The Repairability for Wireless Sensor Network Based on Surviving Edge

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***Abstract*— Wireless sensor network (WSN) is a self-organizing network which is composed of a large number of intelligent sensor nodes. Since the energy exhaustion of some nodes, the network topology is also changing with the disappearance of some sensor nodes, and even induces the disconnection. The system reliability of the network is also affected by the change of network topology. Although some sensor nodes are dead and impossible to be reactivated, the connectivity of the network can be repaired by adding some new nodes and continue to work in high reliability, this is so-called the repairabililty of WSN. This paper proposes an idea to redefine the reliability for repairability of disconnected network. A new reliability is proposed as the probability that the edge-induced subgraph is connected. Different from traditional all-terminal reliability, this new reliability measure focuses on residual edge connectedness and is able to distinguish the reliabilities of different tree topologies. Furthermore, we give the repairablilty measure by considering a reliability and disconnected network. The examples illustrate the effectiveness of the proposed measure and method.**

Keywords - reliabiity;repairablity; tree topology; edge failure

I. INTRODUCTION

Wireless sensor network is a self-organized network, which is formed by a group of tiny devices. The recent research on WSN has made great progress, especially, its wide-range applications, such as battlefield surveillance, habitat monitoring, biological detection, inventory tracking etc. [1]-[4]. With consideration of the malicious environment, it is impossible to repair or maintenance the sensor nodes. One of challenges in developing technique is the problem of maintaining data transmission and reliability. Amirhosein T., Majid T. and Mohsen S.[1] proposed the consideration of dependability, this is, a system is able to deliver services that can be trusted justifiably. Ordinarily the notion of dependability has six fundamental properties: (1) reliability, (2) availability, (3) safety, (4) confidentiality, (5) integrity and (6) maintainability. As to maintainability, it is impossible to have some nodes/products repaired for WSN in the malicious environment, but repairability can be understood by re-adding some new nodes and improving the connection of the network. However, traditional reliability faces to many problems. For example, a classical model of network reliability is based on an undirected graph where sensor nodes are perfect and edges operate with independent probability [5], [6]. Generally, two-terminal [7], K-terminal [8] and all-terminal reliability [9] are three different reliability measures in this model. Literature [10] proposed a reliability measure for WSN as the probability that there exists an operational path from at least one operational sensor in each target cluster to the sink node. But, for different tree topology, these three measures are not able to distinguish the difference of reliability very efficiently. For example, the all-terminal reliabilities of two trees are equal if the two trees are in the same size which means that the number of nodes and edges are the same. Hence, all-terminal reliability measure can not distinguish the reliabilities of trees in the same size. On the other hand, it is necessary to find the difference among trees and disconnected networks. Thus a new reliability measure needs to be studies so as to distinguish the reliabilities of different trees. Maintainability is used in designing the software and some special machine. Alf Inge Wang, Erik Arisholm present results form a quasi-experiment that investigates how the sequence in which maintenance tasks are performed affects the time required to perform them and the functional correctness of the changes made [11]. M.F. Wani, O.P. Gandhi focuses on those software development factors which may possibly affect software maintainability[12]. Maintainability of mechanical systems is modeled in terms of "Maintainability Attributes Digraph" which considers their maintainability attributes and degree of facilitation among one another [13]. Based on the concept of maintainability, we consider the repairability of the disconnected sensor network by adding a few sensors. The repairability measure of single node is defined as the probability of performing a successful repair action within a given time. In other words, maintainability measures the ease and speed with which a system can be restored to operational status after a failure occurs. We hope to measure the repaired different network system. Then the system repaired measure can help us to select the best way to repair.

This paper proposes an idea to redefine the reliability for repairability of disconnected network. A new reliability is proposed as the probability that the edge-induced subgraph is connected. Different from traditional all-terminal reliability, this new reliability measure focuses on residual edge connectedness and is able to distinguish the reliabilities of different tree topologies. Furthermore, we give the repairability measure by considering the reliability and disconnected network. The examples illustrate the effectiveness of the proposed measure and method.

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The rest of this paper is organized as follows. Section II gives the new definition of reliability and a proof of uniformly best tree. Section III gives the repairability measure of a new repaired network. Section IV shows the simulation results. Section V concludes the paper.

II. RELIABILITY MEASURE OF TREE AND DISCONNECTED NETWORK

Since wireless sensor networks are more fragile than the traditional network, maintenance is becoming more important. To keep high reliability of WSN, the repairability depends on the topology of the network and the reliability of the network after repairing. Hence, we consider the following simple topology of disconnected networks.

A. *Disconnected Network Model*

Wireless sensor network can be described mathematically as a graph $G = (V, E)$, where each node represents a sensor node. An edge exists if two nodes can communicate with each other directly. Let Ω be the set of disconnected network with n nodes and m edges. $R_{\text{all}}(T, p)$ denotes the all-terminal reliability, $T \in \Omega(n, m)$. Suppose that E' is a nonempty subset of E. $T[E']$ is the subgraph of T whose node set is the set of endpoints of edges in E', where edge set E' is called the subgraph of T induced by E'. $T[E']$ is an edge-induced subgraph of T [14].

For the convenience of the investigation, the assumptions are listed as follows:

1) The disconnected topology of sensor networks is known.

2) The nodes are perfectly reliable and the edges fail independently of each other with probability $1-p$.

We list the model of disconnected network and propositions directly from [15].

Definition 2.1 The reliability of disconnected network $T(n, m)$ is the probability that the edge-induced subgraph of T induced by surviving edges is connected, also named as residual edge connectedness reliability and denoted as $R(T, p)$. It can be written mathematically as follows:

$$R(T,p) = \sum_{i=1}^{m} E_i(T)p^i(1-p)^{m-i} \qquad (1)$$

where $E_i(T)$ is the number of connected edge-induced subgraphs of T that contain exactly i edges.

Proposition 2.2 For any disconnected graph $T \in \Omega(n, m)$, $E_1(T) = m$ and

$$E_2(T) = \sum_{v \in V} \binom{d(v)}{2},$$

where $d(v)$ is the degree of a node v in T.

Proposition 2.3 For disconnected graph T, if $R(T, p)$ denotes residual edge connectedness reliability, then we have $R_{\text{all}}(T, p) \le R(T, p)$.

B. *Comparison among Disconnected Networks*

Different from the all-terminal reliability, the residual edge connectedness reliability is able to distinguish the reliabilities of disconnected networks, such as two trees in the same class, $\Omega(n, m)$.

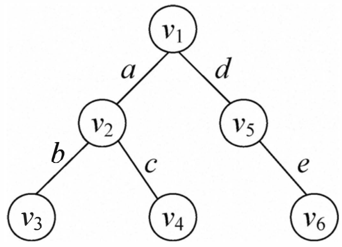

Figure1. Tree T_1

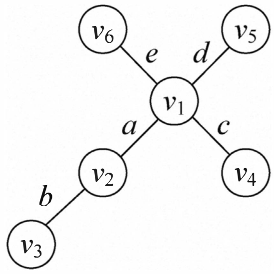

Figure 2. Tree T_2

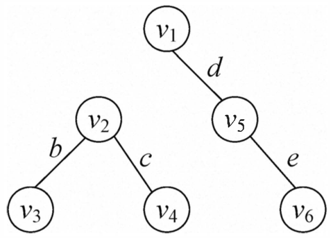

Figure 3. Disconnected graph G

Example 2.4 Two trees T_1 and T_2 are in the class $\Omega(6, 5)$ as Fig.1 and Fig.2. All-terminal reliability of tree T_1 is the same to that of tree T_2, i.e.

$$R_{\text{all}}(T_1, p) = R_{\text{all}}(T_2, p) = p^5$$

From Definition 2.1, the reliability of tree T_1 and T_2 are different. As to T_1, all connected edge-induced subgraphs of tree T_1 are a connected edge-induced subgraph, {*a*, *b*, *c*, *d*, *e*, *ab*, *ac*, *ad*, *bc*, *de*, *abc*, *abd*, *acd*, *ade*, *abcd*, *abde*, *acde*, *abcde*}, in Fig.1. And $E_1(T_1)=5$, $E_2(T_1)=5$, $E_3(T_1)=4$, $E_4(T_1)=3$ and $E_5(T_1)=1$.Thus, we have

$$R(T_1,p) = 5p^1(1-p)^4+5p^2(1-p)^3+4p^3(1-p)^2+3p^4(1-p)^1+1p^5(1-p)^0 \qquad (2)$$

Similarly, we have

$$R(T_2, p) = 5p^1(1-p)^4+7p^2(1-p)^3+7p^3(1-p)^2+4p^4(1-p)^1+1p^5(1-p)^0 \qquad (3)$$

Obviously, $R_{\text{all}}(T_1, p) < R(T_1, p)$, $R_{\text{all}}(T_2, p) < R(T_2, p)$, and

$R(T_2, p) - R(T_1, p) = 2p^2(1-p)^3+3p^3(1-p)^2+p^4(1-p)^1$.

Therefore tree T_2 is more reliable than tree T_1 based on residual edge connectedness reliability.

As to a disconnected network G as Fig 4, All-terminal reliability of G is zero. Since the connected edge-induced subgraphs of G are $\{b, c, d, e, bc, de\}$. So $E_1(G)=4$, $E_2(G)=2$. Thus, we have

$$R(G, p)=4p^1(1-p)^3+2p^2(1-p)^2 \quad (4)$$

The comparison shows that the new definition can well estimate the reliability of connected and disconnected networks.

III. REPAIRABILITY MEASURE FOR DISCONNECTED NETWORK

For the disconnected network, the important problem is how to repair and reach the ideal result of maintenance. Also the effectiveness and economy are also meaningful. Based on the above sections, for the corrected measure for repairing a sensor node depends on the reliability. The repairability is defined as the probability of restoring within a given time. For example, if it is said that a network has a 90% repairability in one hour, this means that there is a 90% probability that the network will be repaired within an hour. First we can add a few sensor nodes to make the network operate again. For the same disconnected network, the different position of nodes can make the network reliability different. Thus, repairability measure of system should consider through comparing the reliability of new repaired network and the reliability of chain.

In a connected network, the chain is the least reliability of network and every chain have the same reliability. So we consider the value that is given by subtracting the reliability of chain form the reliability of new repaired network in the same $\Omega(n, m)$.

Definition 3.1 The system repairability measure of disconnected network is the probability that the reliability of new repaired network G subtract the reliability of chain C in the same $\Omega(n, m)$. It is denoted as $M(G)$ and can be written mathematically as follows:

$$M(G)=R(G)-R(C) \quad (9)$$

Example 3.2 The disconnected network and the repaired network are shown in Fig. 4 and Fig.5.

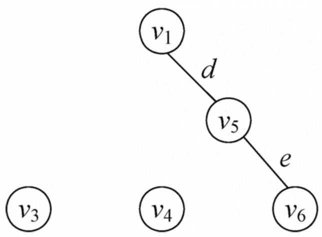

Figure 4. Disconnected network G

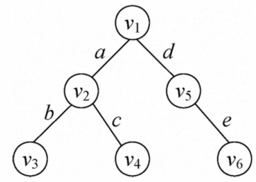

Figure 5. Repaired Network K

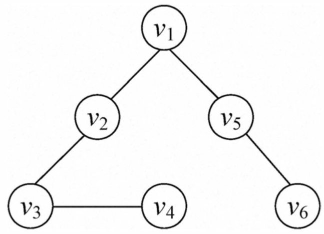

Figure 6. Chain C

From the Definition 2.1 and 3.1, we got the repairability measure by subtracting the reliability of chain C in Fig. 6

$R(K)=5p^1(1-p)^4+5p^2(1-p)^3+4p^3(1-p)^2+3p^4(1-p)^1+1p^5(1-p)^0$.

$R(C)= 5p^1(1-p)^4+4p^2(1-p)^3+3p^3(1-p)^2+2p^4(1-p)^1+1p^5(1-p)^0$.

$M(K)= R(K)- R(C)= p^2(1-p)^3+p^3(1-p)^2+p^4(1-p)^1$.

For $p = 0.9$, $M(K) = 0.0737$.

IV. SIMULATION

In order to compare repairability measures of new repaired connected networks, we give five different repaired networks. Let G_1, G_2, G_3, G_4 and G_5 be the repaired network by different senor nodes, respectively. And let C_1 be the chain in the same class with the repaired network.

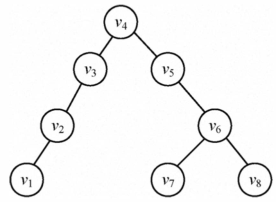

Figure 7. Repaired network G_1

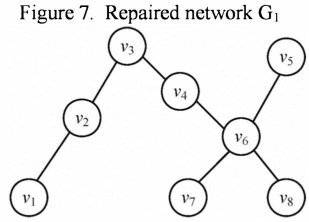

Figure 8. Repaired network G_2

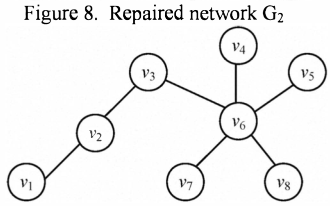

Figure 9. Repaired network G_3

From the Definition 2.1, we obtain

$R(G_1)=7p^1(1-p)^6+7p^2(1-p)^5+6p^3(1-p)^4+5p^4(1-p)^3+4p^5(1-p)^2+3p^6(1-p)^1+1p^7(1-p)^0$;

$R(G_2)=7p^1(1-p)^6+9p^2(1-p)^5+9p^3(1-p)^4+8p^4(1-p)^3+7p^5(1-p)^2+4p^6(1-p)^1+1p^7(1-p)^0$;

$R(G_3)=7p^1(1-p)^6+12p^2(1-p)^5+15p^3(1-p)^4+15p^4(1-p)^3+11p^5(1-p)^2+5p^6(1-p)^1+1p^7(1-p)^0$.

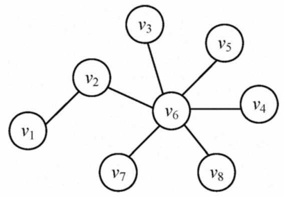

Figure 10. Repaired network G_4

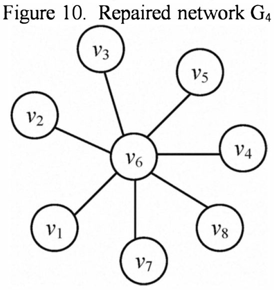

Figure 11. Repaired network G_5

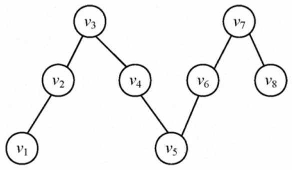

Figure 12. Chain C_1

$R(G_4)=7p^1(1-p)^6+16p^2(1-p)^5+25p^3(1-p)^4+25p^4(1-p)^3+16p^5(1-p)^2+6p^6(1-p)^1+1p^7(1-p)^0$;

$R(G_5)=7p^1(1-p)^6+21p^2(1-p)^5+35p^3(1-p)^4+35p^4(1-p)^3+21p^5(1-p)^2+7p^6(1-p)^1+1p^7(1-p)^0$;

$R(C_1)=7p^1(1-p)^6+6p^2(1-p)^5+5p^3(1-p)^4+4p^4(1-p)^3+3p^5(1-p)^2+2p^6(1-p)^1+1p^7(1-p)^0$.

From Definition 3.1, we have

$M(G_1)= R(G_1)- R(C_1)=$
$p^2(1-p)^5+p^3(1-p)^4+p^4(1-p)^3+p^5(1-p)^2+p^6(1-p)^1$;

$M(G_2)= R(G_2)- R(C_1)=$
$3p^2(1-p)^5+4p^3(1-p)^4+4p^4(1-p)^3+4p^5(1-p)^2+2p^6(1-p)^1$;

$M(G_3)= R(G_3)- R(C_1)=$
$6p^2(1-p)^5+10p^3(1-p)^4+11p^4(1-p)^3+8p^5(1-p)^2+3p^6(1-p)^1$;

$M(G_4)= R(G_4)- R(C_1)=$
$10p^2(1-p)^5+20p^3(1-p)^4+21p^4(1-p)^3+13p^5(1-p)^2+4p^6(1-p)^1$;

$M(G_5)= R(G_5)- R(C_1)=$
$15p^2(1-p)^5+30p^3(1-p)^4+31p^4(1-p)^3+18p^5(1-p)^2+5p^6(1-p)^1$.

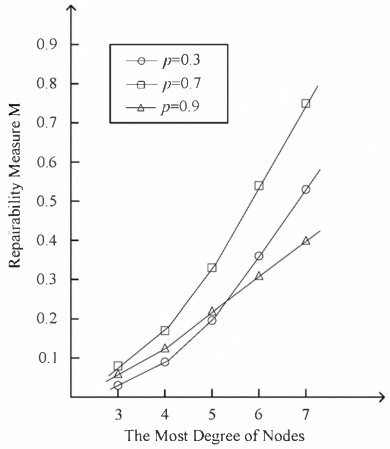

Figure 13. The repairability of different network

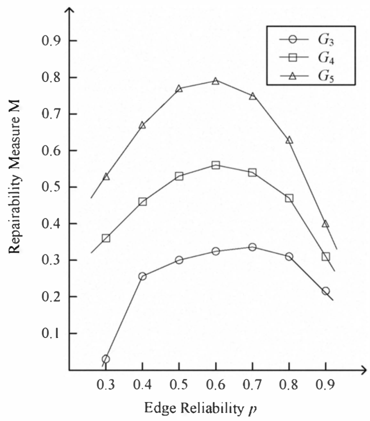

Figure 14. The repairability of different edge reliability

Fig. 13 is for the most degree of nodes and the repairablity. As the most degree of nodes increases in repaired networks G, the repairability measure $M(G)$ of repaired network G increases. Fig. 14 shows the repairability measure change from different reliability p. In a repaired network, as p increases, the repairability measure is not always increases. The repairability measure will reduces when the p reaches the fixed value, because the repairability measure increases initially and then decreases. The repairability value reaches the most value at interval (0.4, 0.7) of p.

Let G_1 be a disconnected network in Fig.15. Then we add sensor nodes to repair the network. And we give two different repaired ways to compare the reliability of new networks in Fig. 16 and Fig. 17, respectively. In the first way, we add three sensor nodes to connect the network as Fig 16. Then H_1 is becoming a connected tree H_2. On the other hand, we only add one sensor node to repair the network as shown in Fig 17. The reliabilities of repaired two graphs are different. We can see the relations in the following Table I.

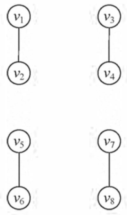

Figure 15. Disconnected graph H_1

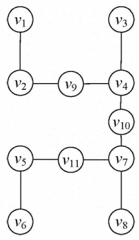

Figure 16. Repaired graph H_2

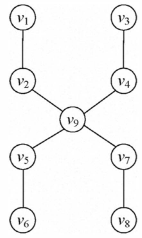

Figure 17. Repaired graph H_3

From Table 1, to add more nodes can not make the reliability of reparied graph higher. Thus the key node is very important for the disconnected network, sometimes.

Table I. Comparison of different repaired networks

P	$R(H_1)$	$R(H_2)$	$R(H_3)$
0.9	0.0036	0.5403	0.6731
0.7	0.0756	0.1613	0.3475

Note: $H_i(i$=1,2,3) is disconnected graphs

V. CONCLUSION

This paper considers the repairability of WSN by proposing a new reliability of networks, named residual edge connectedness reliability. The proposed reliability is able to distinguish different trees in the same size and reconsidered to use for the repairability measure. Examples illustrate the effectiveness of the new reliability definition and the effectiveness of repairability measure. The concept of reliability and repairability based on residual edge connectedness may be applied to the WSN design in the future.

ACKNOWLEDGMENT

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