all n dimensional probability vectors. Thus, $\mathcal{P}$ is the set of all feasible strategies for the Scanner as well as for the Invader. Then, the expected payoff to the Scanner if the rivals employ strategy $\boldsymbol{x}$ and $\boldsymbol{y}$ is given as follows:

$$V(\boldsymbol{x}, \boldsymbol{y}) = \sum_{i=1}^{n} \gamma_i x_i y_i. \tag{1}$$

This payoff reflects detection probability of the Invader. The Scanner wants to maximize this detection probability, while the Invader wants to minimize it. Thus, this is a zero sum game, and we look for its equilibrium. Recall that $(\boldsymbol{x}, \boldsymbol{y})$ is an equilibrium if and only if for any $(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{y}}) \in \mathcal{P} \times \mathcal{P}$ the following inequalities hold:

$$V(\tilde{\boldsymbol{x}}, \boldsymbol{y}) \leq V(\boldsymbol{x}, \boldsymbol{y}) \leq V(\boldsymbol{x}, \tilde{\boldsymbol{y}}) \tag{2}$$

This implies that $\boldsymbol{x}$ and $\boldsymbol{y}$ are equilibrium strategies if and only if they are the best response to each other, i.e., they are solutions of the following best response equations:

$$\boldsymbol{x} = \mathrm{BR}_S(\boldsymbol{y}) := \operatorname*{argmax}_{\boldsymbol{x} \in \mathcal{P}} V(\boldsymbol{x}, \boldsymbol{y}), \tag{3}$$

$$\boldsymbol{y} = \mathrm{BR}_I(\boldsymbol{x}) := \operatorname*{argmin}_{\boldsymbol{y} \in \mathcal{P}} V(\boldsymbol{x}, \boldsymbol{y}). \tag{4}$$

This is a zero sum game with diagonal payoff matrix and its equilibrium strategies will be given in closed form in Corollary 1 of Sect. 4.

3 Basics of Prospect Theory

In this section, we will briefly review the basic concepts associated with the PT solution. Prospect theory, which was introduced by Tversky and Kahneman [17], is a method for describing decisions under a *subjective* factor. In particular, it is revealed in PT that agents use subjective probabilities ("decision weight") $w(p)$, rather than the objective probabilities p, to weight the values of possible outcomes. Moreover, agents tend to over-weight low probability outcomes and under-weight higher probability outcomes. This feature is captured by weighting the probability distribution by the *weighting function* $w(p)$. Such weighting functions satisfy four basic properties in the plane (p, w) with $p \in [0, 1]$ [22]: (a) regressive intersecting the diagonal from above, (b) asymmetric with fixed point at about 1/3, (c) S-shaped concave on an initial interval and convex beyond that, and (d) reflective assigning equal weight to a given loss-probability as to a given gain-probability. Although all of our obtained analytical results hold for any weighting function, as a basic example of the probability weighting function we consider the Prelec function in our numerical examples, which is as follows [22]:

$$w(p) := e^{-(-\ln(p))^{\kappa}} \tag{5}$$

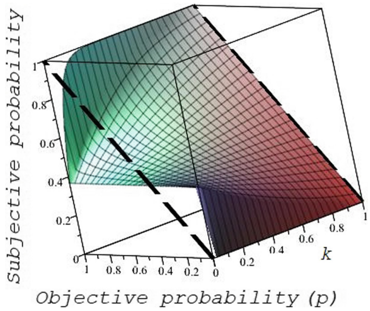

Fig. 1. Subjective probability as function on objective probability p and probability weighting parameter κ.

with $\kappa \in (0, 1]$ is the *probability weighting parameter.* The probability weighting parameter reflects the distortion from the true objective probability that is caused by the subjective evaluation of the agent. In other words, this parameter allows us to measure how rational (or subjective) an agent is. Namely, $\kappa = 1$ corresponds to rational agent, while smaller κ corresponds to a less rational agent, or in other words, a more subjective agent. Figure 1 illustrates the dependence of the subjective probabilities given by (5) on the objective probability p and the probability weighting parameter κ. For boundary case $\kappa \downarrow 0$, the subjective probability $w(p)$ approximates a step function, flat everywhere except near the endpoints of the probability interval. While, for the other boundary case of $\kappa = 1$, this subjective probability $w(p)$ coincides with objective probability, i.e.,

$$w(p) \equiv p \text{ for } \kappa = 1, \tag{6}$$

i.e., $\kappa = 1$ corresponds to full rationality of the agent. Finally, in particular, $w(p)$ has the following monotonicity property:

$$w(p) \text{ is strictly increasing from 0 for } p = 0 \text{ to 1 for } p = 1. \tag{7}$$

4 Subjective Rivals

Denote by $w_S(p)$ and $w_I(p)$ the probability weighting functions employed by the Scanner and the Invader correspondingly. Then, the PT-utilities for the rivals associated with the zero sum game with payoff (21) are given as follows:

$$u_S^{PT}(\boldsymbol{x},\boldsymbol{y}) := \sum_{i=1}^{n} \gamma_i x_i w_S(y_i) \text{ and } u_I^{PT}(\boldsymbol{x},\boldsymbol{y}) := \sum_{i=1}^{n} \gamma_i w_I(x_i) y_i. \tag{8}$$

Then, under PT equilibrium, we consider the solution of the following PT best response equations

$$\boldsymbol{x} = \mathrm{BR}_S^{PT}(\boldsymbol{y}) := \operatorname*{argmax}_{\boldsymbol{x}\in\mathcal{P}} u_S^{PT}(\boldsymbol{x},\boldsymbol{y}), \tag{9}$$

$$\boldsymbol{y} = \mathrm{BR}_I^{PT}(\boldsymbol{x}) := \operatorname*{argmin}_{\boldsymbol{y}\in\mathcal{P}} u_I^{PT}(\boldsymbol{x},\boldsymbol{y}). \tag{10}$$

Theorem 1. *The game has the unique PT equilibrium* $(\boldsymbol{x},\boldsymbol{y})$*, where*

$$x_i = x_i(\omega) = w_I^{-1}\left(\frac{\omega}{\gamma_i}\right) \text{ and } y_i = y_i(\nu) = w_S^{-1}\left(\frac{\nu}{\gamma_i}\right), \tag{11}$$

where $\omega \in (0, \min \underline{\gamma})$ *and* $\nu \in (0, \min \underline{\gamma})$ *with* $\underline{\gamma} = \min_i \gamma_i$ *uniquely defined as solutions of the equations:*

$$\sum_{i=1}^{n} w_I^{-1}\left(\frac{\omega}{\gamma_i}\right) \text{ and } \sum_{i=1}^{n} w_S^{-1}\left(\frac{\nu}{\gamma_i}\right) = 1. \tag{12}$$

Proof. Since $u_S^{PT}(\boldsymbol{x},\boldsymbol{y})$ is linear in $\boldsymbol{x}$, $\boldsymbol{x}$ is the best response strategy to $\boldsymbol{y}$ if and only if there is a ν such that

$$\gamma_i w_S(y_i) \begin{cases} = \nu, & x_i > 0, \\ \leq \nu, & x_i = 0. \end{cases} \tag{13}$$

Since $u_I^{PT}(\boldsymbol{x},\boldsymbol{y})$ is linear on $\boldsymbol{y}$, $\boldsymbol{y}$ is the best response strategy to $\boldsymbol{x}$ if and only if there is ω such that

$$\gamma_i w_I(x_i) \begin{cases} = \omega, & y_i > 0, \\ \geq \omega, & y_i = 0. \end{cases} \tag{14}$$

The assumption that there is an i such that $x_i = 0$ leads to a contradiction, since, by (14), $\omega = 0$, and so $x_i = 0$ for any i. Thus,

$$x_i > 0 \text{ for any } i. \tag{15}$$

Assume, now that there is an i such that $y_i = 0$. Then, by (13), $x_i = 0$. This contradicts (15). Thus, also $y_i > 0$ for any i. Thus, by (13) and (14),

$$\gamma_i w_S(y_i) = \nu \text{ and } \gamma_i w_I(x_i) = \omega \text{ for any } i. \tag{16}$$

This and (17) imply (11). Finally (17), (11) and the fact that $\boldsymbol{x}$ and $\boldsymbol{y}$ are probability vectors yield (12). ∎

Remark 1. *The inverse of the weighting function (5) is given as follows:*

$$w^{-1}(p) := e^{-(-\ln(p))^{1/\kappa}}. \tag{17}$$

It is apparent that the scenario with rational rivals is a boundary case of the subjective behavior with $\kappa = 1$, and it corresponds to zero-sum game with diagonal payoff matrix [7]. Namely, for rational rivals the following result holds.

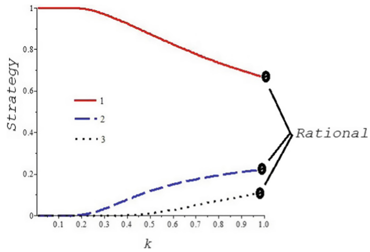

Fig. 2. PT equilibrium strategy as a function of the probability weighting parameter κ.

Corollary 1. *In the game with a rational Scanner and Invader, the equilibrium strategies* $(\boldsymbol{x}, \boldsymbol{y})$ *are unique and given as follows:*

$$x_i = y_i = \frac{1/\gamma_i}{\sum_{j=1}^{n}(1/\gamma_j)} \text{ for } i = 1, \ldots, n. \tag{18}$$

Proof: For rational rivals $w_I(p) \equiv w_S(p) \equiv p$. Thus, by (11), $x_i(\omega) = \omega/\gamma_i$ and $y_i(\nu) = \nu/\gamma_i$. Then, by (12), $\omega = \nu = 1/\sum_{j=1}^{n}(1/\gamma_j)$, and the result follows. ■

Thus, if both rivals are rational their equilibrium strategies coincide and both of them equalize the detection probability at each band. Meanwhile, the PT equilibrium strategies can differ if the subjective factors for the rivals differ, and the strategies are then given by water-filling equations (12).

Let us illustrate dependence of PT equilibrium strategies on κ by an example with $n = 3$ bands and detection probabilities $\gamma = (0.1, 0.3, 0.6)$. Figure 2 illustrates that the more subjective agent (which is reflected by smaller probability weighting parameter κ) focuses more effort (scanning for the Scanner and intrusion for the Invader) on the band with the smallest detection probability, i.e., band 1, and reduces its effort on all of the other bands. On the other hand, the more objective agent would spread its effort amongst the bands in such a way to equalize the detection probability when it becomes to be rational. An important property of these strategies is that each band will be scanned with positive probability, and likewise any band may be employed to sneak in with positive probability although these probabilities could approximately be zero when the probability weighting parameter tends to zero. As a point of reference, one can observe similar strategy behavior in the design of α-fair solutions as one increases the fairness coefficient [1].

5 Bayesian Game with Rational Rivals

In this section we return to the scenario involving rational rivals. Recall that the detection probability of the Invader depends on its SINR [21], and thus, in particular, on the power of the Invader's signal as well as on the distance between the Invader and Scanner. With each combination of transmitted signal by the Invader and its allocation, we can associate a type for the Invader and the corresponding detection probabilities in bands. In this paper, we will assume the Invader can be one of two types, an agent performing *regular intrusion* (denoted by type-1) and an agent performing *low intensity intrusion* (denoted by type-2). Let, with a priori probability α^t, a type-t Invader occurs, then $\alpha^1 + \alpha^2 = 1$. Let the detection probability be γ_i^t for the type-t Invader to be detected when it sneaks in band i and the Scanner scans this band. A (mixed) strategy for the type-t Invader is $\boldsymbol{y}^t = (y_1^t, \dots, y_n^t)$, where y_i^t is the probability that he sneaks in band i. Thus, $\sum_{i=1}^n y_i^t = 1$ and $y_i^t \geq 0$, $i = 1, \dots, n$. Let $\boldsymbol{Y} = (\boldsymbol{y}^1, \boldsymbol{y}^2)$. Then, we consider the probability of detection of the Invader if the rivals employ strategy $\boldsymbol{x}$ and $\boldsymbol{Y}$ as the expected payoff to the Scanner:

$$V(\boldsymbol{x}, \boldsymbol{Y}) = \sum_{i=1}^n \left(\alpha^1 \gamma_i^1 x_i y_i^1 + \alpha^2 \gamma_i^2 x_i y_i^2\right), \tag{19}$$

while the probabilities to be detected are considered as cost functions of the corresponding Invader's type:

$$V^t(\boldsymbol{x}, \boldsymbol{y}^t) = \sum_{i=1}^n \gamma_i^t x_i y_i^t. \tag{20}$$

We look for Bayesian equilibrium, i.e., for such strategies $(\boldsymbol{x}, \boldsymbol{Y})$ that for any $(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{Y}})$ the following inequalities hold:

$$\begin{aligned} V(\tilde{\boldsymbol{x}}, \boldsymbol{Y}) &\leq V(\boldsymbol{x}, \boldsymbol{Y}), \\ V^t(\boldsymbol{x}, \boldsymbol{y}^t) &\leq V^t(\boldsymbol{x}, \tilde{\boldsymbol{y}}^t), t = 1, 2. \end{aligned} \tag{21}$$

This implies that $\boldsymbol{x}$ and $\boldsymbol{Y}$ are equilibrium strategies if and only if they are the best response to each other, i.e., they are solutions of the following best response equations:

$$\boldsymbol{x} = \mathrm{BR}_S(\boldsymbol{Y}) := \operatorname*{argmax}_{\boldsymbol{x} \in \mathcal{P}} V(\boldsymbol{x}, \boldsymbol{Y}), \tag{22}$$

$$\boldsymbol{y}^t = \mathrm{BR}_I^t(\boldsymbol{x}) := \operatorname*{argmin}_{\boldsymbol{y}^t \in \mathcal{P}} V^t(\boldsymbol{x}, \boldsymbol{y}^t) \text{ for } t = 1, 2. \tag{23}$$

Unique equilibrium strategies of this Bayesian game will be given in Corollary 2 of Sect. 8.

6 Bayesian Game with Subjective Rivals

In this section, we formulate the PT extension of the Bayesian game from the previous section. We now let the rivals be subjective. Thus, to define such PT extension, first, we have to define PT-utilities for the rivals associated with the Bayesian game with payoff (19) and cost functions (20) given as follows:

$$\begin{aligned} u_S^{PT}(\boldsymbol{x}, \boldsymbol{Y}) &:= \sum_{i=1}^n \left(\alpha^1\gamma_i^1 x_i w_S(y_i^1) + \alpha^2\gamma_i^2 x_i w_S(y_i^2)\right), \\ u_I^{PT,t}(\boldsymbol{x}, \boldsymbol{y}^t) &:= \sum_{i=1}^n \gamma_i^t w_I(x_i) y_i^t. \end{aligned} \tag{24}$$

Then, under the PT equilibrium $(\boldsymbol{x}, \boldsymbol{Y})$ of the Bayesian game, we consider the solution of the following PT best response equations:

$$\boldsymbol{x} = \mathrm{BR}_S^{PT}(\boldsymbol{Y}) := \operatorname*{argmax}_{\boldsymbol{x}} u_S^{PT}(\boldsymbol{x}, \boldsymbol{Y}), \tag{25}$$

$$\boldsymbol{y}^t = \mathrm{BR}_I^{PT,t}(\boldsymbol{x}) := \operatorname*{argmin}_{\boldsymbol{y}^t} u_I^{PT,t}(\boldsymbol{x}, \boldsymbol{y}^t), t = 1, 2. \tag{26}$$

In Theorem 2 of Sect. 8, it will be proven that these PT best response equations have a unique solution and the PT equilibrium strategies will be found in closed form.

7 Auxiliary Assumption, Notations and Results

In this section, we introduce auxiliary notations and results. First, to avoid bulkiness in formulas, we assume that there are no two different bands with the same ratio of detection probabilities, i.e., $\gamma_i^2/\gamma_i^1 \neq \gamma_j^2/\gamma_j^1$ for $i \neq j$. Then, without loss in generalization we can assume that the bands are arranged in such an order that

$$\gamma_1^2/\gamma_1^1 > \gamma_2^2/\gamma_2^1 > \ldots > \gamma_n^2/\gamma_n^1. \tag{27}$$

Let us now extend the definition of inverse functions $w_S^{-1}(\xi)$ and $w_J^{-1}(\xi)$ as follows:

$$w_S^{-1}(\xi) := \begin{cases} w_S^{-1}(\xi), & \xi \leq 1, \\ 1, & \xi > 1 \end{cases} \text{ and } w_I^{-1}(\xi) := \begin{cases} w_I^{-1}(\xi), & \xi \leq 1, \\ 1, & \xi > 1. \end{cases} \tag{28}$$

Let

$$\underline{\Psi}_i(\nu) := \sum_{j=1}^{i-1} w_S^{-1}\left(\frac{\nu}{\alpha^1\gamma_j^1}\right), \tag{29}$$

$$\overline{\Psi}_i(\nu) := \sum_{j=i+1}^{n} w_S^{-1}\left(\frac{\nu}{\alpha^2\gamma_j^2}\right), \tag{30}$$

and

$$\underline{A}_i^1 := \min_{j\leq i-1} \alpha^1\gamma_j^1 \text{ and } \overline{A}_i^2 := \min_{j\geq i+1} \alpha^2\gamma_j^2. \tag{31}$$

Proposition 1. **(a)** *For a fixed i, $\underline{\Psi}_i(\xi)$ is strictly increasing on ξ from zero for $\xi = 0$ to $\underline{\Psi}_i(\underline{A}_i^1) > 1$ for $\xi = \underline{A}_i^1$.*
(b) *For a fixed $\xi > 0$, $\underline{\Psi}_i(\xi)$ is increasing on i.*
(c) *For a fixed i, $\overline{\Psi}_i(\xi)$ is strictly increasing on ξ from zero for $\xi = 0$ to $\overline{\Psi}_i(\overline{A}_i^2) > 1$ for $\xi = \overline{A}_i^2$.*
(d) *For a fixed $\xi > 0$, $\overline{\Psi}_i(\xi)$ is decreasing on i.*

Proof: (a) and (c) follow from (17) and (28), while (b) and (d) follow from the following relations

$$\underline{\Psi}_{i+1}(\xi) - \underline{\Psi}_i(\xi) = w_S^{-1}\left(\xi/(\alpha^1\gamma_i^1)\right) > 0,$$
$$\overline{\Psi}_{i+1}(\xi) - \overline{\Psi}_i(\xi) = -w_S^{-1}\left(\xi/(\alpha^2\gamma_{i+1}^2)\right) < 0,$$

and the result follows. ■

Based on Proposition 1, we can define two auxiliary finite sequences ν_i^1, ν_i^2, $i = 1, \ldots, n$ as follows:

$$\nu_i^1 = \begin{cases} \infty, & i = 1, \\ \text{the unique root in } (0, \underline{A}_i^1) \text{ of equation } \underline{\Psi}_i(\nu_i^1) = 1, & i = 2, \ldots, n \end{cases} \tag{32}$$

and

$$\nu_i^2 = \begin{cases} \text{the unique root in } (0, \overline{A}_i^2) \text{ of equation } \overline{\Psi}_i(\nu_i^2) = 1, & i = 1, \ldots, n-1, \\ \infty, & i = n. \end{cases} \tag{33}$$

These sequences have the following monotonicity properties:

Proposition 2. **(a)** $\nu_1^1 > \nu_2^1 > \ldots > \nu_{n-1}^1 > \nu_n^1$,
(b) $\nu_1^2 < \nu_2^2 < \ldots < \nu_{n-1}^1 < \nu_n^1$.

Proof: (a) follows from Proposition 1(a) and (b), while (b) follows from Proposition 1(c) and (d). ■

Proposition 3. *There exists the unique $m_* \in \{1, \ldots, n\}$ such that*

$$\nu_j^1 \begin{cases} > \nu_j^2, & j < m_*, \\ \geq \nu_j^2, & j = m_*, \\ < \nu_j^2, & j > m_*. \end{cases} \tag{34}$$

Proof: Since $\nu_1^1 = \infty > \nu_1^2$ and $\nu_n^1 < \infty = \nu_n^2$, the result straightforward follows from Proposition 2. ■

Proposition 4. *There is a unique i_* such that one of the following relations holds:*

$$\nu_{i_*}^1 = \nu_{i_*}^2, \tag{35}$$

$$\nu^2_{i_*-1} \le \nu^1_{i_*} < \nu^2_{i_*} \tag{36}$$

or

$$\nu^1_{i_*+1} \le \nu^2_{i_*} < \nu^1_{i_*}. \tag{37}$$

Moreover,

$$i_* = \begin{cases} m_* + 1, & \textit{if } (36) \textit{ holds}, \\ m_*, & \textit{if } (37) \textit{ or } (35) \textit{ holds}. \end{cases} \tag{38}$$

Proof: First assume that $\nu^1_i \ne \nu^2_i$ for any i. Thus, (35) cannot hold. Then, by Proposition 3,

$$\nu^2_i < \nu^2_{m_*} < \nu^1_{m_*} < \nu^1_j \text{ for any } i, j < m_* \tag{39}$$

and

$$\nu^1_j < \nu^1_{m_*+1} < \nu^2_{m_*+1} < \nu^2_i \text{ for any } i, j > m_* + 1. \tag{40}$$

Thus, neither (36) nor (37) can hold for any $i_* \notin \{m_*, m_* + 1\}$. So, if i_* exists then $i_* \in \{m_*, m_* + 1\}$. Moreover, by (39), if (37) holds then $i_* = m_*$. While, by (40), if (36) holds then $i_* = m_* + 1$. This proves that if i_* exists then (38) has to hold. Thus, to complete the proof we have to prove that for i_* given by (38) only one of two relations (36) and (37) always holds. Let us consider separately two cases: (i) $\nu^1_{m_*+1} \le \nu^2_{m_*} < \nu^1_{m_*}$ and (ii) $\nu^2_{m_*} < \nu^1_{m_*}$ and $\nu^2_{m_*} < \nu^1_{m_*+1}$. It is clear that (i) is equivalent to (37) with $i_* = m_*$. Let (ii) hold. Then, by (40), (ii) is equivalent to $\nu^2_{m_*} < \nu^1_{m_*+1} < \nu^2_{m_*+1}$, and this coincides with (36) with $i_* = m_* + 1$.

The case that there exists an i such that $\nu^1_i = \nu^2_i$ can be considered similarly, and the result follows. ■

8 Solution of the PT Bayesian Game

In this Section, we prove the uniqueness of the PT Bayesian equilibrium and find it in closed form. To prove the result we are going to employ a constructive approach. Namely, we first derive the necessary and sufficient conditions for the strategies $(\boldsymbol{x}, \boldsymbol{Y})$ to be the PT equilibrium, and establish what structure these strategies must have to satisfy these conditions. Then, taking into account that the strategies are probability vectors, we show that only one pair of strategies can have such a structure, and we design this pair of strategies in closed form.

Theorem 2. *The game has a unique Bayesian PT equilibrium* $(\boldsymbol{x}, \boldsymbol{Y})$*. Moreover, the Scanner's strategy* $\boldsymbol{x}$ *is given as follows*

$$x_j = x_j(\omega) = \begin{cases} w_I^{-1}\left(\dfrac{\omega}{\gamma^1_j}\right), & j \le i_*, \\ w_I^{-1}\left(\dfrac{\gamma^2_{i*}\omega}{\gamma^1_{i*}\gamma^2_j}\right), & j \ge i_* + 1, \end{cases} \tag{41}$$

where i_* is given by Proposition 4, while ω is the unique root in $(0, \overline{\omega}_{i_*})$ of the equation

$$\sum_{j=1}^{i_*} w_I^{-1}\left(\frac{\omega}{\gamma_j^1}\right) + \sum_{j=i_*+1}^{n} w_I^{-1}\left(\frac{\gamma_{i*}^2\omega}{\gamma_{i*}^1\gamma_j^2}\right) = 1, \tag{42}$$

where $\overline{\omega}_i := \min\{\underline{\gamma}_i^1, \frac{\gamma_i^1}{\gamma_i^2}\overline{\gamma}_i^2\})$ with $\underline{\gamma}_i^1 = \min_{j\le i}\gamma_j^1$ and $\overline{\gamma}_i^2 = \min_{j>i}\gamma_j^2$.

This ω can be found by the bisection method since the left side of Eq. (42) is an increasing function of ω from zero for $\omega = 0$ and becomes greater than one for $\omega = \overline{\omega}$.

The Invader's strategy $\boldsymbol{Y}$ is given as follows:

$$y_i^1 = y_i^1(\nu) := \begin{cases} w_S^{-1}\left(\dfrac{\nu}{\alpha^1\gamma_i^1}\right), & i \le i_* - 1, \\ 1 - \sum\limits_{j=1}^{i_*-1} w_S^{-1}\left(\dfrac{\nu}{\alpha^1\gamma_j^1}\right), & i = i_*, \\ 0, & i \ge i_* + 1, \end{cases} \tag{43}$$

$$y_i^2 = y^2(\nu) := \begin{cases} 0, & i \le i_* - 1, \\ 1 - \sum\limits_{j=i_*+1}^{n} w_S^{-1}\left(\dfrac{\nu}{\alpha^2\gamma_j^2}\right), & i = i_*, \\ w_S^{-1}\left(\dfrac{\nu}{\alpha^2\gamma_i^2}\right), & i \ge i_* + 1, \end{cases} \tag{44}$$

where ν is the unique root in $(0, \underline{\nu}_{i*}]$ with $\underline{\nu}_i := \min\{\nu_i^1, \nu_i^2\}$ of the equation

$$\Phi_{i_*}(\nu) = \nu \tag{45}$$

with

$$\begin{aligned} \Phi_i(\nu) = \alpha^1\gamma_i^1 w_S &\left(1 - \sum_{j=1}^{i-1} w_S^{-1}\left(\frac{\nu}{\alpha^1\gamma_j^1}\right)\right) \\ &+ \alpha^2\gamma_i^2 w_S\left(1 - \sum_{j=i+1}^{n} w_S^{-1}\left(\frac{\nu}{\alpha^2\gamma_j^2}\right)\right). \end{aligned} \tag{46}$$

Due to function $\Phi_{i_*}(\nu)$ is decreasing on ν such that $\Phi_{i_*}(0) > 0$ for $\nu = 0$ and $\Phi_{i_*}(\underline{\nu}_{i*}) \le \underline{\nu}_{i*}$ for $\nu = \underline{\nu}_{i*}$, the unique root of (45) in $(0, \underline{\nu}_{i*}]$ can be found by bisection method.

Finally, note that ν is the probability that the Invader is detected by the Scanner, ω is the detection probability for the type-1 Invader, while $\gamma_{i*}^2\omega/\gamma_{i*}^1$ is detection probability for the type-2 Invader.

Proof. Note that $u_S^{PT}(\boldsymbol{x}, \boldsymbol{Y})$ is linear on $\boldsymbol{x}$. Then, $\boldsymbol{x}$ is the best response strategy to $\boldsymbol{Y}$ if and only if there is a ν such that

$$\alpha^1\gamma_i^1 w_S(y_i^1) + \alpha^2\gamma_i^2 w_S(y_i^2) \begin{cases} = \nu, & x_i > 0, \\ \le \nu, & x_i = 0. \end{cases} \tag{47}$$

Thus, in particular, $\nu > 0$, since $\boldsymbol{x}$, $\boldsymbol{y}^1$ and $\boldsymbol{y}^2$ are probability vectors.

Note that $u_I^{PT,t}(\boldsymbol{x}, \boldsymbol{y}^t)$ is linear on $\boldsymbol{y}^t$. Thus, $\boldsymbol{y}^t$ is the best response strategy to $\boldsymbol{x}$ if and only if there is an ω^t such that

$$\gamma_i^t w_I(x_i) \begin{cases} = \omega^t, & y_i^t > 0, \\ \geq \omega^t, & y_i^t = 0, \end{cases} \tag{48}$$

where $t = 1, 2$. Thus, in particular, $\omega^1 > 0$ and $\omega^2 > 0$, since $\boldsymbol{x}$, $\boldsymbol{y}^1$ and $\boldsymbol{y}^2$ are probability vectors.

Assuming that there is an i such that $x_i = 0$ (48) implies that $\omega^t = 0$ for $t = 1, 2$. Thus, since $\boldsymbol{y}^1 = 0$ and $\boldsymbol{y}^2$ are probability vectors, (48) implies that $\boldsymbol{y}^1 \equiv \boldsymbol{y}^2 \equiv 0$. This contradiction yields that $x_i > 0$ and

$$\alpha^1 \gamma_i^1 w_S(y_i^1) + \alpha^2 \gamma_i^2 w_S(y_i^2) = \nu \text{ for } i = 1, \dots, n. \tag{49}$$

While, by (47), the assumption that there is an i such that $y_i^1 = 0$ and $y_i^2 = 0$ yields that $x_i = 0$. This contradiction implies that there is no i such that $y_i^1 = 0$ and $y_i^2 = 0$. Then, by (47) and (48) we have that

$$y_i^1 = \begin{cases} w_S^{-1}\left(\dfrac{\nu}{\alpha^1 \gamma_i^1}\right), & i \in I_{10}, \\ \alpha^1 \gamma_i^1 w_S(y_i^1) + \alpha^2 \gamma_i^2 w_S(y_i^2) = \nu, & i \in I_{11}, \\ 0, & i \in I_{01}, \end{cases} \tag{50}$$

$$y_i^2 = \begin{cases} 0, & i \in I_{10}, \\ \alpha^1 \gamma_i^1 w_S(y_i^1) + \alpha^2 \gamma_i^2 w_S(y_i^2) = \nu, & i \in I_{11}, \\ w_S^{-1}\left(\dfrac{\nu}{\alpha^2 \gamma_i^2}\right), & i \in I_{01}, \end{cases} \tag{51}$$

where

$$\begin{aligned} I_{10} &:= \{i : y_i^1 > 0, y_i^2 = 0\}, \\ I_{11} &:= \{i : y_i^1 > 0, y_i^2 > 0\}, \\ I_{01} &:= \{i : y_i^1 = 0, y_i^2 > 0\}. \end{aligned} \tag{52}$$

By (48), we have that:

(a) if $i \in I_{10}$ then

$$\gamma_i^1 w_I(x_i) = \omega^1 \tag{53}$$

and

$$\gamma_i^2 w_I(x_i) \geq \omega^2 \tag{54}$$

Dividing (54) by (53) implies

$$\gamma_i^2 / \gamma_i^1 \geq \omega^2 / \omega^1 \text{ for } i \in I_{10}. \tag{55}$$

(b) if $i \in I_{01}$ then

$$\gamma_i^1 w_I(x_i) \geq \omega^1 \tag{56}$$

and

$$\gamma_i^2 w_I(x_i) = \omega^2. \tag{57}$$

Dividing (56) by (57) implies

$$\omega^2/\omega^1 \geq \gamma_i^2/\gamma_i^1 \text{ for } i \in I_{01}. \tag{58}$$

(c) if $i \in I_{11}$ then

$$\gamma_i^1 w_I(x_i) = \omega^1 \tag{59}$$

and

$$\gamma_i^2 w_I(x_i) = \omega^2. \tag{60}$$

Dividing (60) by (59) implies

$$\omega^2/\omega^1 = \gamma_i^2/\gamma_i^1 \text{ for } i \in I_{11}. \tag{61}$$

Then, by (59), (60), (61) and assumption (27), there exists a unique i such that

$$I_{11} = \{i\}, I_{10} = \{1, \dots, i-1\} \text{ and } I_{01} = \{i+1, \dots, n\}.$$

Then, by (48),

$$x_j = x_j(\omega_1, \omega_2) = \begin{cases} w_I^{-1}\left(\dfrac{\omega^1}{\gamma_j^1}\right), & j \leq i-1, \\ w_I^{-1}\left(\dfrac{\omega^1}{\gamma_i^1}\right) = w_I^{-1}\left(\dfrac{\omega^2}{\gamma_i^2}\right), & j = i, \\ w_I^{-1}\left(\dfrac{\omega^2}{\gamma_i^2}\right), & j \geq i+1. \end{cases} \tag{62}$$

Let $\omega = \omega^1$. Then, by (61), $\omega^2 = \gamma_i^2 \omega/\gamma_i^1$. Thus, by (62), $\boldsymbol{x}$ has to be given by (41) with i instead of i_*. Since $\boldsymbol{x}$ is probability vector, then ω is uniquely defined by (42).

Next, we have to prove that $i = i_*$ and find ν. By (50) and (51), taking into account notations (29) and (30), and the fact that $\boldsymbol{y}^1$ and $\boldsymbol{y}^2$ are probability vectors, we have that i and ν are given by the following conditions:

$$\Phi_i(\nu) = \nu, \tag{63}$$

$$\underline{\Psi}_i(\nu) \leq 1 \tag{64}$$

and

$$\overline{\Psi}_i(\nu) \leq 1. \tag{65}$$

Note that

$$\Phi_i(\nu) - \nu \text{ is continuously decreasing for } \nu \in [0, \overline{\nu}_i]. \tag{66}$$

Moreover,

$$(\Phi_i(\nu) - \nu)\Big|_{\nu=0} = \alpha^1\gamma_i^1 + \alpha^2\gamma_i^2 > 0, \tag{67}$$

Thus, by (66) and (67), Eq. (63) has the root (and it is unique) in $[0, \overline{\nu}_i]$ if and only if

$$(\Phi_i(\nu) - \nu)\Big|_{\nu=\overline{\nu}_i} \le 0. \tag{68}$$

To derive the necessary and sufficient conditions for (68) to hold, let us consider three cases separately: (i) $\nu_i^1 = \nu_i^2$, (ii) $\overline{\nu}_i = \nu_i^1 < \nu_i^2$ and (iii) $\overline{\nu}_i = \nu_i^2 < \nu_i^1$.

(i) Let $\nu_i^1 = \nu_i^2$. Then,

$$(\Phi_i(\nu) - \nu)\Big|_{\nu=\nu_i^1} = -\nu_i^1 = -\nu_i^2 < 0. \tag{69}$$

Thus, in case (i), (68) always holds. Moreover, by Proposition 1(b) and Proposition 1(d), (64) and (65) hold for each $\nu \le \nu_i^1 = \nu_i^2$.

(ii) Let $\overline{\nu}_i = \nu_i^1 < \nu_i^2$. Then,

$$(\Phi_i(\nu) - \nu)\Big|_{\nu=\nu_i^1} = \alpha^2\gamma_i^2 w_S \left(1 - \sum_{j=i+1}^{n} w_S^{-1}\left(\frac{\nu_i^1}{\alpha^2\gamma_j^2}\right)\right) - \nu_i^1, \tag{70}$$

Thus, (68) with $\nu = \nu_i^1$ is equivalent to

$$1 \le \sum_{j=i}^{n} w_S^{-1}\left(\frac{\nu_i^1}{\alpha^2\gamma_j^2}\right), \tag{71}$$

Moreover, taking into account notation (30) and combining (71) with the fact that (64) and (65) have to hold, we have that the following condition must hold

$$\overline{\Psi}_i(\nu_i^1) < 1 \le \overline{\Psi}_{i-1}(\nu_i^1) \text{ with } \nu_i^1 < \nu_i^2 \tag{72}$$

By Proposition 1(d), (72) is equivalent to

$$\nu_{i-1}^2 < \nu_i^1 < \nu_i^2. \tag{73}$$

(iii) Let $\overline{\nu}_i = \nu_i^2 < \nu_i^1$. Then,

$$(\Phi_i(\nu) - \nu)\Big|_{\nu=\nu_i^2} = \alpha^1\gamma_i^1 w_S \left(1 - \sum_{j=1}^{i-1} w_S^{-1}\left(\frac{\nu_i^2}{\alpha^1\gamma_j^1}\right)\right) - \nu_i^2. \tag{74}$$

Thus, (68) with $\nu = \nu_i^2$ equivalent to

$$1 \le \sum_{j=1}^{i} w_S^{-1}\left(\frac{\nu_i^2}{\alpha^1\gamma_j^1}\right). \tag{75}$$

Moreover, taking into account notation (30) and combining (71) with the fact that (64) and (65) must hold, we have that the following condition has to hold

$$\underline{\Psi}_i(\nu_i^2) \le 1 < \underline{\Psi}_{i+1}(\nu_i^2) \text{ with } \nu_i^2 < \nu_i^1. \tag{76}$$

By Proposition 1(b), (76) is equivalent to

$$\nu_{i+1}^1 < \nu_i^2 < \nu_i^1 \tag{77}$$

Then, the result follows from (73), (77) and Proposition 4. ■

Of course, the scenario with rational rivals is a boundary case for subjective behavior with $\kappa = 1$. Namely, for rational rivals the following result holds.

Corollary 2. *In the Bayesian game with a rational Scanner and rational Invader, the equilibrium strategies* $(\boldsymbol{x}, \boldsymbol{Y})$ *are unique and given as follows:*

$$x_i = \frac{1}{\sum_{j \le i_*} \frac{1}{\gamma_j^1} + \sum_{j \ge i_*+1} \frac{\gamma_{i*}^2}{\gamma_{i*}^1 \gamma_j^2}} \begin{cases} \frac{1}{\gamma_i^1}, & j \le i_*, \\ \frac{\gamma_{i*}^2}{\gamma_{i*}^1 \gamma_i^2}, & j \ge i_* + 1, \end{cases} \tag{78}$$

and

$$y_i^1 = \begin{cases} \frac{\nu}{\alpha^1 \gamma_i^1}, & i \le i_* - 1, \\ 1 - \sum_{j=1}^{i_*-1} \frac{\nu}{\alpha^1 \gamma_j^1}, & i = i_*, \\ 0, & i \ge i_* + 1, \end{cases} \tag{79}$$

$$y_i^2 = \begin{cases} 0, & i \le i_* - 1, \\ 1 - \sum_{j=i_*+1}^{n} \frac{\nu}{\alpha^2 \gamma_j^2}, & i = i_*, \\ \frac{\nu}{\alpha^2 \gamma_i^2}, & i \ge i_* + 1, \end{cases} \tag{80}$$

where

$$\nu = \frac{\alpha^1 \gamma_{i*}^1 + \alpha^2 \gamma_{i*}^2}{1 + \sum_{j \le i_*-1} \frac{\gamma_{i*}^1}{\gamma_j^1} + \sum_{j \ge i_*+1} \frac{\gamma_{i*}^2}{\gamma_j^2}} \tag{81}$$

and i_* *is given by Proposition 4 with*

$$\nu_i^1 = \frac{\alpha^1}{\sum_{j \le i-1} (1/\gamma_j^1)} \text{ and } \nu_i^2 = \frac{\alpha^2}{\sum_{j \ge i+1} (1/\gamma_j^2)}. \tag{82}$$

Proof. For rational rivals $w_I(p) \equiv w_S(p) \equiv p$. Thus, by (29) and (30),

$$\underline{\Psi}_i(\nu) = \sum_{j=1}^{i-1} \frac{\nu}{\alpha^1 \gamma_j^1} \text{ and } \overline{\Psi}_i(\nu) = \sum_{j=i+1}^{n} \frac{\nu}{\alpha^2 \gamma_j^2}. \tag{83}$$

This, jointly with (32) and (33), implies (82).
By

$$x_j = x_j(\omega) = \omega \times \begin{cases} \dfrac{1}{\gamma_j^1}, & j \le i_*, \\ \dfrac{\gamma_{i*}^2}{\gamma_{i*}^1 \gamma_j^2}, & j \ge i_* + 1. \end{cases} \tag{84}$$

Then, taking into account that $\boldsymbol{x}$ is a probability vector we obtain (78). Finally, by (85)

$$\Phi_i(\nu) = \alpha^1 \gamma_i^1 \left(1 - \sum_{j=1}^{i-1} \frac{\nu}{\alpha^1 \gamma_j^1}\right) + \alpha^2 \gamma_i^2 \left(1 - \sum_{j=i+1}^{n} \frac{\nu}{\alpha^2 \gamma_j^2}\right). \tag{85}$$

Substituting this $\Phi_i(\nu)$ into (45), and solving this equation we obtain (81), and the result follows. ■

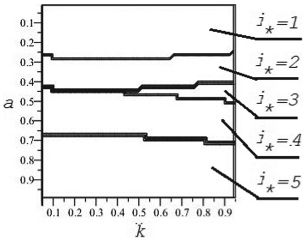

Fig. 3. Switching band i_* as function on a priori probability and probability weighting parameter κ.

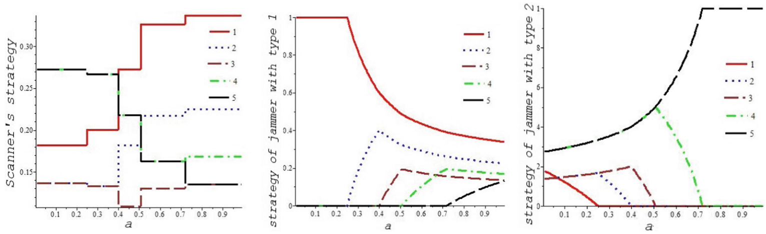

Fig. 4. The user's strategy (left), strategy of type-1 jammer (middle) and (c) strategy of type-2 jammer (right) for $\kappa = 1$.

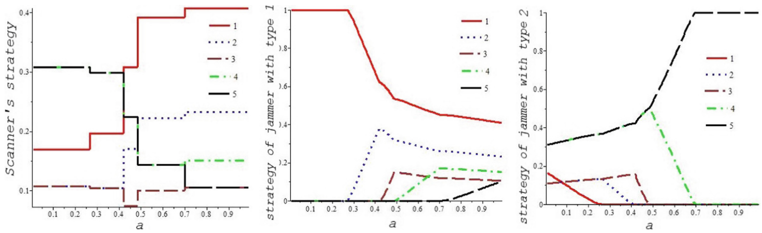

Fig. 5. The user's strategy (left), strategy of type-1 jammer (middle) and (c) strategy of type-2 jammer (right) for $\kappa = 0.75$.

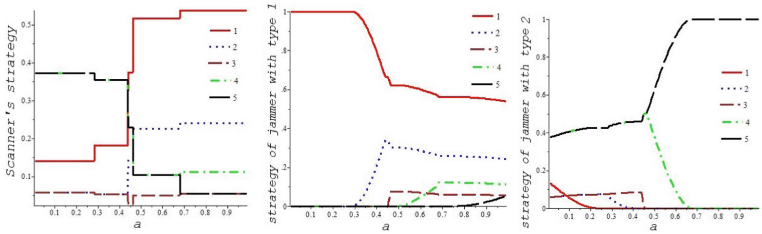

Fig. 6. The user's strategy (left), strategy of type-1 jammer (middle) and (c) strategy of type-2 jammer (right) for $\kappa = 0.5$.

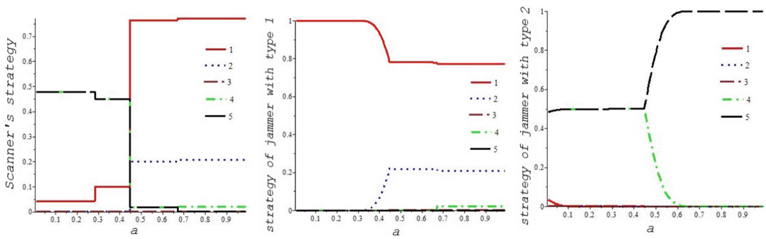

Fig. 7. The user's strategy (left), strategy of type-1 jammer (middle) and (c) strategy of type-2 jammer (right) for $\kappa = 0.25$.

Let us illustrate the obtained result by a bandwidth scanning example consisting of $n = 5$ bands, where

$$\gamma^1 = (0.2, 0.3, 0.5, 0.4, 0.5) \text{ and } \gamma^2 = (0.3, 0.4, 0.4, 0.2, 0.2).$$

Figure 3 illustrates the dependence of the switching band i_*, i.e., the band where both types of Invader sneak through with positive probability, on a priori probability $\alpha^1 = \alpha$ (thus, $\alpha^2 = 1 - \alpha$) and probability weighting parameter κ.

Figures 4, 5, 6 and 7 illustrate the dependence of the equilibrium PT strategy of the Scanner and the Invader on the a priori probability α and probability weighting parameter $\kappa \in \{0.25, 0.5, 0.75, 1\}$. The Scanner's equilibrium strategy has water-filling form (41) with water-filling equation (42). Each band is scanned with positive probability although such probabilities could tend to zero as the probability weighting parameter decreases. By (41), the Scanner's strategy $\boldsymbol{x}$ is piecewise constant with respect to a priori probability, while by (43) and (44), the Invader's strategies $\boldsymbol{y}^1$ and $\boldsymbol{y}^2$ are continuous with respect to a priori probability. Thus, due to such piecewise constant structure, the Scanner strategy is less sensitive to a priori probabilities compared with the Invader's strategy, except for a finite set S of a priori probabilities where the Scanner strategy has jump discontinuities. Within this finite set S, the Scanner strategy is over-sensitive to a priori probability (which is reflected by the corresponding jumps) to compensate for the lack of such sensitivity outside of the set S. Finally, the Scanner's strategy and Invader's strategies are continuous with respect to the probability weighting parameter and, in this sense, they are equally sensitive to the subjective factor reflected by the probability weighting parameter.

9 Conclusions

In this paper we have investigated the impact of subjectivity on the rival's behavior in a bandwidth scanning problem. Namely, we have formulated a prospect theoretic extension of a Bayesian game between a Scanner and an Invader where the Scanner knows only a priori probabilities about what type of intrusion (regular intensity or low intensity) occurs in bandwidth. Existence and uniqueness of the PT Bayesian equilibrium is proven. Moreover, these PT Bayesian equilibrium strategies are derived in closed form as functions of the detection probabilities. Waterfilling equations were found that allows one to derive these detection probabilities. In particular, the waterfilling equations provide a means to identify the bands where the Invader's strategies have band-sharing form, and to establish equal sensitivity of the Scanner strategy and the Invader strategy to the subjective factor reflected by the probability weighting parameter. It is worth remarking that, the rival strategies can have different sensitivity with respect to a priori probabilities about the intrusion type. Namely, the Scanner strategy is piecewise constant while the Invader's strategy is continuous with respect to such a priori probability. Thus, the Scanner strategy combines non-sensitive behavior (when it is constant) with over sensitive behavior (when it has jumps). The goal of our future research is to generalize the obtained result for more general cases regarding the intrusion types.

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