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Joint Encoding and Decoding Optimization of LT Codes over Noise Channels

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Abstract

The low-degree variable nodes and high residual errors of Belief Propagation (BP) decoding are key factors of high error floor of LT Codes over noise channels. A joint encoding and decoding scheme are proposed to lower this error floor. The Regularized Variable-Node (RVN) encoding method is adopted to maximize the minimum degree and avoid the low-degree variable node. By investigating the characteristic of error patterns in error frames and the relationship between error bits and their Log-likelihood Ratio (LLR) values, the most unreliable bits are identified after BP decoding. Thanks to the Cyclic Redundancy Check (CRC) detector, the residual error bits of BP decoder are identified and corrected by bit flip. The simulation results demonstrate that the proposed method can dramatically reduce the frame error rate in the error floor region at the cost of almost negligible extra computation.

Keywords: LT Codes, RVN, Belief Propagation, CRC, Error Floor.

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1. Introduction

Fountain codes also known as rateless codes, which were originally designed for reliable transmission over the binary erasure channel, exhibit excellent performance over time-varying channels. Luby Transform (LT) Codes [1] is the first practical realization of fountain codes. Raptor Codes [2], which concatenated by LT Codes and low-density parity check (LDPC) Codes, has better performance. At present, improving the performance of fountain codes is still a research hotspot. For example, much attention has been focused on the design of degree distribution, encoding method and decoding method [3-5]. In addition, the application of fountain codes in different scenes has also attracted great research focus. Fountain codes are adopted to improve efficiency of encoding/decoding in distributed storage systems in [6]. A distributed computing scheme based on LT Codes is designed to lower the delay and overhead in [7].

Due to rateless feature and flexibility of encoding structure, the research and application of fountain codes represented by LT Codes over the noise channel of physical layer have gained more and more attention. In [8], a downlink coordinated joint transmission scheme based on fountain codes is proposed. In [9], the data transmission scheme based on fountain codes for the 5G downlink is studied. It is theoretically analyzed and proved that fountain codes can effectively improve the data throughput and cell coverage of the 5G downlink compared with the conventional fix-rate channel codes. In [10], it is theoretically proved that fountain codes based adaptive transmission scheme in physical layer has better performance than the conventional adaptive power transmission scheme with fix-rate codes. Furthermore, due to their inherent self-applicability, fountain codes are regarded as a promising solution to URLLC in 5G [11-12].

Although the fountain codes represented by LT Codes have considerable prospects in noise channel, their poor decoding efficiency and high error floor greatly limit its application. Improving the decoding performance of LT Codes and lowering its error floor are of great importance for the application of fountain codes in practice.

The degree distribution is one of the primary factors for the high error floor of LT Codes. Since the fountain codes were originally designed for erasure channel, the degree distribution applicable to the erasure channels does not apply to the noise channels. In [13], the influence of the degree distribution for decoding performance over noise channels is analyzed. In [14], a ripple-based degree design framework is presented to improve the performance of LT Codes over binary input additive white Gaussian noise (BIAWGN) channel. In [15], it was pointed out that speed up log-likelihood ratio (LLR) value transfer between the variable nodes (VNs) and the check nodes (CNs) can improve the decoding performance, and the corresponding degree optimization method is proposed. In [16-17], the degree distribution of systematic fountain codes in the noise channels is optimized.

The random encoding process is another key factor for the high error floor of LT Codes in the noise channels. The corresponding error floor is predominantly caused by low-degree variable nodes [18]. And the regularized variable-node (RVN) encoding method for LT Codes is proposed to lower the error floor in [18].

In addition to the degree distribution and the random encoding method, the belief production (BP) algorithm is also the major reason for the poor decoding performance of LT Codes in noise channels. But up to now, most attention has just been paid to the optimization of degree distribution and encoding method for lowering error floor of LT Codes in noise channels. Only a few literatures have studied the decoding method of LT Codes. [19] and references do research into the BP algorithm. However, their purposes are to reduce decoding complexity, and do not improve decoding performance. Although a soft-On-the-Fly Gaussian Elimination (sOFG) method is proposed to improve decoding performance of LT Codes over noise channels in [20], its working condition is very limited due to high complexity.

The Cyclic Redundancy Check (CRC) is the most common way to verify data frame of the channel decoding in communication system. In general, CRC detection failure means channel-decoding failure. When the CRC detection fails, by some post-processing according to the characteristics of the encoding structure and decoding algorithm, the performance of channel decoding could be further improved. In [20-21], two CRC-aided enhanced decoding methods are proposed to improve the performance of Polar Codes and Turbo Codes, respectively. And the post-processing algorithm proposed in [20] has been adopted by the 5G.

Through the analysis of a large number of simulations, it is found that the key factor that causes the frame error is a very small number of residual error bits of BP in error floor region. When the CRC detection of BP decoding fails, if the residual errors could be corrected by further treatment according to the characteristics of BP, which can effectively improve the decoding performance and lower the error floor.

In this paper, we propose a joint encoding and decoding optimization scheme, which combines the RVN encoding and CRC-aided BP decoding to lower the error floor of LT Codes. At the encoder side, RVN encoding is employed. By maintaining a degree look-up table and keeping track of the degrees of the VNs, the degree of VNs are maximized and low-degree VNs are avoided, which can improve the decoding performance considerably. At the receiver side, post-processing of BP decoding resting on CRC detection is adopted to correct the BP residual error bits and further lower the error floor. When the CRC fails, a set of n bits most unreliable bits in error frame are identified by LLR values of VNs. And then, the actually m $(m \leq n)$ error bits can be localized resting on the CRC detection. At last, the residual error bits are corrected by bit-flipping. The proposed scheme just needs some look-up table operations at the encoder and few times CRC verification operations. The simulation results demonstrated that the proposed method can lower the error floor efficiently compared with conventional encoding/decoding method with negligible extra complexity. It is expected that the proposed method will improve the performances of a class of fountain codes, which with LT Codes as internal code, such as Raptor Codes.

The remainder of the paper is organized as follows. A brief description of LT codes is presented in Section II. Section III presents the RVN encoding method and analyses performance of LT Codes and RVN LT Codes. Section IV details the CRC-aided post-processing of BP decoding. Section V discusses the numerical results and insights. Section VI concludes the paper.

2. Conventional LT Codes over Noise Channel

2.1. Conventional Encoding Algorithm

As in erasure channels, a random coding mode is adopted for the encoding of LT Codes in noise channels. The encoding process is divided into two steps: 1) Randomly sample a degree j according to the degree distribution function, and 2) uniformly-at-random select j distinct information symbols to generate the encoding symbol by xor operation.

2.2. Conventional BP Decoding Algorithm

Unlike in erasure channels where the decoding of LT Codes is completed by gradually removing the edges of the bipartite graph, in BIAWGN channel, iterative decoding is adopted, by which the estimation of the LLR value between VNs and CNs is performed iteratively through BP algorithm. The encoded bits are transmitted using binary phase shift keying (BPSK) over an AWGN channel. The received vector is $y = x + w$, where x is the modulated codeword, and w is a vector of zero-mean white Gaussian noise sample with variance ${\sigma_n}^2 = N_0/2$, where N_0 is the noise power spectra density. The initial LLR value for the ith CN is calculated as $L(c_i) = 2y_i/\sigma_n$. $L_{c_i \to v_j}$ denotes the LLR value passed from the ith CN to the jth VN, where D_{c_i} is the neighbor nods of the ith CN,

$$L_{c_i \to v_j} = 2artanh\left(tanh\frac{L(c_i)}{2} \cdot \prod_{t \in D_{c_i}, t \neq j} tanh\frac{L_{v_t \to c_i}}{2}\right). \quad (1)$$

$L_{v_j \to c_i}$ denotes the LLR value passed from the jth VN to the ith CN, where D_{v_j} is the neighboring node of the jth VN,

$$L_{v_j \to c_i} = \sum_{h \in D_{v_j}, h \neq i} L_{c_h \to v_j}. \quad (2)$$

After some iterations, the LLR value of the jth VN $L(v_j)$, according to which the decoding symbol is finally determined by hard-decisions, is calculated as

$$L(v_j) = \sum_{j \in D_{v_j}} L_{c_i \to v_j}. \quad (3)$$

3. The Regularized Variable-Node LT Codes

3.1. RVN Encoding

The conventional encoding method of LT Codes can cause some low-degree VNs. In [18], the RVN LT Codes is proposed, by which the degree value of VNs becomes regular. By establishing a degree look-up table, the degree of VNs are tracked and the VNs with minimum degrees are selected first in every encoding pass. The encoding structure is shown in Fig.1, where VNs represented by $v = (v_1, \dots, v_K)$ and CNs represented by $c = (c_1, \dots, c_K, \dots)$. As shown in the Fig.1, the VNs at decoder side has the same degree.

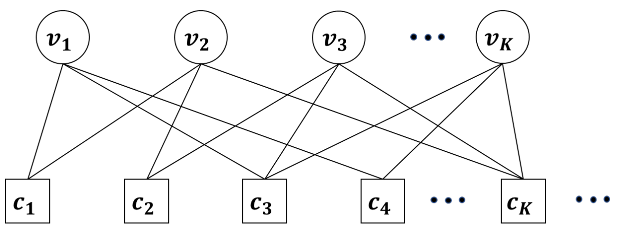


Figure 1. The structure of the RVN LT Codes

3.2. The Lower Bound Analysis

Let $\Delta(x)$ denote the CNs degree distribution, where j represents CNs degree and d_j is the fraction of degree j,

$$\Delta(x) = \sum_{j=1}^{j_{max}} d_j x^j. \quad (4)$$

Let $\Lambda(x)$ denote the VNs degree distribution, where i represents VNs degree and l_j is the fraction of degree i,

$$\Lambda(x) = \sum_{i=0}^{i_{max}} l_i x^i. \quad (5)$$

With the same degree distribution and code rate R, the conventional LT Codes and the RVN LT Codes have the same average VNs degree value μ, show in (6), where β is the average CNs degree value,

$$\mu = \beta/R. \quad (6)$$

(i) The Lower Bound of Conventional LT Codes

As the information bits are selected uniformly at random by the CNs in conventional LT Codes, the VNs degrees are Poisson distributed as $K \to \infty$. Consequently, l_i, as the fraction of VNs of degree i, can be approximated by

$$l_i = \frac{e^{-\mu}\mu^i}{i!}. \quad (7)$$

According to (7), it can be concluded that there are some low-degree VNs. And as pointed out in [18], those low-degree VNs are of great important to the high error floor. The bit error rate (BER) lower bound of VNs whose degree is i is given by (8) as [18], where $m_i = 2i/{\sigma_n}^2$, ${\sigma_i}^2 = 4i/{\sigma_n}^2$, σ_n is variance and Q-function is the right-tail probability of a standard normal distribution.

$$P_{e,i} = Q\left(\frac{m_i}{\sigma_i}\right). \quad (8)$$

According to (7) and (8), the BER lower bound P_e of overall VNs is computed as [18].

$$P_e \geq \sum_{i=0}^{i_{max}} l_i \cdot P_{e,i}. \quad (9)$$

(ii) The Lower Bound of RVN LT Codes

The VNs degree tends to be the same value with RVN encoder. Since the degree value is an integer, actual VN degree is either $\lceil\mu\rceil$ or $\lceil\mu\rceil - 1$. Consequently, the VNs degree distribution is represented by

$$\Lambda(x) = l_{\lceil\mu\rceil-1} x^{\lceil\mu\rceil-1} + l_{\lceil\mu\rceil} x^{\lceil\mu\rceil}. \quad (10)$$

Where $l_{\lceil\mu\rceil-1}$ and $l_{\lceil\mu\rceil}$ are computed as

$$\begin{cases} l_{\lceil\mu\rceil-1} \cdot (\lceil\mu\rceil - 1) + l_{\lceil\mu\rceil} \cdot \lceil\mu\rceil = \mu \\ l_{\lceil\mu\rceil-1} + l_{\lceil\mu\rceil} = 1 \end{cases}. \quad (11)$$

The corresponding BER lower bound can be approximated by

$$P_e \geq Q\left(\frac{m_\mu}{\sigma_\mu}\right). \tag{12}$$

The theoretical BER lower bounds of conventional and RVN LT Codes for $R = 1/2$ and different signal-to-noise ratios (SNR), with degree distributions $\Phi(x)$ [18] and $\Omega(x)$ [15] are shown in Fig. 2. It can be seen that the BER lower bound of RVN LT Codes is significantly lower than the conventical ones.

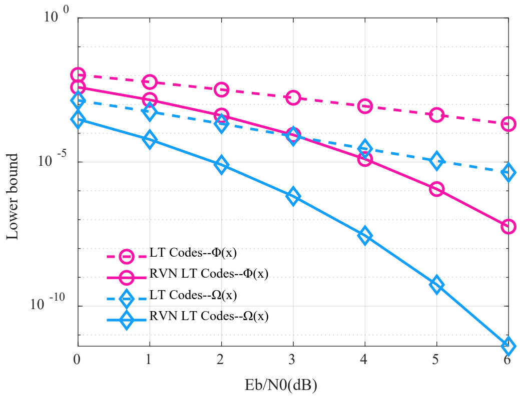


Figure 2. BER lower bound

4. Post Processing of BP Decoder of LT Resting on CRC Detector

Apart from the degree distribution and the encoding structure, the decoding algorithm is also one of the primary reasons which result in high error floor of LT Codes. BP algorithm are the most widely used for decoding LT Codes in noise channels. Although few other decoding algorithms are proposed in literature, their applications are extremely limited. By analysing the characteristics of error patterns and error bits in the error frames after BP decoding, we propose a post processing scheme for BP decoder resting on CRC detector to further improve decoding performance and lower the error floor of LT Codes.

4.1. The Error Pattern Analysis of BP Decoder in Error Floor Region

In the modern communication system, the information bits are encoded and transmitted in frame. Let assume K bits in one frame, any one or more bits errors after channel decoding will result in the whole frame useless. We categorize the data frame at decoder into K categories according to the number of error bits, and there are k error bits in k-category error frame. Let P_e denote the average BER of frames after decoding, and assuming that the error bit uniformly and randomly distributed in frames, the probability of the error frame with k error bits denoted by P_{f_k} ,is computed as

$$P_{f_k} = C_K^k (1 - P_e)^{K-k} \cdot {P_e}^k. \tag{13}$$

Let P_f denote the frame error rate (FER), and P_f is computed as

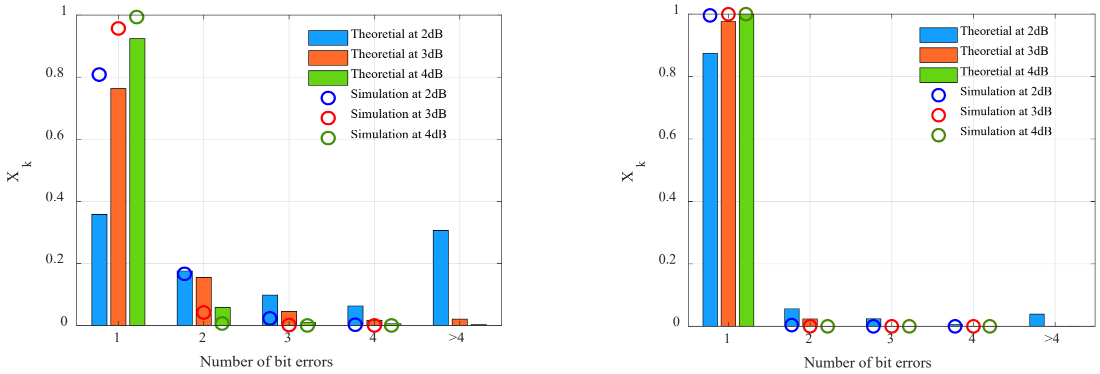
$$P_f = \sum_{k=1}^{K} P_{f_k}. \tag{14}$$

Let X_k denote the ratio of the k-category frames to all error frames,

$$X_k = P_{f_k}/P_f. \tag{15}$$

The simulation and theoretical results of X_k for degree distributions $\Phi(x)$ and $\Omega(x)$ is detailed in Fig. 3. The bar plot is obtained with Monte Carlo simulations where an BIAWGN channel is selected with frame length $K = 1000$ bits, code rate $R = 1/2$, RVN encoding and BP decoding is adopted at encoder and decoder side. At each SNR value, results are average for 100,000 frames. One can observe that the theoretical results are fairly close to the simulation results although the equation (13) is only true for uniform and random error bits distribution. And whether the theoretical values or the simulation values, almost all the error frames just only have a few error bits in the error floor region. For example, when the SNR is 3dB, the theoretical values of X_1 in figure (a) and figure (b) are 95.70 % and 99.97 %, and the simulation values are 76.31% and 97.62%. Therefore, when the BP decoding process fails, the decoded frame is very close to the transmitted one. Based on this characteristic, the error frames could be reprocessed to further lower the error floor of BP decoder.

4.2. The Error Bit Characteristics Analysis



(a) $\Phi(x)$ (b) $\Omega(x)$

Figure 3. Ratio of error frames with different error bits

The ith decoding bit is obtained by hard-decision of the ith VN LLR value $L(v_j)$, so the decoding bits are directly related to the LLR values outputted by BP decoder. By exhaustive experimental analyses, it is found that the LLR values of VNs vary greatly at the end of iterations, and almost all the BP residual error bits have a very small absolute LLR value. We order all VNs in every error frame by their absolute LLR value $|L(v_j)|$ from lowest to highest, then classify order numbers into 4 classes. The BP residual errors bits distribution for two studied degree distribution over 4 classes is detailed in Tab.1, in which RVN encoding and BP decoding is adopted with $K = 1000$, $R = 1/2$ and the $Eb/N_0 = 3$dB. As shown in Tab.1, the error bits locate in the lowest class with extremely high probability, e.g. 88.02% and 97.67% error bits are with order number less than 10 for $\Phi(x)$ and $\Omega(x)$ separately. In other words, the lower the order number is, the higher the error probability of decoded bit gets. Therefore, the most unreliable decoding bits can be identified by the absolute LLR value with high confidence.

Table 1. The distribution of BP residual error bits

Order number class	[1,10]	(10,20]	(20,30]	(30,1000]
$\Phi(x)$	88.02%	3.92%	1.74%	6.32%
$\Omega(x)$	97.67%	2.33%	0	0

4.3. CRC Aided Bit Flip of BP Residual Error Bits

According to the previous analysis, we propose a CRC Aided Bit Flip (CRC-BF) scheme to reprocess the BP decoder to lower the error floor, which corrects BP residual error bits by bit flip with the help of CRC detector. Let $\mathrm{U} = (u_1, \ldots, u_K)$ represent decoding bit sequence of BP decoder. If the CRC is not verified after the last iteration, the CRC-BF scheme starts. All VNs are ordered according to their absolute LLR values $|L(v_j)|$, and select n bits with smallest $|L(v_j)|$ as the most unreliable bits. Assumed that there are no more than $m(m \leq n)$ error bits in one error frame, and all error bits are occurring in the selected n most unreliable bits. There are up to $\sum_{i=1}^{m} C_n^i$ different m-bit error patterns, and the totally number of error patterns with one up to n-bit error is $\sum_{i=1}^{n} C_n^i = 2^n - 1$, denoted by the set $\mathrm{E} = (e_1, \cdots e_{2^n-1})$. Flipping the bits in U according to one error patterns in E to genenrate a new decoding bit sequence U′, if the CRC of U′ is verified, the process is completed. Otherwise, the next candidate decoding bit sequence is generated until all $2^n - 1$ candidate decoding bit sequences have considered or the CRC is verified. The pseudocode of CRC-BF is given in Tab.2.

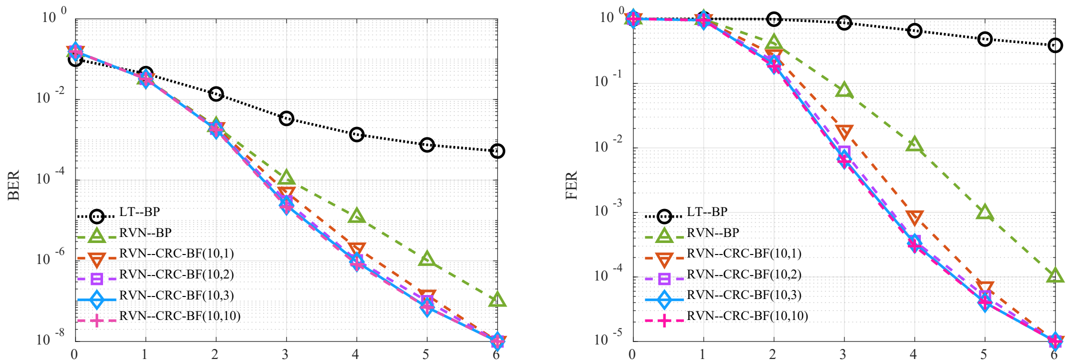
Table 2. CRC-BF Algorithm

Step 1: U=BP Decoder ();
Step 2: When CRC Check (U)=False **Then**
Select n most unreliable bits according their absolute LLR value and calculate the error pattern to form set E;
For i=1 to $\sum_{j=1}^{n} C_n^j$

Computer a new decoding bit sequence U′ by bit flip according to e_i in E
If CRC Check (U′)= False **Then**
Continue;
Else
U = U′;
Break;
End If
End For
Return U.

5. Numerical Results

To demonstrate the effectiveness of the proposed combined optimization method, a large number of simulations have been performed over the BIAWGN channel. In our simulations, we used two kind of different degree distributions $\Phi(x)$ [18] and $\Omega(x)$ [15] with frame length $K = 1000$(include CRC check bits) and code rate $R = 1/2$ and all the results are average over 100,000 times independent simulations. We present simulations of BER and FER performance of the proposed RVN LT Codes with CRC-BF scheme in comparison with conventional LT Codes and RVN LT Codes with classical BP decoder.



(a) BER (b) FER

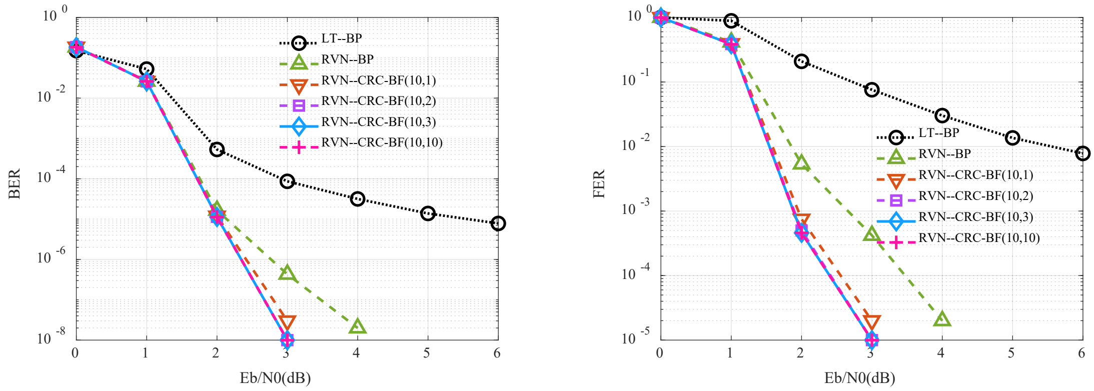
Figure 4. Performance comparison for $\Phi(x)$

5.1. Performance Comparation

Fig.4 gives the BER and FER performance comparation over various SNR using $\Phi(x)$. Fig.4 (a) details BER performance for three schemes. One can see that the conventional LT Codes has poor BER performance and high error floor when SNR larger than 3dB. By modifying the encoding mechanism, RVN encoder can thoroughly avoid the low degree VNs, so the error floor is lowered sharply by RVN encoder. By reprocessing the classical BP decoder, the proposed scheme which combine RVN LT Codes and CRC-BF scheme further lowers the error floor. Compared with the conventional LT Codes with BP decoder, the gains in terms of BER archive more than five orders of magnitude at most, and the most SNR gain reach up to 3.5dB. Compared with the RVN LT Codes with BP decoder, the gains in terms of BER archive more than one order of magnitude at most, and the most SNR gain reach up to more than 1dB. Fig.4 (b) details the FER performance for three schemes. As the BER in Fig.4 (b), the proposed scheme also outperforms the other two schemes in terms of FER.

Compared with the classical BP decoding, the CRC-BF needs more CRC verifications. The larger the range of most unreliable bits are selected, the more verifications are needed. From the previous analysis, it can be seen that almost all the error bits occur within the lowest class with the order number not larger than 10. Moreover, when the range of most unreliable bits is selected, the number of CRC verifications is determined by the assumed number of max error bits denoted by m as shown in Fig.3, the number of error bit in one frame is no more the 3 bits in the error floor region when SNR>3dB. Therefore, we considered $n = 10$, $m = 1$-3 for CRC-BF scheme, and $m = 10$ as reference in our simulations. The curve labeled by CRC-BF(m) are with the number of max error bits m and the curve CRC-BF(1,10) are with $m = 10$ as reference.

As shown in Fig.4, when n is fixed to 10 and $m = 1$,the proposed scheme can get most of the gain. when the number of maximum error bits m increases from 1 to 3, the BER and FER performance is improved gradually. However, increasing m barely gets performance gain when $m > 3$, but induces a computational complexity CRC verification operations and some bit flip operations. However, CRC-BF is mainly designed to reprocess the low-category error fames with extremely a few bits, so the complexity overhead coming from bit flip is very limited. So, the complexity overhead of the proposed scheme is mainly coming from CRC operations.



(a) BER (b) FER

Figure 5. Performance comparison for $\Omega(x)$

overhead. Taking FER at SNR=4dB for example, the total number of BP residuals error frames is 1082, in which the number of frames with error bits less than 3bits is 1073. The CRC-BF scheme can correct 1049 frames with $m = 3$. By increasing m up to 10, the number of corrected frames is not increasing. So $m = 3$ seems to provide a good trade-off between performance and computational complexity.

In order to verify adaptivity of the proposed scheme over other degree distributions, Fig.5 gives the BER and FER performance using $\Omega(x)$ which is one of the most optimal degree distribution for LT Codes over noise channels. As in Fig.4, the performance of the proposed scheme outperforms the other two schemes drastically. It can be observed that the performance of the three schemes using $\Omega(x)$ outperforms that using $\Phi(x)$. That is to say the performance of the proposed scheme is also related to degree distribution as the other two schemes. It can be observed that the error bit distribution in error frames by using $\Omega(x)$ is more conciliant with the theoretical analysis than that using $\Phi(x)$, and more error frames is low-category frames in which error bits is no more than 3. The proposed scheme is more efficient in resolving low-category error frames.

5.2. Complexity Comparation

A comparison the computational complexity is discussed. Compared with the conventional LT Codes, by maintaining a memory list, the RVN LT Codes can regularize the VNs degree with low complexity. Therefore, the complexity overhead of the proposed scheme compared with conventional LT Codes is mainly due to CRC-BF scheme in decoding process. The complexity over of CRC-BF scheme consists in more

From the above analysis, the maximum number of added CRC operations is related to the selected number of most unreliable bits n and the assumed number of error bits m. The greatest number of CRC operations is $\sum_{i=1}^{m} C_n^i$ when $n = 10$, $m = 3$. In practice, the majority the error frames just have one error bit, so the actual number of CRC operations is much less than the maximal number. Tab.3 listed the average number of actual CRC operations in simulations for CRC-BF scheme using two different degree distributions. One can observe that just a few numbers of CRC operations is needed in the error floor region. Above all, the complexity overhead of the proposed scheme is negligible, compared with the conventional LT Codes.

Table 3. Average number of CRC operations by CRC-BF scheme

Eb/N_0	1dB	2dB	3dB	4dB	5dB	6dB
$\Phi(x)$	171.11	90.00	20.42	7.27	9.27	18.80
$\Omega(x)$	51.63	6.84	2.52	1.00	—	—

6. Conclusion

High error floor is the vital disadvantage of LT Codes over noise channels. Excepting degree distribution, encoding structure and the conventional BP decoding algorithm are also the keys factors inducing poor performance of LT Codes. Based on this observation, this proposed a combined scheme to lower the error floor of LT Codes, in which RVN encoder is adopted to avoid the low-degree VNs and a CRC-BF scheme is devised to cope with the residual error of BP decoder. The simulation results confirm that the proposed method can lower the error floor significantly compared with conventional LT Codes with negligible extra computation overhead.

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