Combined Turbo Coding and Non-Coherent Space-Time Modulation in Rapid Rayleigh Fading Channels

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***Abstract*—Unitary space-time modulation (USTM) can perform well without channel state information (CSI) in fast fading channels when signal-to-noise ratio (SNR) is large enough. However, it loses its advantages at low SNR or the channel coherence interval is $T = 1$. In this condition, the information should be carried by multilevel structured constellations with a single transmit antenna. Even though, the bit error rate (BER) of this kind of signalling scheme is still too high to accept. In this paper, we propose a simple non-coherent space-time modulation scheme, combine it with turbo coding, and derive the corresponding decoding algorithm. Simulation results show that it can greatly improve the system performance. With multiple receive antennas, the BER can be further decreased.**

I. INTRODUCTION

The development of wireless communication systems are driven by high reliability applications. Fading is traditionally regarded as a nuisance by the designers of wireless communications systems. Exploiting space diversity by using multiple antennas at the transmitter and the receiver in wireless communication systems has been researched by different approaches.

Typical space-time codes [1 - 4] fit for slowly fading channels, where the fading coefficients between the transmitter antennas and the receive antennas remain approximately constant for many symbol intervals, therefore the transmitter can send training signals that allow the receiver to accurately estimate the fading coefficients. However, in fast fading circumstances, fading coefficients change into new and independent values before being learned by the receiver through training signals. Unitary space-time modulation was proposed fitting for conditions where the channel coherence interval T and the number of transmit antennas M satisfy $T \geq 2M$, and the SNR ρ is large enough [5], [6].

However, at low SNR, or for small values of coherence interval, the unitary constellations can not provide an acceptable performance. Based on the Kullback-Leibler (KL) distance between conditional distributions, in [7], the authors proposed an abstruse constellation scheme for low SNR and rapid fading scenarios [8]. The BER of this multilevel non-coherent space-time constellation scheme alone is too high to accept, it must be combined with channel coding to be put into use.

In this paper, we propose a simple multilevel non-coherent space-time modulation (NCSTM) scheme and then combine it with turbo coding in rapid Rayleigh fading channels. We present a simple iterative decoding technique that suits for the system, and we show by simulations that when the fading coefficients change fast and the SNR is low, turbo-coded multilevel NCST scheme can give a significantly better performance compared with no channel coding versions.

II. COMBINED TURBO CODES WITH NON-COHERENT SPACE-TIME MODULATION

A. *Non-coherent Space-Time System Description*

When the fading coefficients keep approximately constant for a long time so as the CSI can be obtained by training signals, the typical space-time codes [1 - 4] can provide either multiplex gains or diversity gains. If T no less than $2M$, and the SNR ρ is large enough, unitary space-time modulation, which uses the phases of the elements of the transmitted matrices to carry information, can provide a fairly good performance. However, when $T = 1$ and in low SNR scenarios, USTM can not be used any longer. In this condition, the transmit power should be concentrated on one transmit antenna and the information should be represented by the magnitude of the carriers, but not only the phases [9]. They further mentioned that USTM can be regarded as a special case of their space-time signaling schemes.

Throughout this paper, we will assume that $T = R = 1$ and so there are only two signal points in the constellation, *i.e.*, $L = 2^{TR} = 2$. As the information is carried by transmitted signals through the form of $\boldsymbol{X}\boldsymbol{X}^{\dagger}$ where "$\dagger$" denotes the conjugate transpose, there is no gain by employing more transmit antennas than T [10]. Therefore, we also assume that $M = 1$. With this assumption, the transmitted matrix is simply a complex scalar. The probability density of the received symbol $\boldsymbol{Y}$ conditioned on the transmitted symbol can be represented as

$$p(\boldsymbol{Y}|x) = \frac{1}{\pi^N (1+|x|^2)^N} \cdot \exp\left(-\frac{\| \boldsymbol{Y} \|_{\mathrm{F}}^2}{1+|x|^2}\right) \qquad (1)$$

where $\| \cdot \|_{\mathrm{F}}$ denotes the Frobenius norm operation. The transmitted symbol can be denoted by $x = \sqrt{2} \cdot b$, where $b(0) = 0$ and $b(1) = 1$. The numbers in the brackets denote the indices of the symbols in constellation. From (1), we can derive the maximum likelihood (ML) detection by

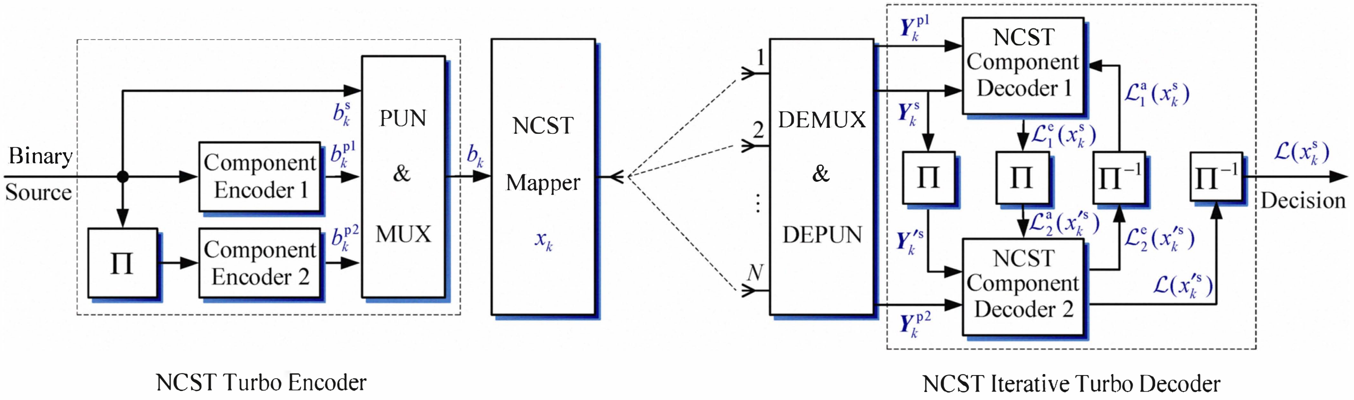

Fig. 1. Block diagram of encoding and decoding scheme for Turbo coding with non-coherent multilevel space-time modulation

$$
\begin{aligned}
x_{\mathrm{ML}} =& \arg \max_{l\in\{0,1\}} p[\boldsymbol{Y}|x(l)] \\
=& \arg \min_{l\in\{0,1\}} \left\{ N \cdot \ln\left[1+|x(l)|^2\right] + \frac{||\boldsymbol{Y}||_{\mathrm{F}}^2}{1+|x(l)|^2} \right\}
\end{aligned} \tag{2}
$$

B. Encoding

The block diagram of the proposed transmitter is given in the left part of Fig. 1. The data is partitioned into blocks of K bits to be encoded by a binary turbo encoder. The turbo encoder consists of two recursive, systematic convolutional (RSC) encoders, concatenated in parallel via a turbo interleaver. The puncturer (noted as PUN) deletes some of the parity bits, then we can obtain different spectral efficiencies. After multiplexing (noted as MUX), each symbol b_k is composed with both the systematic bit and the parity bits. For example, if without puncturing, b_k can be represented as $b_k = \{b_k^{\mathrm{s}}, b_k^{\mathrm{p1}}, b_k^{\mathrm{p2}}\}$. At the non-coherent space-time modulator, it would be mapped into x_k.

C. Decoding

The receiver is equipped with N antennas. The received signals are first demultiplexed by the demultiplexer (De-MUX). If the symbols are punctured, the depuncturer (De-PNC) set the initial values of the corresponding received matrices with all zero elements. Then the received symbols $\boldsymbol{Y}^{\mathrm{s}}$ and $\boldsymbol{Y}^{\mathrm{p1}}$ are imported into component decoder 1. After interleaving by the interleaver (noted as "Π") whose permutation factor is the same as the one at the transmitter, we get $\boldsymbol{Y'}^{\mathrm{s}}$, and $\boldsymbol{Y'}^{\mathrm{s}}$ and $\boldsymbol{Y}^{\mathrm{p2}}$ are imported into component decoder 2.

In order to decode the received signal, each NCST component decoder computes the log-likelihood ratios (LLR) of transmitted bits and use them as if they are the LLR of the observations from a pulse magnitude modulation (PAM) over an AWGN channel. Clearly, this is a suboptimal decoding algorithm. With each received NCST symbol we can get the LLR for each transmitted bits

$$
\mathcal{L}(x_k^{\mathrm{a}}) = \ln\left[\frac{\Pr(x_k=1|\boldsymbol{Y}_k)}{\Pr(x_k=0|\boldsymbol{Y}_k)}\right] = \ln\left[\frac{\Pr(x_k=1,\boldsymbol{Y}_k)}{\Pr(x_k=0,\boldsymbol{Y}_k)}\right] \tag{3}
$$

Assuming that all the constellation points are with equiprobabilities, from (1), we have

$$
\begin{aligned}
\mathcal{L}(x_k^{\mathrm{a}}) &= \ln\left\{\frac{\Pr[\boldsymbol{Y}_k|x_k=x(1)]}{\Pr[\boldsymbol{Y}_k|x_k=x(0)]}\right\} \\
&= \ln\left[\frac{\frac{1}{\pi^N}\cdot\exp\left(-\left\{N\cdot\ln[1+|x(1)|^2]+\frac{||Y_k||_{\mathrm{F}}^2}{1+|x(1)|^2}\right\}\right)}{\frac{1}{\pi^N}\cdot\exp\left(-\left\{N\cdot\ln[1+|x(0)|^2]+\frac{||Y_k||_{\mathrm{F}}^2}{1+|x(0)|^2}\right\}\right)}\right]
\end{aligned} \tag{4}
$$

In binary case, because of $x(0) = 0$ and $x(1) = \sqrt{2}$, we have

$$
\mathcal{L}(x_k^{\mathrm{a}}) = \frac{2}{3}\cdot \parallel \boldsymbol{Y}_k \parallel_{\mathrm{F}}^2 - \ln(3)\cdot N \tag{5}
$$

With the NCST demodulated LLRs, we can perform iterative decoding using the classical turbo decoding algorithms. The likelihoods in (3) for each bit are for component decoder 1 of the first decoding iteration, assuming that the *a priori* information of all possible the symbols in the constellation are all the same. During the first iteration, after interleaving, the extrinsic information $\mathcal{L}^{\mathrm{e}}(x^{\mathrm{s}})$ of component decoder 1 are used as the *a priori* information $\mathcal{L}^{\mathrm{a}}(x'^{\mathrm{s}})$ for component decoder 2. Therefore, the LLR calculation except for component decoder 1 of the first iteration should be performed by

$$
\mathcal{L}(x_k^{\mathrm{a}}) = \ln\left[\frac{\Pr(\boldsymbol{Y}_k|x_k=1)\cdot\Pr(x_k=1)}{\Pr(\boldsymbol{Y}_k|x_k=0)\cdot\Pr(x_k=0)}\right] \tag{6}
$$

Similarly, the extrinsic information produced by component decoder 2 are used as the *a priori* information for component decoder 1 through the deinterleaver ("Π^{-1}").

III. SIMULATION RESULTS AND PERFORMANCE ANALYSIS

In this section, we present several examples of turbo coded noncoherent space-time modulation scheme for multiple antennas and illustrate their performance via simulations. As coherent modulation and USTM do not suit for $T = 1$ condition, we can not compare the BER performance of the proposed scheme with others. The turbo code consists of two

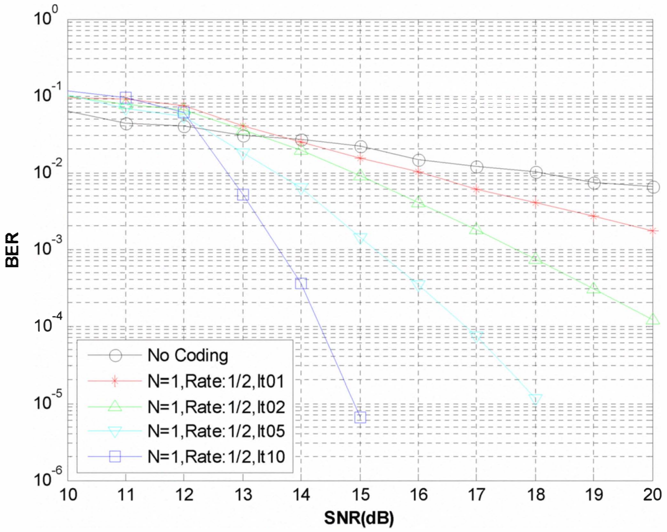

Fig. 2. BER performance of the turbo coded noncoherent space-time modulation with one transmit antenna and one receive antenna where the code rate is $1/2$.

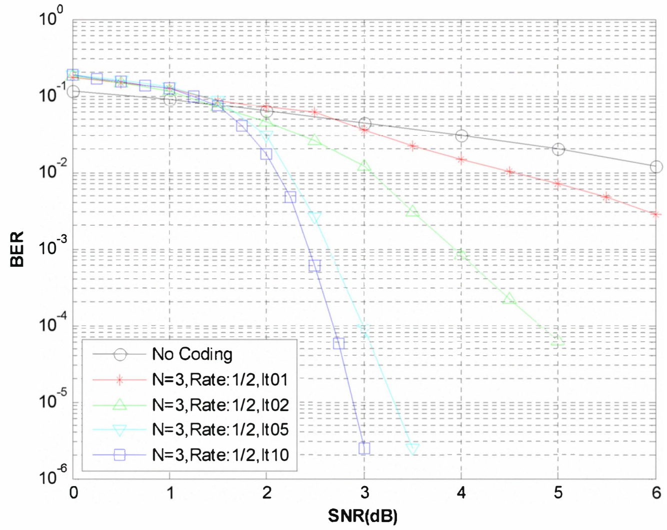

Fig. 3. BER performance of the turbo coded noncoherent space-time modulation with one transmit antenna and three receive antenna where the code rate is $1/2$.

feedback systematic convolutional codes described by (g_1, g_2), where g_1 and g_2 are polynomials respectively. It uses a pseudorandom interleaver, and the parity bits are punctured to obtain desired bandwidth efficiency. We choose the generating polynomials as $(g_1, g_2) = (7, 5)_8$. For a fair comparison, the SNR is defined as the SNR divided by the code rate. As our aim is to show the effectivity of the proposed combination scheme, we only use the Max-Log-MAP decoding algorithm because it needs the smallest calculation amount compared with other candidates.

When $T = 1$, both the typical space-time codes and USTM can not be used any more. As the multilevel signaling scheme fits for low SNR, increasing the SNR independently to decrease the BER is not a promising method. In fig. 2, we compare the performance of the turbo-coded modulation system with the uncoded one (*i.e.*, multilevel noncoherent space-time modulation alone) with single transmit antenna and single receive antenna where the code rate is $1/2$. The most attractive feature of turbo code is its iterative processing to further improve the BER performance. Although the BER of the first decoding iteration is also not acceptable, we can use more iterations to improve the performance. With 5 iterations, the system can give a BER of 10^{-5} at 18 dB. With 10 iterations, it can further improve the performance by 3 dB. Because the multilevel NCST system is suitable for up links, multi antennas at the receiver are feasible. Fig. 3 shows that we only need a no more than 3 dB SNR to get a 10^{-5} BER.

IV. CONCLUSION

In this paper, we proposed a combination scheme of turbo codes and multilevel non-coherent space-time modulation for low SNR and rapid Rayleigh fading scenarios. We presented a simple iterative decoding algorithm that is suitable for the system, and we showed by simulations that even when USTM loses its feasibility, turbo-coded multilevel non-coherent space-time modulated signal can significantly improve the system performance with iterative decoding.

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