Precoding and Power Allocation in Cognitive Radio Networks

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Abstract—**Cognitive Radio (CR) has attracted much attention for its ability allowing secondary users (SUs) to share the spectrum with licensed users (PUs). The precondition that PUs bear the coexistence of SUs is that the interference caused by SUs should be under a given threshold. Therefore the main challenge of CR is interference control or even interference cancellation. In this work we employ zero-forcing precoding at the base station of the secondary network to ensure no interference from SUs to PUs. On the other hand, to maximize the sum rate of the secondary network, optimal power allocation strategy is performed at the secondary base station. The uncoded bit error rate (BER) performance of the SUs and PUs with the optimal power allocation strategy are investigated and compared with that when employing average power assignment. Simulation results show that the proposed precoding scheme completely eliminates the interference from SUs to PUs and the optimal power allocation strategy enhances the sum rate of secondary network considerably.**

I. INTRODUCTION

Radio spectrum is limited resource for wireless communication. With the rapid development of various wireless networks, fixed spectrum assignment policy is not suitable any longer because the spectrum usage is concentrated on certain portions of the spectrum and most of the assigned spectrum is underutilized [1]. Cognitive radio (CR) proposed in 2000 [2] has attracted much attention in recent years for its innovation of spectrum sharing. In CR, the secondary users (SUs) can coexist with the primary users (PUs) by spectrum sharing. The PUs are licensed users who have the primary right to access the spectrum while the SUs can occupy the spectrum in a dynamic way to ensure they do not interrupt the communications of PUs. Therefore, the main challenge of CR lies in how to ensure the quality-of-service (QoS) of PUs while maximizing the sum rate of SUs.

Applying multiple antennas at the base station yields significant improvement either in channel capacity or in reliability of data transmission through spatial multiplexing with no additional power or bandwidth cost [3], [4]. The advantages of applying multiple antennas at transmitter and/or receiver have been well investigated. However, due to the power and cost limitations, the use of multiple antennas on the mobile stations may not be practical. In our study, we assume the base stations of the secondary network and primary network are equipped with multiple antennas while the SUs and PUs each has a single antenna.

Precoding and power allocation are two approaches to improve the link performance by appropriately controlling multiuser interference. Transmit precoding mitigates the distortion caused by channel fading, combats the multiple access interference [5], [6]. In [7], four linear precoding schemes are studied for the downlink of multiple-input single-output (MISO) CR system. It deals with one SU and one PU scenario. Joint beamforming and power allocation for multiple access channels in CR networks is studied in [8]. It studies the sum rate maximization and SINR balancing problems for the uplink of CR system under interference constraints for the PUs. For the uplink of CR system with multiple SUs and multiple PUs, joint power control and beamforming to minimize the total transmit power of the CR system is studied in [9], the received interferences at PUs are ensured to remain below a threshold while the interference plus noise ratio (SINR) requirements of SUs are guaranteed.

Comparing with previous work on CR system, in this paper, we consider the downlink of a CR system with multiple SUs and multiple PUs. The base stations are equipped with multiple antennas while the users each has a single antenna. Precoding is preformed at the base station of the secondary network to maximize the received signal power at the SUs while ensuring no interference to the PUs. To design precoders and allocate power, the base station should know the channel state information (CSI). In this study, we assume perfect CSI is perfectly known at the base stations of secondary network and primary network. We employ a precoding scheme at the base station of secondary network to eliminate the interference from SUs to PUs. On the other hand, to maximize the sum rate of secondary network, optimal power allocation is performed for the secondary network under total transmit power constraint. We study the uncoded bit error rate (BER) of SUs and PUs and the sum rate performance of the secondary network.

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Simulation results show that the proposed schemes maximize the sum rate of secondary network while completely remove the interference from SUs to PUs.

Throughout this paper, matrices and column vectors are denoted in bold upper and lower case letters, respectively. $\mathrm{E}[\cdot]$, $\mathrm{Tr}(\cdot)$, $(\cdot)^{\mathrm{H}}$, $(\cdot)^{\mathrm{T}}$, $|\cdot|$ and $\|\cdot\|_2$ denote the expectation operator, trace of a matrix, complex conjugate transpose, transpose, absolute value and Euclidian norm, respectively. $\mathbf{I}_m$ denotes the $m \times m$ identity matrix.

II. SYSTEM MODEL

Consider the downlink of a CR system which consists of a secondary network and a primary network as shown in Fig.1. In the secondary network, a base station and K unlicensed users operate in a frequency band allocated to M primary users. Since the PUs and SUs operate in the same spectrum band, the channels are inherently interference channels. We assume that the base stations of the secondary network and the primary network have N_{s} and N_{p} transmit antennas respectively, $N_{\mathrm{s}} \geq M$, and the SUs and PUs each has a single antenna. Thus both the secondary network and the primary network are MISO systems. Denote $\boldsymbol{h}_k \in \mathbb{C}^{1\times N_{\mathrm{s}}}$, $k \in [1, K]$, and $\boldsymbol{g}_m \in \mathbb{C}^{1\times N_{\mathrm{s}}}$, $m \in [1, M]$, as channel gains between the secondary base station and the kth SU and that between the secondary base station and the mth PU, respectively. The channel between the primary base station and the mth PU and that between the primary base station and the kth SU are represented by $\hat{\boldsymbol{h}}_m \in \mathbb{C}^{1\times N_{\mathrm{P}}}$ and $\hat{\boldsymbol{g}}_k \in \mathbb{C}^{1\times N_{\mathrm{P}}}$, respectively. We also assume block fading channels. The received signals at the SU_k and PU_m are written as

$$y_k = \boldsymbol{h}_k \sum_{i=1}^{K} \sqrt{p_i}\boldsymbol{v}_i s_i + \hat{\boldsymbol{g}}_k \sum_{j=1}^{M} \sqrt{\hat{p}_j}\hat{\boldsymbol{v}}_j \hat{s}_j + n_k, \tag{1}$$

$$\hat{y}_m = \hat{\boldsymbol{h}}_m \sum_{j=1}^{M} \sqrt{\hat{p}_j}\hat{\boldsymbol{v}}_j \hat{s}_j + \boldsymbol{g}_m \sum_{i=1}^{K} \sqrt{p_i}\boldsymbol{v}_i s_i + \hat{n}_m, \tag{2}$$

respectively. p_i and $\hat{p}_j$ are the allocated power for SU_i and PU_j, respectively. $\boldsymbol{v}_i \in \mathbb{C}^{N_{\mathrm{s}}\times 1}$ and $\hat{\boldsymbol{v}}_j \in \mathbb{C}^{N_{\mathrm{P}}\times 1}$ is the precoding vectors of the ith SU and jth PU respectively. s_i and $\hat{s}_j$ denote the transmitted signals to SU_i and PU_j, respectively. n_k and $\hat{n}_m$ is independent, identically distributed (i.i.d.) gaussian noise with zero mean and variance σ_k^2 and $\hat{\sigma}_m^2$, respectively. Therefore, the first terms in the above two equations are the desired signals and multiuser interference from the same network, the second terms are the co-channel interference from another network.

Without loss of generality, we assume the transmitted symbols are independent with zero mean and unit variance. The precoding vectors are normalized, i.e.,

$$\mathrm{E}\left[\|\boldsymbol{v}_k s_k\|_2^2\right] = \mathrm{Tr}\left(\boldsymbol{v}_k \boldsymbol{v}_k^{\mathrm{H}}\right) \mathrm{E}\left[|s_k|^2\right] = 1, \tag{3}$$

$$\mathrm{E}\left[\|\hat{\boldsymbol{v}}_m \hat{s}_m\|_2^2\right] = \mathrm{Tr}\left(\hat{\boldsymbol{v}}_m \hat{\boldsymbol{v}}_m^{\mathrm{H}}\right) \mathrm{E}\left[|\hat{s}_m|^2\right] = 1. \tag{4}$$

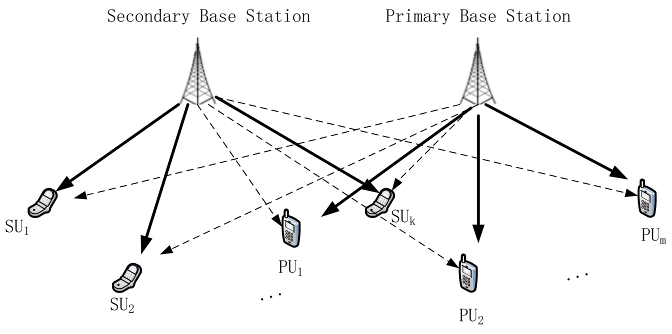

Fig. 1. The cognitive radio system model. Solid line denotes transmission channel and dotted line represents interference channel.

Thus, the SINR for the kth SU can be written as

$$\mathrm{SINR}_k = \frac{p_k |\boldsymbol{h}_k \boldsymbol{v}_k|^2}{\sum\limits_{i=1, i\neq k}^{K} p_i |\boldsymbol{h}_k \boldsymbol{v}_i|^2 + \sum\limits_{m=1}^{M} \hat{p}_m |\hat{\boldsymbol{g}}_k \hat{\boldsymbol{v}}_m|^2 + \sigma_k^2}.$$

III. PRECODING DESIGN

In the considered CR system, the secondary network coexists with the primary network. Therefore, the interference powers received by PUs from SUs should be below certain thresholds which are usually dependent on the QoS of PUs. It is usually solved by controlling the transmission power of secondary network. This obviously will degrade the performance of the secondary network. But, if the interference introduced by SUs can be eliminated there will be no such kind of limitation on the power of the secondary networks. This can be done by employing a precoding scheme at the secondary base station. In this section, we derive the zero-forcing precoding vectors for SUs which maximize the received signal powers at SUs while guaranteeing no interference to PUs. The problem can be formulated as

$$\begin{aligned} \boldsymbol{v}_k = \quad & \arg\max_{\boldsymbol{v}_k} \quad |\boldsymbol{h}_k \boldsymbol{v}_k|^2, \ k = 1, \cdots, K && (5) \\ \text{s. t.} \quad & |\boldsymbol{g}_m \boldsymbol{v}_k|^2 = 0, \ m = 1, \cdots, M && \\ & \mathrm{Tr}(\boldsymbol{v}_k \boldsymbol{v}_k^{\mathrm{H}}) = 1. && (6) \end{aligned}$$

The above optimization problem is unfortunately nonconvex and can not be solved by convex optimization method. From the expressions of the formulated problem, we get the key point of it is to find a vector $\boldsymbol{v}_k$ which is orthogonal to the interference channels $\boldsymbol{g}_m$ while closest to the transmission channels $\boldsymbol{h}_k$. Therefore, we get the precoding vectors as

$$\boldsymbol{v}_k = \beta \left(\left(\mathbf{I}_{N_{\mathrm{s}}} - \boldsymbol{G}^{\mathrm{H}} \left(\boldsymbol{G}\boldsymbol{G}^{\mathrm{H}} \right)^{-1} \boldsymbol{G} \right) \boldsymbol{h}_k^{\mathrm{H}} \right), \tag{7}$$

where β is chosen to ensure $\|\boldsymbol{v}_k\|_2^2 = 1$. $\boldsymbol{G} \in \mathbb{C}^{M\times N_{\mathrm{s}}}$ is the subspace spanned by the interference channels $\boldsymbol{g}_m$, i.e. $\boldsymbol{G} = \mathrm{span}\{\boldsymbol{g}_1, \cdots, \boldsymbol{g}_M\}$.

By employing the precoding scheme at the secondary base station, the interferences from SUs to PUs could be completely removed, which will be confirmed by simulations.

IV. Power Allocation

In CR system, to guarantee the QoS of PUs, power allocation in the secondary network should be appropriately determined to optimize the performance metrics of SUs, such as sum rate or SINR. As analysis made in the last section, the interference has been eliminated, meaning that the power allocation scheme of secondary system will not affect the PUs. Therefore, the objective of power allocation scheme in secondary network is only to optimize the performance of itself. In this section, we will derive the optimal power allocation strategy to maximize the sum rate of secondary network under total transmit power constraint of secondary network. The sum rate of secondary network is described as $R = \sum\limits_{k=1}^{K} \log\left(1+\mathrm{SINR}_k\right)$. The power allocation optimization problem reads

$$\begin{aligned} p_k = \quad & \arg\max_{p_k} \quad R \\ \text{s. t.} \quad & \sum_{k=1}^{K} p_k \le E_{\mathrm{s}}, \\ & p_k \ge 0, \end{aligned} \tag{8}$$

where E_{s} is the total transmission power of the secondary network. The above optimization problem is a nonlinear nonconvex problem. It can be transformed into a convex optimization problem through a log transformation based on some assumptions. From the precoder design criteria, we get $\boldsymbol{h}_k\boldsymbol{v}_k\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{h}_k^{\mathrm{H}}$ is much larger than $\boldsymbol{h}_k\boldsymbol{v}_i\boldsymbol{v}_i^{\mathrm{H}}\boldsymbol{h}_k^{\mathrm{H}}$. On the other hand, we assume zero-forcing is employed by the primary system to avoid causing interference to the secondary system. Thus, SINR_k is much greater than 1. In this case, the sum rate R can be approximated as $R = \sum\limits_{k=1}^{K} \log\left(\mathrm{SINR}_k\right)$.

Let $\widetilde{p}_k = \log p_k$, we have

$$\begin{aligned} \widetilde{p}_k = \quad & \arg\max_{\widetilde{p}_k} \quad \sum_{k=1}^{K} \log\left(\frac{e^{\widetilde{p}_k} b_{kk}}{\sum\limits_{i=1,i\ne k}^{K} e^{\widetilde{p}_i} b_{ki} + \sigma_k^2}\right) \\ \text{s. t.} \quad & \sum_{k=1}^{K} e^{\widetilde{p}_k} \le E_{\mathrm{s}}. \end{aligned} \tag{9}$$

where $b_{kk} = \boldsymbol{h}_k\boldsymbol{v}_k\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{h}_k^{\mathrm{H}}$ and $b_{ki} = \boldsymbol{h}_k\boldsymbol{v}_i\boldsymbol{v}_i^{\mathrm{H}}\boldsymbol{h}_k^{\mathrm{H}}$. We can verify that the objective function in the above optimization problem is a concave function [10]. Then the globe optimal solution can be got by convex optimization method.

Constructing the Lagrangian function

$$\begin{aligned} L\left(\widetilde{p}_k,\ \lambda\right) = \quad & \sum_{k=1}^{K} \log\left(\frac{e^{\widetilde{p}_k} b_{kk}}{\sum\limits_{i=1,i\ne k}^{K} e^{\widetilde{p}_i} b_{ki} + \sigma_k^2}\right) \\ & -\lambda\left(\sum_{k=1}^{K} e^{\widetilde{p}_k} - E_{\mathrm{s}}\right), \end{aligned} \tag{10}$$

with lagrangian multiplier λ corresponding to the transmit power constraint, and taking the derivation with respect to $\widetilde{p}_k$, we have

$$\frac{\partial L\left(\widetilde{p}_k, \lambda\right)}{\partial \widetilde{p}_k} = 1/\ln 2\left(1 - \sum_{l=1,l\ne k}^{K} \frac{e^{\widetilde{p}_k} b_{lk}}{\sum\limits_{i=1,i\ne l}^{K} e^{\widetilde{p}_i} b_{li} + \sigma_l^2}\right) - \lambda e^{\widetilde{p}_k},$$

where $b_{lk} = \boldsymbol{h}_l\boldsymbol{v}_k\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{h}_l^{\mathrm{H}}$ and $b_{li} = \boldsymbol{h}_l\boldsymbol{v}_i\boldsymbol{v}_i^{\mathrm{H}}\boldsymbol{h}_l^{\mathrm{H}}$.

Then, it is easy to verify that the derivation of $L\left(p_k,\ \lambda\right)$ with respect to p_k is

$$\frac{\partial L\left(p_k, \lambda\right)}{\partial p_k} = 1/\ln 2\left(1/p_k - \sum_{l=1,l\ne k}^{K} \frac{b_{lk}}{\sum\limits_{i=1,i\ne l}^{K} p_i b_{li} + \sigma_l^2}\right) - \lambda, \tag{11}$$

where $L\left(p_k,\ \lambda\right)$ is the Lagrangian function of the original optimization problem, i.e.

$$L\left(p_k,\ \lambda\right) = \sum_{k=1}^{K} \log\left(\frac{p_k b_{kk}}{\sum\limits_{i=1,i\ne k}^{K} p_i b_{ki} + \sigma_k^2}\right) - \lambda\left(\sum_{k=1}^{K} p_k - E_{\mathrm{s}}\right).$$

Setting (11) to zero, the optimal p_k is derived as

$$p_k^* = 1/\left(\ln 2\lambda + \sum_{l=1,l\ne k}^{K} \frac{b_{lk}}{\sum\limits_{i=1,i\ne l}^{K} p_i b_{li} + \sigma_l^2}\right). \tag{12}$$

For the multiplier λ, it can be got by solving the dual problem of (9), i.e.

$$\min_{\lambda} \max_{p_k} \sum_{k=1}^{K} \log\left(\frac{e^{\widetilde{p}_k} b_{kk}}{\sum\limits_{i=1,i\ne k}^{K} e^{\widetilde{p}_i} b_{ki} + \sigma_k^2}\right) - \lambda\left(\sum_{k=1}^{K} e^{\widetilde{p}_k} - E_{\mathrm{s}}\right). \tag{13}$$

This dual problem can be efficiently solved by subgradient method [11] which updates the dual variable λ with step size $\delta(t)$

$$\lambda(t+1) = \left[\lambda(t) + \delta(t)\left(\sum_{k=1}^{K} p_k(t) - E_{\mathrm{s}}\right)\right]^{+}, \tag{14}$$

where t is the iteration time. Since the problem in (9) is a convex optimization problem, solving the dual problem is equivalent to solving the problem in (9).

The algorithm for solving the dual problem to obtain the optimal power allocation strategy for the secondary network is summarized as follows

Algorithm I: The subgradient iteration algorithm for (13)

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• Initialization:
    $\lambda(0) > 0,\ p_k(0) = E_{\mathrm{s}}/K;$
• Repeat: At each iteration $t,\ t = 0, 1, 2, \cdots$
  1) Obtain the optimal power solution based on (12)
      $p_k^*(t+1) = 1\Big/\left(\ln 2\lambda(t) + \sum_{l=1, l\neq k}^{K} \frac{b_{lk}}{\sum_{i=1, i\neq l}^{K} p_i(t) b_{li} + \sigma_l^2}\right);$
  2) Update $\lambda$ in the following way
      $\lambda(t+1) = \lambda(t) + \delta(t)\left(\sum_{k=1}^{K} p_k(t) - E_{\mathrm{s}}\right);$
  3) $t = t + 1$.
• Until: Required precision is satisfied.
• Output: $p_k^*$.
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By employing the precoders derived in section III and the optimal power allocation strategy at the base station of secondary network, we maximize the sum rate of secondary network while introducing no interference to the primary network.

V. Computer Simulations

Simulations are conducted to evaluate the performance of the proposed schemes. Each channel coefficient composes of both small scale fading and path loss components. The small scale fading is assumed to be Rayleigh distributed with zero mean and unit variance and the pass loss exponent is 4. Thus, the channel model is $\mu(l/l_0)^2\boldsymbol{a}$, where μ is a constant, l is the distance from user to base station, l_0 is the reference distance and $\boldsymbol{a}$ is a vector whose elements is Rayleigh distributed with zero mean and unit variance. Assume the primary network is aware of the existence of the secondary network and the zero-forcing scheme is employed by the primary network to avoid causing interference to the secondary network. The numbers of antennas at the base stations of primary network and secondary network are both equal to 4. All results are obtained by taking the average of results over 10000 channel realizations. We assume the transmitter knows the perfect instantaneous channel state information.

Fig. 2 illustrates the average BER performance of the primary network with transmission power $E_{\mathrm{p}} = 5$ dB, 15 dB versus transmission power of the secondary network for different number of SUs. It is shown that with the increase of transmission power of primary network, the BER of PUs decreases as expected. On the other hand, the BER performance of primary network is not affected by the communications of the secondary network, which remains unchanged as the transmission power of secondary network increases or the number of SUs increases. It means that by employing the precoding scheme at the base station of secondary network, the communications of SUs do not cause any interference to PUs.

In Fig. 3, the BER performance of secondary network with different number of PUs versus its transmission power is presented. From this figure, we observe that the BER performance of the secondary network is not acceptable. Take $E_{\mathrm{s}} = 20$ dB for the number of PUs when $M = 1$ as an example, the average BER of SUs is about 0.12. That is because the precoders at the base station of secondary network eliminate only the interference form SUs to PUs, but not the multiuser interferences among SUs within the secondary network. It is shown that as the number of PUs increases, the BER performance of secondary system decreases. That's because the precoders employed try to form orthogonal transmission channels between the secondary base station and those PUs, which will lead to a decrease of received signal power at SUs especially when the number of PUs getting large. This thereby results in a BER performance degradation of secondary networks.

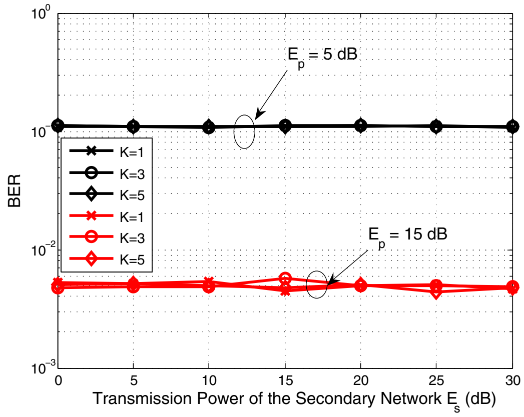

Fig. 2. BER of the primary network versus transmission power of the secondary network. K is the number of SUs and $\mathrm{E_p}$ is transmission power of the primary network. The number of PUs $M = 3$.

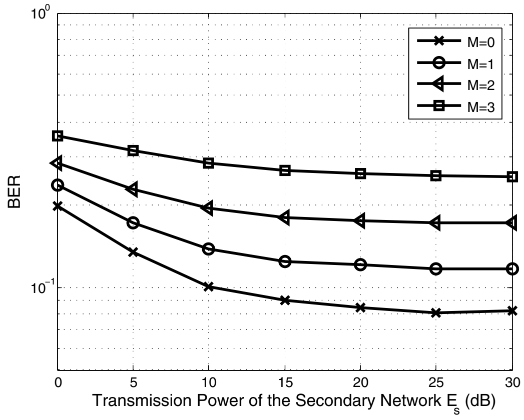

Fig. 3. BER of the secondary network versus transmission power of the secondary network. M is the number of PUs. The number of SUs $K = 3$.

The sum rate of secondary network with different number of PUs versus transmission power of secondary network is presented in Fig. 4. We compare the performance of the proposed optimal power allocation strategy with that of the average power transmission. In the figure, the optimal power allocation strategy and average power transmission in labels are shorten to be “O” and “A”, respectively. The sum rate of

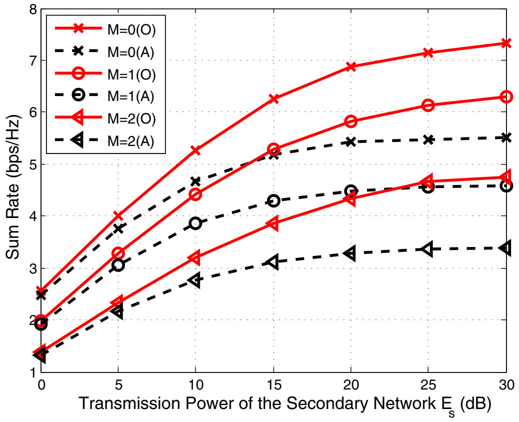

Fig. 4. Sum rate of the secondary network versus transmission power of the secondary network. M is the number of PUs. The number of SUs $K = 3$.

secondary network decreases as the number of PUs increases. The optimal power allocation strategy outperforms average power transmission for all cases. At the number of PUs $M = 1$, $E_{\mathrm{s}} = 20$dB, the sum rate of secondary network with optimal power allocation strategy is improved about 1.35bps/Hz comparing with that of average power allocation strategy. From the figure, we conclude that the optimal power allocation strategy maximize the sum rate of secondary network for all scenarios with different number of PUs.

VI. CONCLUSION

In this paper, we have studied the problem of precoding and power allocation for the secondary network in a CR system. There are multiple PUs and multiple SUs in the CR system. The base stations of the primary network and secondary network are equipped with multiple antennas and each user has a single antenna. To eliminate the interference from SUs to PUs, we employ the zero-forcing precoding scheme at the base station of secondary network. At the same time, optimal power allocation strategy is performed to maximize the sum rate of secondary network. Simulation results have shown that our proposed schemes improve the sum rate of the secondary network greatly while guaranteeing no interference to PUs.

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