

Reducing Crosspoints on Multistage Switching by Using Batcher Banyan Switches

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Abstract: Multistage switching is generally built by using crossbar switches in according to Clos architecture which has small blocking probabilities. However, number of cross points increases significantly to numbers of input output. This paper analyses the implementation of Batcher Banyan configuration to multistage switching system to reduce number of cross points. As Batcher Banyan sorts input to output ports, besides reducing cross points, it avoids collision among its input output. The Clos crossbar multistage switching requires up to 1,560,576 cross points with 6.9×10^{-17} blocking probability, but Batcher Banyan needs only 294,912 cross points, 429.17% lower, with zero blocking probability for the same numbers of input output.

Keywords: Clos Multistage switching, Crossbar switches, Batcher Banyan.

1. Introduction

Switch routes packets from sources to destinations. Switch is used whether for circuit switched network such as telephone exchange or packet switched network such as router (Sani A, 2018). As there are many sources or input ports and destinations or output ports, crossbar switches are employed.

The number of switches increases to number of input output. One way to reduce number of switches is by using certain switch arrangement such as multistage switching. Figure 1 shows the arrangement of switches using Clos architecture (Clos, 1953).

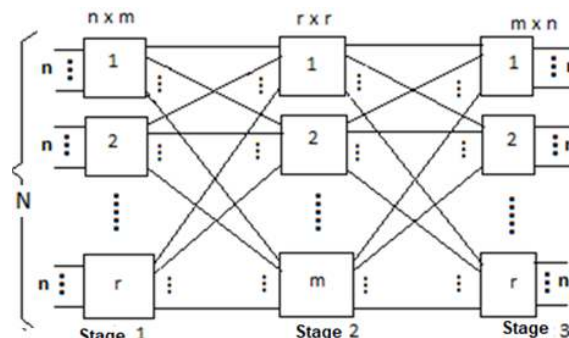


Fig. 1. Multi stage Switching.

The individual switches are arranged into $n \times m$, $r \times r$ and $m \times n$ switch modules. These modules are interconnected forming multistage switching. If p is the probability an input and an output links are busy, then the probability a certain link between switches is:

$$p' = p/m \quad (1)$$

The chance of a path between an input and an output is free is:

$$q' = (1-p')^2 \quad (2)$$

Since there is m possible links that can be used, the blocking probability is:

$$P_b = [1 - (1-p')^2]^m \quad (3)$$

If there are $n-1$ active links in the switching stage 1 input, then the probability of blocking is given by:

$$P_B = [1 - [1 - (n-1)p']^2]^m \quad (4)$$

Clos network is a three stage switching system where each stage consists of some crossbar switches. A symmetric Clos with triple parameters (m, n, r) where m is the number of middle switch, n is number of input or output port in each switch, and r is number input and output switch. The middle switch of Clos network has one link connecting input switch and one link to every output switch.

For instance, Clos network of $(3, 3, 4)$ has $m = 3$, $n = 3$, $r = 4$. At first stage it has 4 switching elements of 3×3 , and at stage 2 has 3 switching elements of 4×4 and at third stage 4 switching elements of 3×3 . Figure 2 shows the Clos $(3, 3, 4)$ network and Figure 3 the Clos $(5, 3, 4)$ (Danny, William J. and Towles, B. 2004).

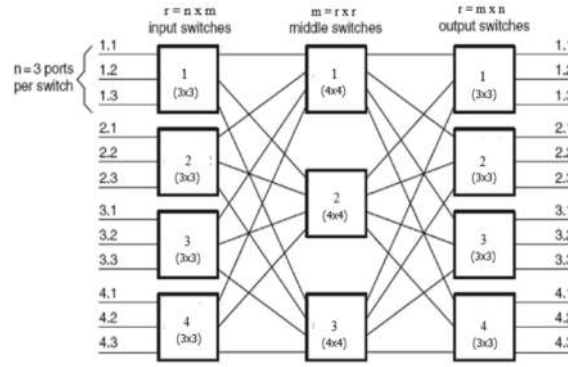


Fig. 2. Switching Clos $(3, 3, 4)$.

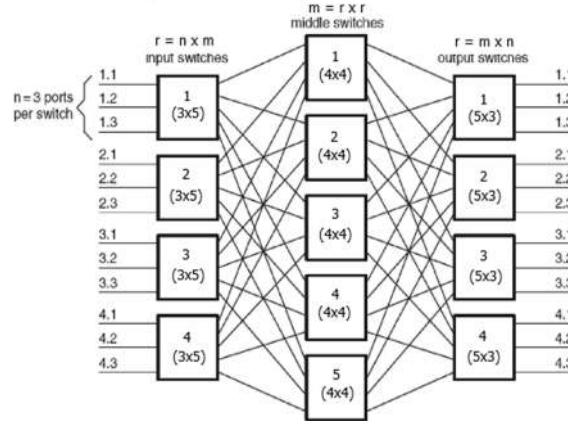


Fig. 3. Switching Clos $(5, 3, 4)$.

Blocking occurs when number of output ports is less than the input ports. Clos multistage switching experiences blocking approximated by the following equation (Lee, 1955):

$$P_B = \frac{(u!)^2 (2-p)^{2u-m} p^m}{m! (2u-m)!} \quad (5)$$

Number of switches required (cross points, CP) to form N x N input output is (Clos, 1953):

$$CP = 2(N/n)nk + k(N/n)^2 \quad (6)$$

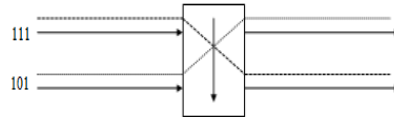
This blocking probability is sometimes accepted but some researchers use additional method, for instance, by adding buffer in front of or at the end of switching module (Garcia, 1990; M. Karol, 1987).

This paper analyses cross point and blocking probability reductions by using Batcher (Narashima, 1988) and Banyan switches to form multistage switching.

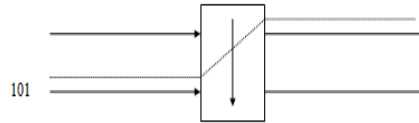
2. Batcher And Banyan

Batcher is a sorting circuit. It is developed by a 2x2 switch that works by using the following rules (Batcher, 1968):

- If there are two messages within its two-inputs, then the information with higher header value will be passed to the upper *output* port and the other to the lower one.
- If there is only one message in one of its input-ports, the message will be passed into the upper output port.



a. Two messages



b. One message

fig. 4. Batcher Switch.

Batcher network with N=8 is shown in Figure 5 (Hunter, 1997). Batcher switch N *input* and N *output* has $\log_2 N (1 + \log_2 N)/2$ stages and $N \log_2 N$ *switching* elements. As each element has 2x2 (4 crosspoints) then, the total CP is:

$$CP = 4N \log_2 N \quad (7)$$

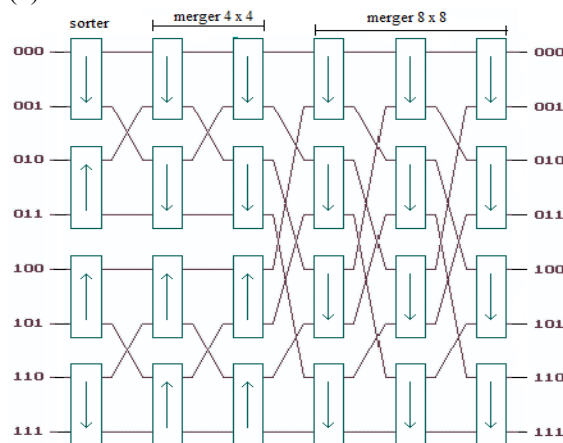


Fig. 5. Batcher Multistage Switching.

Banyan topology is a self routing network; where messages enter this network find the destined output by using their message header. It does not require centralized control. Banyan network connects N inputs to N outputs requires $\log_2 N$ stages and $N/2 \log_2 N$ switching elements. So that the total CP required developing $N \times N$ switching system is (Goke L.R. dan Lipovski, 1973):

$$CP = 2N \log_2 N \quad (8)$$

However, Banyan has weakness as its blocking probability is still not zero. The probability blocking is even higher than Clos switch. Figure 6 illustrates the blocking in the first stage.

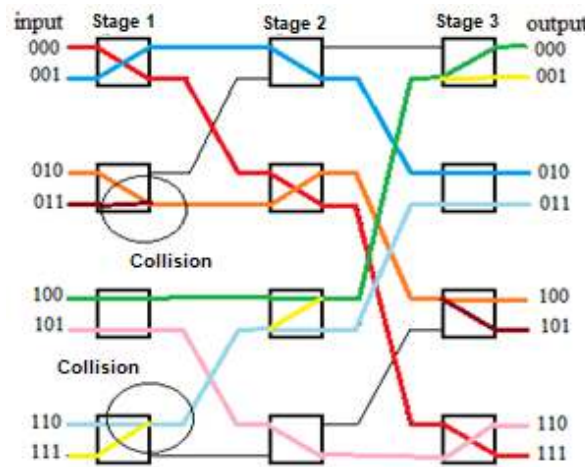


Fig. 6. Banyan Switching Blocking.

3. Switch Combination

In order to reduce cross point number in crossbar switch, Batcher and Banyan switches are combined. To avoid the blocking within Banyan network, Batcher switch is inserted in the first stage of Banyan multistage switching system. To interconnect both systems, link shuffle as shown in Figure 4 is employed (Patel, 1981). The link shuffle is to ensure that input messages are sorted before entering Banyan.

As a result, Batcher-Banyan multistage switching is obtained (Figure 7). Random inputs are sorted before entering Banyan. Packet goes to output 000 arriving at Banyan through shuffle 000. Packet goes to output 001 arriving at Banyan via shuffle 001. By doing this, all packets experience no conflict and no blocking occurred (Zulfin, 2015).

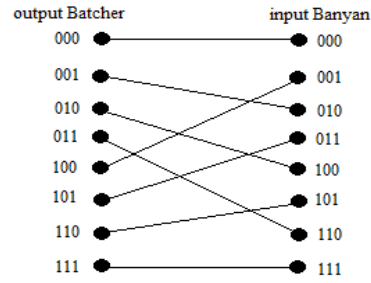


Fig. 7. Link Shuffle.

Figure 8 shows Batcher-Banyan size 8 x 8 (Hunter, 1997).

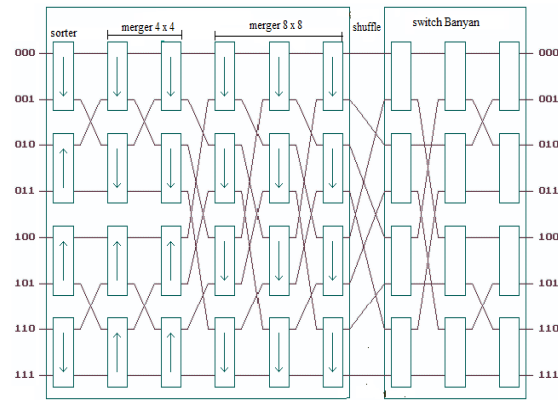


Fig. 8. Batcher-Banyan with link shuffle size 8 x 8.

Batcher-Banyan multistage switching with size of $N \times N$ requires total crosspoint of switch Batcher combined with switch Banyan, which is:

$$CP = 4N \log_2 N + 2N \log_2 N \quad (9)$$

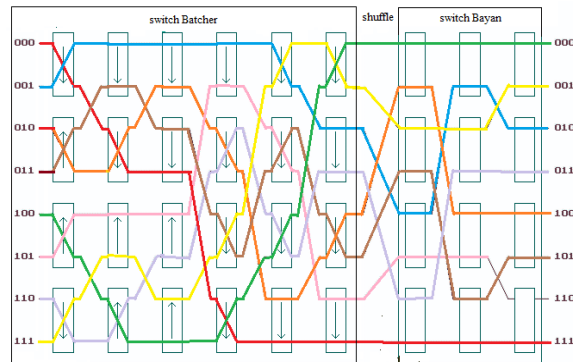


Fig. 9. Batcher-Banyan Multi stage Switching.

4. Switching Comparisons

In order to compare the crossbar multistage switching and Batcher-Banyan multistage switching systems, number of cross points and probability of blocking are compared.

The number of stage is set to be 3, the input or output is set to be 16, 64, 256, 1024 and 4096. Table 1 shows the results.

Table 1. Cross point comparisons

Input-Output (N)	Crosspoint	
	Crossbar	Batcher-Banyan
16	336	256
64	2,880	2,176
256	23,808	12,288
1,024	193,536	61,440
4,096	1,560,576	294,912

The Batcher-Banyan cross points are much lower than the crossbar multistage switching system. For N=16, the number of cross point of Batcher-Banyan is 31.25% lower than crossbar. For N=4,096, the difference rocketed to 429.17%.

These differences increase following the Equation 6, where y is the difference percentage and x is the number of input or output.

$$y = 0.0561x^{0.5146} \quad (10)$$

This equation is obtained from the regression analysis of Figure 9.

Table 2. Blocking probability comparisons.

Input-Output (N)	Blocking probability	
	Crossbar	Batcher-Banyan
16	6.3×10^{-2}	0
64	6.6×10^{-3}	0
256	6.8×10^{-5}	0
1.024	6.8×10^{-9}	0
4.096	6.9×10^{-17}	0

Further, Batcher-Banyan maintains the multistage switching system working without blocking. While the crossbar exerts probability of blocking from 6.3×10^{-2} for N=16 to 6.9×10^{-17} for N=4096 (Table 2).

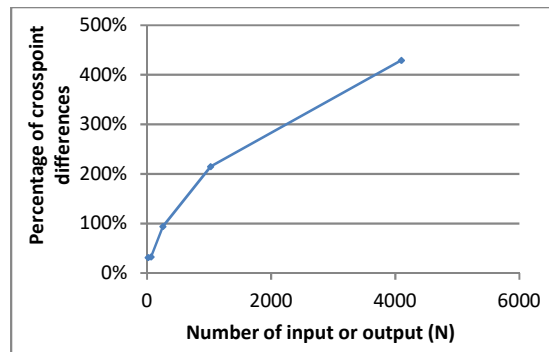


Fig. 10. Batcher-Banyan Multi stage Switching.

5. Conclusion

The Batcher-Banyan multistage switching system has many advantages over the crossbar multistage switching system. Beside Batcher-Banyan switch is a self-routing switching system that does not require complex control, Batcher-Banyan Multistage switching system has lower number of cross point compared to crossbar switch. In addition, Batcher-Banyan is purely non-blocking. Future work may consider digital switch presence in comparing both systems.

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