

# Cooperative sensor solution to enhancing the performance of spectrum sensing

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**Abstract-** Cognitive radio (CR) is a promising technology for future wireless networks. In order to reduce the interference to primary system, CR users have to detect the spectra with low overhead and reliably in sensing slots, which has to occupy part of the available resources. In this paper, we propose a novel spectrum sensing solution that is based on cooperative wireless sensors to detect and collect available channels information and then establish a regional database managing regional spectrum information. We classify wireless sensors into relay wireless sensors (RWSs) and ordinary wireless sensors (OWSs), considering the diversity of multi-sensors and Cooperative Amplify-and-Forward (CAF) spectrum detection and CRC-based Decode-and-Forward (CDF) cooperative result reporting. Soften hard combination mechanism is proposed to further enhance the performance of spectrum sensing with one-bit overhead. If observed signals in OWSs fall in uncertain energy region, OWSs relay the signals to RWSs by CAF, otherwise, OWSs report the detection results to RWSs and then RWSs relay to Cognitive Radio Bastion (CR-BS) by CDF. Simulations show that spectrum sensing performance is improved by means of the two-layer sensing mechanism.

**Keywords-** Cognitive radio, Cooperative spectrum sensing, Cooperative communication, Wireless sensor, Spectrum information database

## I. INTRODUCTION

Cognitive radio (CR) enables much higher spectrum efficiency by dynamic spectrum access [1] and spectrum sharing [2]. Therefore, it is a promising technique for future wireless communications to mitigate the spectrum scarcity issue and to enhance spectrum usage. As unlicensed (secondary) users of the spectrum band, CR operators are allowed to utilize the spectral resources only when it does not cause interference to the primary (licensed) users, which entails continuous spectrum sensing in CR networks. As primary users with CR function, traditional mobile operators can share licensed spectral resource among their different radio access networks. Therefore, it becomes a critical issue for the application of cognitive radio to reliably and quickly detect the available spectra.

The existing spectrum sensing solutions mainly are based on CR users themselves to detect the presence or

absence of primary users. In fact, it is most important for CR cell system to know the information of spectra available within certain area and then to do intra-cell and inter-cell radio resource management. In order to correctly collecting the spectrum information, we need make use of spectrum sensing and detection fusion tools.

As to spectrum sensing techniques, energy detection, matched filter detection, and cyclostationary detection [3] are main-stream. Thereof, energy detection has been widely applied since it does not require any a priori knowledge of the detected signals and has much lower complexity than the other two schemes. When considering shadowing, fading, and time-varying natures of wireless channels, cooperative spectrum sensing schemes have been proposed to obtain the multi-user spatial diversity in CR networks [4], which combines local detection decisions from different CR users to make a final decision.

Hard combination and soft combination are considered in detection fusion tools. It needs only one-bit message regarding whether the observed energy is above a certain threshold for hard combination to feedback; more than one-bit for soft combination having significant performance improvement over the conventional hard combination, but with more complexity and overheads. Chair-Varshney criterion [5] is the optimal fusion rule for one-bit decision, which is naturally equivalent to the log likelihood ratio test. The combined detection probability is based on “AND” or “OR” combination. However, different local decisions have different confidences due to different detection and report channel gains. In general, Dempster-Shafer criterion [6] has better fusion performance, which makes a decision based on local detection result and detection credibility.

Conventional hard combination mechanism reduces the reliability of spectrum detection due to one-bit detection result; however, soft combination increases the overhead and reduces the reliability of spectrum reporting. In this paper, wireless sensors are deployed to conduct spectrum sensing based on two-layer energy detection with one-bit overhead to overcome these problems. The rest of this paper is organized as follows. In Section II, cluster-based and cooperative spectrum sensing is briefly introduced. The wireless sensor spectrum sensing solution is proposed in

Section III, where cooperative sensing solution including AF cooperative detecting and CDF cooperative reporting, and CR-BS fusion scheme are studied. Simulation results are shown and discussed in Section IV. Finally, we draw our conclusions in Section V.

## II. RELATED WORK

Cluster-based mechanism used in wireless sensor networks is introduced in cognitive radio. Cluster-based detection mechanisms are proposed in [7]-[9]. It applies cooperative sensing technology to the scenario that WiMAX system coexists with TD-SCDMA system in [7], trying to solve spectrum sharing problem. WiMAX users, that are secondary users in the overlap area of TD-SCDMA cell and WiMAX cell, are divided into several clusters. The user whose receiving SNR is best in a cluster is selected to be cluster-head. Only cluster-head senses spectra and then reports detection results to WiMAX base station. Finally, WiMAX BS makes a decision.

To enhance reliability of detection, soft data fusion mechanism is used in [7] [10]. There are three thresholds dividing the whole range of the observed energy into four regions. Every threshold is corresponding to one false alarm probability and detection probability.

If detection signal energy from primary system falls in region 1, region 2, the secondary user respectively reports "00", "01" to WiMAX BS, in region 3, region4 reporting "10", "11". WiMAX BS judges primary user is occupying the spectrum channel if receiving one "11" or L "10" or L<sup>2</sup> "01".

Clustering in WiMAX system is assumed to be finished and reporting overhead is two bits. The same assumption is in [8]-[9]. In [8], cluster-heads exchange local decisions each other to get the whole network sensing result. It is centralized and a common receiver is in charge of final decision in [9].

Another cooperative sensing scheme is proposed in [11], which uses relay technology to enhance spectrum sensing. If the signal from primary user is fading seriously, it will take more time for the secondary user to detect. One of solutions is to get better signal from other nodes. Pair of nodes amplifies and forward their own detection signal to each other, so resulting in spatial diversity to promote detection probability.

It can be seen that cluster-based and cooperative spectrum sensing schemes aforementioned separate spectrum detecting from detection result reporting, which results in the performance loss of entire spectrum sensing. In this paper, wireless sensors conduct two-layer spectrum sensing, integrating spectrum detecting based on CAF and detection result reporting based on CDF cooperative communication. Moreover, soften hard combination is proposed to further enhance the performance of spectrum sensing.

## III. COOPERATIVE WIRELESS SENSOR SPECTRUM SENSING

### A. System model

We deploy wireless sensors to conduct spectrum sensing based on spatial location. The coverage of wireless sensor is limited, so CR cell is divided into relay region and non-relay region. A wireless sensor that is deployed in relay region and meets the requirement of channel uncorrelated, denoted as a Relay Wireless Sensor (RWS), not only conducts spectrum sensing based on observed signals from Ordinary Wireless Sensors (OWSs) and Primary Users (PUs) but also is in charge of making data fusions. Based on RWSs, we can specify the OWSs required for the sensing system. Fig.1 shows the relay region is closer to OWSs.

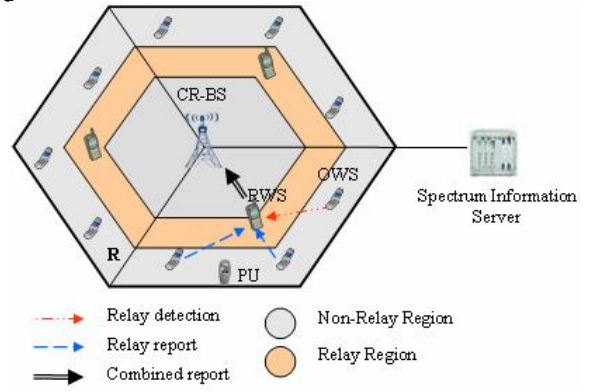


Figure 1. System model for wireless sensor sensing

In relay region, we can find some locations where average receiving Signal-to-Noise Ratio (SNR) of links to CR-BS is more perfect by networks-planning. RWSs are deployed in those locations; meantime, they are required to be not correlative.

OWSs deployed to sense spectrum also needs to meet their independences. What's more, to combat the impact of shadow fading, where to deploy RWSs and OWSs is specified by network-planning considering the features of common control channel and physiognomy in CR cell. The distance among RWSs, and that among OWSs are derived from channel coherence.

From channel correlative parameter [12],

$$p = \exp\left(-\frac{|\Delta d|}{d_{cor}} \ln 2\right) \quad (1)$$

We can get the independent distance, that is, deployment parameter D:

$$D = |\Delta d| = -\frac{d_{cor} \ln p}{\ln 2} \quad (2)$$

where  $\Delta d$  denotes the distance between two nodes;  $d_{cor}$  is the minimal distance that make nodes uncorrelated.

OWSs those are able to specify their local decision report their detection results to corresponding RWSs. Otherwise, OWSs relay their observed signals to the favorable RWSs by CAF.

A RWS Firstly decodes the detection datum from OWSs. If the RWS can not correctly decode certain detection data from a certain OWS, it discards the data. Otherwise, the RWS then make a relay-level decision based on the correct detection results.

If RWSs can make an independent detection, they report to CR-BS their local results and the relay-level decision by cooperative CRC-based DF mode; otherwise, they only relay the relay-level decision to CR-BS by CDF.

CR-BS makes a final decision and broadcasts its available spectrum information to the whole cell on common control channel and sends its available spectrum information to the spectrum information database.

In CDF protocol, Relay Station (RSs) need to decode the data received from Sources firstly and then forward to Destinations, so RSs should be closer to Sources, which can get higher decoding performance. This means RWSs should be closer to OWSs in our solution. On the contrary, RSs should be closer to Destinations in CAF protocol because it will result in serious noise amplification when RSs are closer to Sources. This also means RWSs should be closer to OWSs.

Therefore, our solution combines the advantages of CDF and CAF and can enhance the reliability of spectrum detecting and reporting.

### B. Soften hard detection fusion

To further enhance the reliability of detection and not increase reporting overhead of detection decision, soften hard detection fusion mechanism is proposed in this paper. As Fig.2 shows, observed signal energy range is divided into three energy regions by the two detection parameters  $\lambda_1$  and  $\lambda_2$ . When observed signal energy  $Y$  falls in region 2, wireless sensors are not able to make local detection decisions, and OWSs relay the observed signals to RWSs.

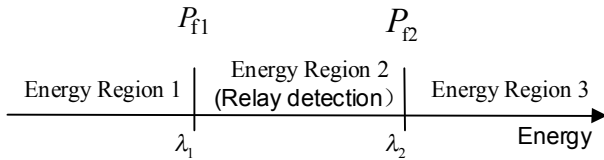


Figure 2. Soften hard detection fusion mechanism

When detection signal energy falls in region 1,  $Y < \lambda_1$ , which suggests that the spectrum is not occupied by primary users, say  $H_0$ , so reporting one bit “0”; when falling in region 3,  $Y > \lambda_2$ , say  $H_1$ , it reports one bit “1”, representing this spectrum is not available, shown in Eq.3.

$$Y = \begin{cases} E_s + E_n & H_1 \\ E_n & H_0 \end{cases} \begin{matrix} > \lambda_2 \\ < \lambda_1 \end{matrix} \quad (3)$$

where  $E_s$  and  $E_n$  are the signal energy of PUs and noise signal energy respectively.

We control the interference to primary system by means of configuring the minimum of detection probability,  $P_{d,Min}$ ; at the same time, we set the maximum of false alarm probability,  $P_{f,Max}$ , so as to make full use of spectra available. From  $P_{f,Max}$ , we can get detection parameter  $\lambda_1$  from [13]:

$$P_{f,Max} = P(Y > \lambda_1 | H_0) = \frac{1}{2} \operatorname{erfc} \left[ \frac{\lambda_1 - 2TW}{2\sqrt{2}\sqrt{TW}} \right] \quad (4)$$

$$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} \exp(-x^2) dx \quad (5)$$

Likewise, we can get detection parameter  $\lambda_2$  from (6):

$$P_{d,Min} = P(Y > \lambda_2 | H_1) = \int_0^{\infty} Q_m(\sqrt{2\gamma}, \sqrt{\lambda_2}) f(\gamma) d\gamma \quad (6)$$

where  $f(\gamma)$  denotes the distribution of SNR, which relies on the fading of signal.  $Q_m(\dots)$  is Marcum Q function:

$$Q_m(a, b) = \int_b^{\infty} \frac{x^m}{a^{m-1}} e^{-\frac{x^2+a^2}{2}} I_{m-1}(ax) dx \quad (7)$$

where  $I_{m-1}()$  is (m-1) rank modificative Bessel function; m is equal to TW.

### C. Two-layer Sensing

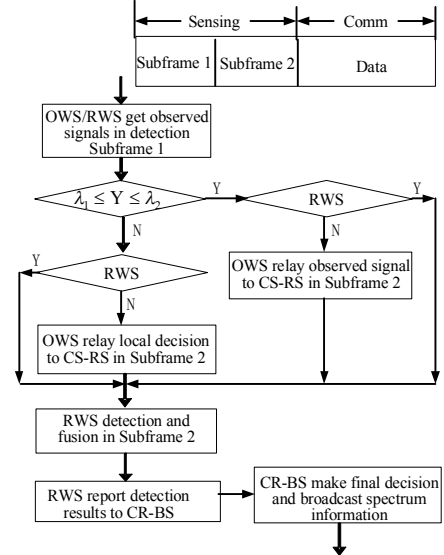


Figure 3. Two-layer Sensing

CR-BS divides a detection frame into two subframes, as shown in Fig.3. In detection subframe 1, OWSs and RWSs conduct spectrum detection and take corresponding action based on (3). If RWSs are not able to make the detection decision, RWSs will continue to detect in subframe 2, combining signals by Equal Gain Combining (EGC), including observed signals from OWSs and signals from PUs. The detection signal of a RWS is given by

$$y(t) = h_p s(t) + \sum_{i=1}^M g_i h_i h_{pi} s(t) + n(t) \quad (8)$$

where  $s(t)$  is the signal from PUs and  $n(t)$  is the noise item including noises from OWSs.  $M$  is the number of OWSs.  $h_p$ ,  $h_{pi}$  and  $h_i$  denote channel gains between the RWS and PUs, the  $i$ -th OWS and PUs, the RWS and the  $i$ -th OWS.

The scaling factor used by the  $i$ -th OWS to relay the observed signal to the RWS,  $g_i$ , is given by [14]:

$$g_i = \sqrt{\frac{P_i}{|h_{pi}|^2 |s(t)|^2 + N_0}} \quad (9)$$

where  $P_i$  is the transmitting power of the  $i$ -th OWS and  $N_0$  is noise power.

In this paper, the signal combination is based on EGC that is known to perform only slightly inferior to Maximal Ratio Combining (MRC) [15]. However, MRC requires the fading-channel gains at different links between RWSs and OWSs, PUs and OWSs to be estimated. Because the channel estimation in EGC is no longer required, it is in favor of energy saving of wireless sensors and then benefits to two-layer sensing.

The output of energy detector and detection decision in a RWS is presented by

$$Y = \left( |h_p|^2 + \sum_{i=1}^M g_i^2 |h_i h_{pi}|^2 \right) E_s + E_n \quad (10)$$

According to (4), (6) and (10), we can get the false alarm probability and the detection probability of a RWS:

$$P_{d,rws} = P(Y > \lambda_2 | H_1) = \int_0^{\infty} Q_{m(M+1)}(\sqrt{2\gamma}, \sqrt{\lambda_2}) f(\gamma) d\gamma \quad (11)$$

$$P_{f,rws} = P(Y > \lambda_1 | H_0) = \frac{1}{2} \operatorname{erfc} \left[ \frac{\lambda_1 - 2TW(M+1)}{2\sqrt{2}\sqrt{TW(M+1)}} \right] \quad (12)$$

where  $\gamma = \frac{1+M}{M+1} \frac{\pi}{4} \left( \gamma_p + \sum_{i=1}^M \frac{\gamma_i \gamma_{pi}}{\gamma_i + \gamma_{pi} + 1} \right)$ ,  $\gamma_p$ ,  $\gamma_{pi}$  and  $\gamma_i$  denote channel gains between the RWS and PUs, the  $i$ -th OWS and PUs, the RWS and the  $i$ -th OWS.

If the detection signal of the RWS falls in energy region 1 or 3, the RWS report the local detection result to CR-BS with detection results received from OWSs.

When CR-BS receives the data fusions from RWSs and OWSs, it makes a final decision based on “K out of N” rule, as (13):

$$B = \begin{cases} H_1, & \text{if } \sum_{i=1}^N S_i \geq K \\ H_0, & \text{otherwise} \end{cases} \quad (13)$$

where  $S_i$  denotes the reporting detection results from the two-layer spectrum sensing. If there are  $K$  “1” among  $N$  local decisions, final decision, denoting to  $B$ , is “1”. It means that the spectrum is not available in the CR cell, because there are  $K$  of  $N$  secondary users sensing the corresponding spectrum is occupied.

Finally, CR-BS broadcasts the detection results piggybacked in available spectrum information message or others system message on the common control channel to the whole cell.

#### IV. PERFORMANCE SIMULATION

In the simulation, we assume that primary users are randomly present. The CR links, including links between sensors and links between CR-BS and sensors, are characterized by Rayleigh fading distribution. The radius of the CR-CELL is 100 meters, and the number of Sensors is fixed to 30. The path loss exponent is set to 3. PU power is assumed to be 150mW. 50 samples are taken for energy detection, which is observed at Nyquist sampling rate. Gaussian noise power is set to -100dBW. In order to focus on the intrinsic benefits of two-layer architecture, a primary user is located around the center of the cell. In order to investigate effects of individual parameters in the proposed scheme, for individual sensing Sensors, the minimum of detection probability is fixed, and the maximum of false alarm probability takes different values to demonstrate the overall detection performance.

##### A. The impact of fading between sensors and PU

In this simulation, we evaluate the impacts of fading channels between Sensors and PUs on the performance of the proposed solution. The SNR is exponentially distributed in Rayleigh fading and logarithm normal distributed in Shadow fading, each of which is respectively given by

$$f_{Ray}(\gamma) = \frac{1}{\gamma} \exp \left( -\frac{\gamma}{\sigma^2} \right), \quad \gamma \geq 0 \quad (14)$$

$$f_{sh}(\gamma) = \frac{1}{\sqrt{2\pi\sigma\gamma}} \exp \left( -\frac{\ln^2 \gamma}{2\sigma^2} \right) \quad (15)$$

The detection probability at CR-BS is showed in Fig. 4, where  $P_d$  denotes the detection probability and  $P_f$  denotes the false-alarm probability.

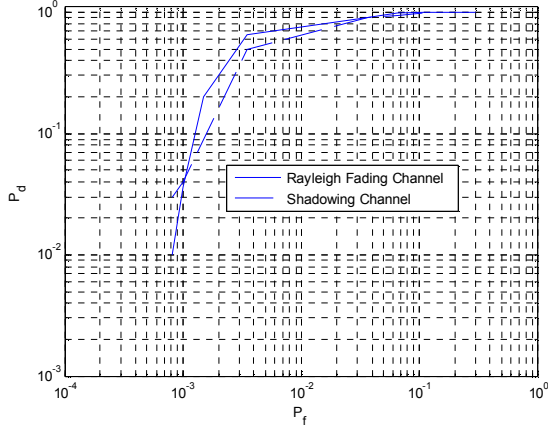


Figure 4. Detection performance in Rayleigh & Shadowing Fading

It can be seen that the proposed scheme has similar performance for wireless channels characterized by Rayleigh fading and shadow fading. Both scenarios have wide range of Receiver Operating Characteristic (ROC) Curve [16], so we can easily obtain the optimal point when designing spectrum access policy.

### B. The impact of AF Relay Parameters

Although RWSs have perfect channels to report detection results to CR-BS, RWSs can not guarantee their detections due to random presence of PUs. Fig.5 shows the impacts of CAF, where three cases are tested.

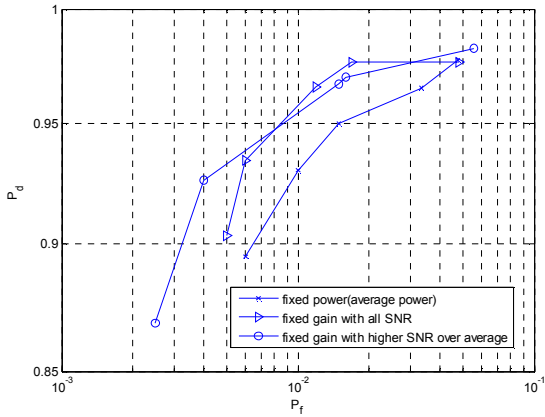


Figure 5. Impact of AF parameters

In Case 1, OWSs relay observed signals to RWSs with the same transmit power, indicating different amplification factors are used by each OWS. In Case 2, OWSs apply the same amplification factor for relaying. Case 3 is similar to Case 2, but only those OWSs having detected SNR above a

predefined threshold will relay the observed signals. It can be seen that the proposed scheme has relatively poor performance for Case 1, and the reason is that using the same transmit power for all nodes implies a greater weight for nodes with low detection SNR than that for nodes with high detection SNR. As a result, nodes with low detection SNR amplify the embedded noise to a great extent, leading to serious contamination of signals observed at RWSs. The performances for Case 2 and Case 3 are quite similar in the high detection probability zone (higher than 0.95 on the Y-axis). As the detection probability descends, the performance for Case 3 is better than that for Case 2, suggesting benefits brought by the use of threshold in determining whether to relay or not. In practice, the merits of use of threshold will be valuable, since it improves the detection performance while reducing the amount of information reported to RWSs.

### C. Comparison with cluster-based sensing

In the cluster-based sensing scheme [7], users with the highest detection SNR sense PUs, and report to CR-BS. For a fair comparison, similar situation is constructed for cluster-based sensing, where 6 CR users are uniformly distributed in the cell. Three well-separated users having highest detection SNR in their respective clusters are selected to be cluster-heads. The minimum distance ensuring un-correlation between cluster-headers is set to 50 meters. Cluster-heads take charge of sensing PUs, and report the two-bit results to CR-BS. The medium-to-low-rate and high-to-medium-rate [7] are set to 0.14 and 0.07, respectively. Wireless channels between cluster-heads and CR-BS are all characterized as Rayleigh fading, and transmit powers of all cluster-headers are assumed to be 10 watt. Other simulation parameters are the same as in [7].

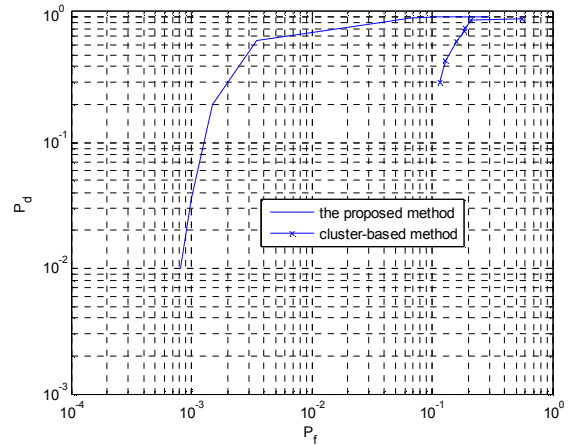


Figure 6. Performance Comparison with cluster-based sensing

The benefit that two-layer sensing scheme brings is shown in Fig.6 where the ranges of detection probability and false alarm probability are relatively small for the cluster-based scheme. Each point of the curve is determined by a triplet containing three thresholds, and the curve could be

interpreted as a search processing for an appropriate triplet of thresholds. The cluster-based scheme intends to give a good detection probability, but the tradeoff is that the false alarm probability is relatively high. On the contrary, the proposed scheme has a higher degree of freedom for trading off the detection probability against the false alarm probability. We can see the proposed scheme has better performance of the detection probability than that of the cluster-based scheme due to two-layer detection by CAF.

Even in the high detection probability area, the proposed scheme outperforms the cluster-based scheme in terms of the false alarm performance. This phenomenon could be explained by that fact that, firstly two-bit transmission is more likely to encounter error than one-bit and CDF in the proposed scheme, and secondly, the fusion rule taken in the cluster-based scheme is too conservative that it tends to declare the presence of primary users. The proposed scheme is much more flexible by providing a better tradeoff between the detection probability and the false alarm probability. In the proposed scheme, we can find a reasonable overall detection probability with a low false alarm probability, which suggests that the scheme can benefit limiting interference to the primary system as well as improving spectrum usage.

## V. CONCLUSIONS

Cognitive radio is an important technology that will be applied in the network of future, especially in future wireless network. The critical problems that include fast sensing and reliable report with lower overheads need to be solved in practice. In certain geometrical region, parts of spectra available should not change too dynamically. In the other hand, CR users have to have some prior and specific spectrum information to access the network. In this paper, we propose a novel sensing solution based on deploying wireless sensors to cooperatively set up quasi-static spectrum information database to enhance the performance of CR networks. Wireless sensor sensing solution can reduce sensing overhead by means of soften hard fusion mechanism and sensing period of CR users augment due to quasi-static spectrum information and enhance sensing reliability based on cooperative Amplify-and-Forward and CRC-based Decode-and-Forward. Simulations show that wireless sensor sensing scheme can improve detection performance compared with traditional cooperative sensing schemes.

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