# Performance Analysis Based on MGF Fading Approximation in Hybrid Cooperative Communication

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**Abstract.** Wireless communications and power line communications (PLC) are essential components for smart grid communications. In this study, the performance of dual-hop wireless/power line hybrid fading system is analyzed from two aspects of outage probability and bit error rate (BER). The system adopts a hybrid fading model based on the general Nakagami-m wireless and lognormal (LogN) power line fading based on amplify and forward (AF) relay protocol, and the Bernoulli Gaussian noise model attached to the PLC channel. Since the LogN distribution has a certain similarity with the Gamma, the key parameters of PDF with approximated LogN distribution from Gamma distribution are determined by using the moment generating function (MGF) equation and the approximation of LogN variable sum. Then the exact closed-form expression of the system outage probability and BER are obtained by integral variation. Finally, Monte Carlo simulation is used to verify the correctness of the theory and analyze the influence of different system parameters on the performance.

**Keywords:** Cooperative communication · Nakagami-m distribution · Log-normal distribution · PDF approximate · MGF

## 1 Introduction

With the development of power supply network, the coverage area of power lines is becoming wider and wider. How to make full use of the existing power supply network resources to achieve reliable information transmission on power lines is gradually attracting extensive attention and research [1]. Therefore, the combined PLC and wireless dual-media cooperative communication technology can integrate resources, optimize and complement each other. The latest research results include multipath transmission [2, 3], relay forwarding [5, 6, 10], parallel communication [12, 14], and multimedia collaboration [10, 13] and other PLC collaboration technologies.

For the Cooperative technology of wireless and PLC, many works such as the literature [4, 5] have utilized the relay-assisted scheme of PLC and wireless communication, which has been proved to improve reliability and expand communication range. The research of relay is mainly divided into double hop [6, 7] and multi hop [8, 9], both of which are mainly concentrated on the decoding and forwarding protocol. Mathur analyzed the average error rate performance of power line and wireless hybrid


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cooperative communication systems using dual-hop parallel communication and decode and forward (DF) protocols in [11]. In [12], the indoor wireless and power line channel models of LogN distribution are used to study the reliability of indoor wireless and power line dual-media cooperative communication systems based on AF and DF protocols. In addition, DF protocol using hard decision will lose useful information and may cause error codes, and its performance is obviously worse than AF protocol [14]. Thus, these ideas have inspired us to study the wireless-power line mixed system model.

Contribution: Most of the works in the literature focus on the performance of a dual-hop wireless system or a dual-hop PLC system, performance analysis in such systems is simple due to the symmetry of the system model. In order to solve the problem that there is no closed expression in the communication theory performance under the Nakagami-m/LogN hybrid fading condition, which leads to the key technical performance analysis is overly dependent on computer simulation. We apply approximation of the sum of LogN distribution variables, integral variation, Mehta algorithms and MGF equation to determine the PDF parameters of approximated distribution. Finally, Monte Carlo simulation is used to verify the versatility and accuracy of the proposed method, which provides a new perspective and necessary theoretical support for dual-media cooperative communication applications.

## 2 System Model

We consider a dual-hop, two-medium hybrid communication system, as shown in Fig. 1. The source (S) conducts wireless communication with the relay (R) using transmitting power $P_S$, R is used to amplify and forward (AF) the received data and transmit the data to destination (D) through power line communication (PLC) with power $P_R$.

In the first time slot, the signal received by R is

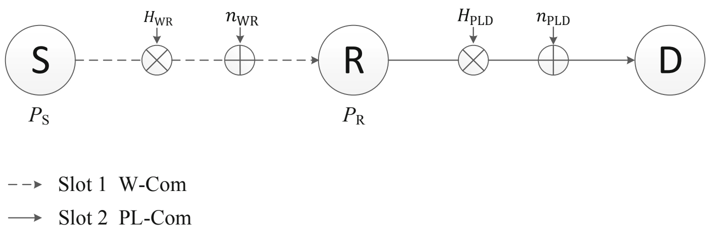


**Fig. 1.** System model for a dual-hop wireless-power line hybrid communication system with AF relay.

$$y_{GR} = H_{GR}\sqrt{P_S}X_S + n_{GR} \tag{1}$$

where noise $n_{GR}$ satisfies normal distribution $N(0, N_W)$; $H_{GR}$ is the wireless fading coefficient, subject to Nakagami-m distribution. $\Omega_R$ = $E(|H_{GR}|^2)$ is the variance of the fading amplitude, and $\Omega_R = 1$ is used to ensure that the fading does not change the average power of the received signal.

Let $\Delta_{GR} = P_S/N_W$ denote the channel average signal to noise ratio. Then, the instantaneous signal to noise ratio of the wireless channel relay R is

$$\gamma_{GR} = H_{GR}^2 \Delta_{GR} = \frac{P_S |H_{GR}|^2}{N_W} \tag{2}$$

It is known that $|H_{GR}|^2$ satisfies the Gamma distribution $G(\alpha_R, \beta_R)$ and has the following form [14]:

$$G(H_{GR}, \alpha_R, \beta_R) = \frac{(H_{GR})^{\alpha_R - 1}}{\beta_R^{\alpha_R} \Gamma(\alpha_R)} \exp(-\frac{H_{GR}}{\beta_R}) \tag{3}$$

The parameter relationship between $G(\alpha_R, \beta_R)$ and Nakagami-m distribution in the formula satisfies $\alpha_R = m_R$, $\beta_R = \Omega_R / m_R$.

In the second time slot the signal received by the receiver is

$$y_{LD} = H_{LD} \sqrt{P_R} X_R + n_{LD} \tag{4}$$

where $H_{LD}$ is the power line fading coefficient, which satisfies the $\mathrm{LogN}(\mu_D, \sigma_D^2)$ distribution [14]:

$$f_{H_{LD}}(H_{LD}, \mu_D, \sigma_D) = \frac{1}{H_{LD} \sigma_D \sqrt{2\pi}} \exp(-\frac{(\ln H_{LD} - \mu_D)^2}{2}) \tag{5}$$

where $\mu_D$ and $\sigma_D$ are the mean and mean variance of ln, respectively. The $E(|H_{LD}|^2) = \exp(2\mu_D + 2\sigma_D^2) = 1$, that is $\mu_D = -\sigma_D^2$. The noise types are modeled using the two-term Bernoulli Gaussian noise model.

Let $\Delta_{LD} = P_R/N_P$, the instantaneous SNR of the power line receiving end can be expressed as

$$\gamma_{LD} = |H_{LD}|^2 \Delta_{LD} \tag{6}$$

Therefore, the total SNR of the dual-hop AF relay protocol communication system is

$$\gamma_{GL} = \frac{\gamma_{GR} \gamma_{LD}}{\gamma_{GR} + \gamma_{LD} + 1} \tag{7}$$

In the case of high SNR ($P_S/N_W$, $P_R/N_P$), Formula (7) can be approximate to

$$\gamma_{GL} \approx \frac{\gamma_{GR} \gamma_{LD}}{\gamma_{GR} + \gamma_{LD}} = \frac{1}{1/\gamma_{GR} + 1/\gamma_{LD}} \tag{8}$$

# 3 Performance Analysis

## 3.1 System Fading Approximation Based on PDF and MGF Equation

Because the PDF of the hybrid fading system D cannot be known, it increases the difficulty of system performance analysis. Therefore, there is a certain similarity between the Gamma distribution and the LogN distribution [15], this study proposes a PDF approximation algorithm based on the MGF equation, which can approximate the $|H_{\mathrm{GR}}|^2$ of the $G(\alpha_{\mathrm{R}}, \beta_{\mathrm{R}})$ to distribution $\mathrm{LogN}(2\mu_{\mathrm{R}}, 4\sigma_{\mathrm{R}}^2)$ distribution.

It is known that $|H_{\mathrm{GR}}|^2$ satisfies the $G(\alpha_{\mathrm{R}}, \beta_{\mathrm{R}})$ distribution. Then, the parameters $\mu_{\mathrm{R}}$ and $\sigma_{\mathrm{R}}$ after the approximation can be solved:

$$(1+\beta_R s_i)^{-\alpha_R} = \sum_{n=1}^{N} \frac{w_n}{\sqrt{\pi}} \exp(-s_i \exp(\sqrt{2}2\sigma_R a_n + 2\mu_R)) \tag{9}$$

Under the condition of channel fading normalization $(\mathrm{E}(|H_{\mathrm{GR}}|^2) = 1)$. Figure 2 shows the PDF curve comparison before and after channel distribution approximation when $m_{\mathrm{R}} = 2.8$.

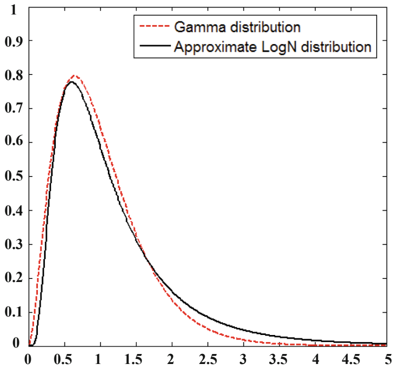


**Fig. 2.** The PDF of for different approximations

Since $\gamma_{\mathrm{GR}} = H_{\mathrm{GR}}^2 \Delta_{\mathrm{GR}}$, the variable $\gamma_{\mathrm{GR}}$ can be approximately expressed as the key parameter of $\mathrm{LogN}(\mu'_{\mathrm{R}}, (\sigma'_{\mathrm{R}})^2)$ according to $\Delta_{\mathrm{GR}}$

$$(\sigma'_{\mathrm{R}})^2 = 4\sigma_{\mathrm{R}}^2 \tag{10}$$

$$\mu'_{\mathrm{R}} = \ln \Delta_{\mathrm{GR}} + 2\mu_{\mathrm{R}} \tag{11}$$

### 3.2 Performance Analysis of LogN-LogN System

We convert the system performance analysis under hybrid fading conditions into the performance analysis problem under the same LogN distribution condition, and the instantaneous mutual information $I$ of the system is

$$I = \frac{1}{2}\log(1+\gamma_{\mathrm{LL}}) \tag{12}$$

Since $P_{\mathrm{S}}|H_{\mathrm{LR}}|^2/N_{\mathrm{W}}$ and $P_{\mathrm{R}}|H_{\mathrm{LD}}|^2/N_{\mathrm{P}}$ are satisfied LogN distribution when $P_{\mathrm{S}}/N_{\mathrm{W}}$ and $P_{\mathrm{R}}/N_{\mathrm{P}}$ are constant. Since the reciprocal of the LogN variable still satisfies LogN distribution, $1/(P_{\mathrm{S}}|H_{\mathrm{LR}}|^2/N_{\mathrm{W}})$ and $1/(P_{\mathrm{R}}|H_{\mathrm{LD}}|^2/N_{\mathrm{P}})$ also satisfy LogN distribution. Therefore, the inverse of the variable sum of LgN distribution.

Let $G = 1/(P_{\mathrm{S}}|H_{\mathrm{LR}}|^2/N_{\mathrm{W}}) + 1/(P_{\mathrm{R}}|H_{\mathrm{LD}}|^2/N_{\mathrm{P}})$, then $\gamma_{\mathrm{LL}} \sim \mathrm{LogN}(-\mu_{\mathrm{G}}, \sigma_{\mathrm{G}}^2)$. When using the Mehta algorithm, the following relations are used for the parameters $\mu_{\mathrm{G}}$ and $\sigma_{\mathrm{G}}$ of the variable G:

$$\begin{aligned}&\sum_{n=1}^{N}\frac{w_n}{\sqrt{\pi}}\exp(-s_i\exp(\sqrt{2}\sigma_G a_n+\mu_G)) = \\ &\sum_{n=1}^{N}\frac{w_n}{\sqrt{\pi}}\exp(-s_i\exp(\sqrt{2}\sigma_A a_n+\mu_A))\cdot\sum_{n=1}^{N}\frac{w_n}{\sqrt{\pi}}\exp(-s_i\exp(\sqrt{2}\sigma_B a_n+\mu_B))\end{aligned} \tag{13}$$

$P_{\mathrm{S}}|H_{\mathrm{LR}}|^2/N_{\mathrm{W}} \sim \log N(\mu_{\mathrm{A}}, \sigma_{\mathrm{A}}^2)$, $P_{\mathrm{R}}|H_{\mathrm{LD}}|^2/N_{\mathrm{P}} \sim \mathrm{logN}(\mu_{\mathrm{B}}, \sigma_{\mathrm{B}}^2)$. The two equations in the simultaneous Eq. (13) can be solved by fsolve function of MATLAB to obtain $\mu_{\mathrm{G}}$ and $\sigma_{\mathrm{G}}$.

Let the threshold be $R_{\mathrm{th}}$ and $\gamma = \exp(2R_{\mathrm{th}}) - 1$, then the system's outage probability $P_{out}^{\mathrm{LL}}$ is

$$P_{\mathrm{out}}^{\mathrm{LL}} = \sum_{j=0}^{1} p_j Q(-\frac{\ln\gamma + \mu_{Gj}}{\sigma_{Gj}}) \tag{14}$$

where $p_0 = 1-p$, $p_1 = p$, which represent the probability of whether impulse noise exists in power line channel respectively. The BER of the system has the following expression:

$$P_{BER}^{\mathrm{LL}} = -\sum_{j=0}^{1}\sum_{k=1}^{4} p_j \frac{Z_{kj}}{X_{kj}} Q(\frac{Y_{kj}}{\sqrt{1+X_{kj}^2}}) \tag{15}$$

which includes

$$
\begin{aligned}
X_{kj} &= \sqrt{2\sigma_{Gj}^2 / R_{3k}^2} \\
Y_{kj} &= \frac{4\sigma_{Gj}(\mu_{Gj} + \ln 0.5 - R_{2k}) - \mu_{Gj} R_{3k}^2}{2X_k R_{3k}^2} \\
Z_{kj} &= 0.5R_{1k}\sigma_{Gj} \exp(\frac{-\mu_{Gj} + Y_k^2}{2}) \exp[-(\frac{-\mu_{Gj} + \ln 0.5 - R_{2k}}{R_{3k}})^2]
\end{aligned} \tag{16}
$$

## 4 Numerical Results and Discussions

In this section, we performed Monte Carlo computer simulation experiments with MATLAB software to verify the reliability and accuracy of the theoretical formula. In all simulations, unless it mentioned otherwise, the simulation process adopts the following default settings: (1) the total system power is 2, $P_S = P_R = 1$. (2) In order to highlight the influence of channel fading and noise on performance, assume that the average SNR of the system channel is Δ, $N_W = N_p = 1/\Delta$, i.e. $\Delta_{GR} = \Delta_{LD} = \Delta$. (3) System interruption threshold $R_{th} = 0.2$. (4) Bernoulli Gaussian noise parameter: $p = 0.1$, $T = 10$.

Figure 3 compares the outage and BER performance of the simulated and theoretical calculations for two different fading parameters. The fading parameters are set to $\{m_R, \sigma_D\} = \{2.8, 2.6\}$ and $\{2.2, 2.8\}$. As we see, the outage probability and BER of the two sets of parameters decrease with increasing SNR of the system, and theory curves match well with Monte Carlo simulations in wider range of SNR at the low SNR, implying the validity of the derived analytical expressions, and the choice of $s_1$ and $s_2$ will also affect the approximation accuracy when determining the approximate parameters.

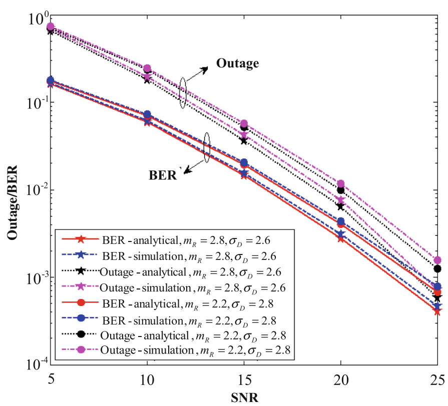


**Fig. 3.** System outage probability and BER against per hop average SNR with different Fading factors.

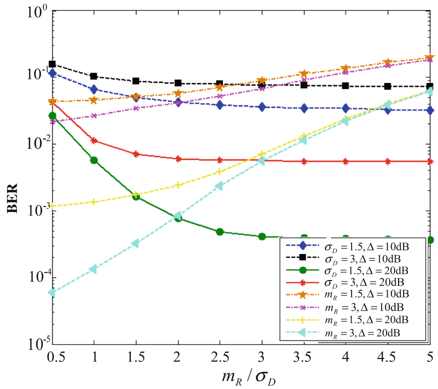


**Fig. 4.** Impact of parameters $m_{\mathrm{R}}/\sigma_{\mathrm{D}}$ on theory BER performance

Figure 4 compares the relationship between the system BER and another fading parameter $m_R$ ($\sigma_D$) under different fading parameters $\sigma_D$ ($m_R$) and the average signal to noise ratio Δ. We assume that $m_R$ and $\sigma_D$ are 1.5 and 3 respectively, and the Δ is 10 dB and 20 dB. For vertical and horizontal analysis of Fig. 4, we can draw the following conclusions: (1) Fixing arbitrary fading parameters, it can be obviously seen that increasing Δ then system BER will decrease. (2) At the same $m_R$ and Δ, the system BER will decrease as $\sigma_D$ decreases, the system BER will decrease as $m_R$ increases when the same $\sigma_D$ and Δ. (3) The four downward curves of the fixed $\sigma_D$ have no obvious trend of the four upward curves of the fixed $m_R$, that is to say the influence of $\sigma_D$ on the system BER is higher than the influence of $m_R$ on the BER.

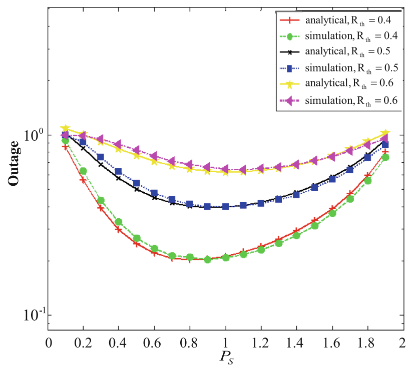


**Fig. 5.** Impact of transmitting power on outage performance

Figure 5 compares the relationship between outage probability and power for different thresholds. Here, only one set of fading parameters is set in order to more clearly show the effect of $R_{\mathrm{th}}$ on outage, which is 0.4, 0.5, 0.6. The point to be emphasized is that the total system power is 2. It can be seen from the figure that the system performance becomes better as the outage decrease, and the theoretical performance of the system under different $R_{th}$ is consistent with the simulation results. The optimal power transmission factor is used to optimize the system performance. Therefore, the system can design the optimal transmit power according to different interrupt thresholds to achieve green energy saving.

## 5 Conclusions

In this study, a novel dual-hop wireless/power line hybrid communication system based on AF protocol is analyzed. The difficulty in analyzing the system performance increases because the PDF at the end of the hybrid-fading system is difficult to solve. Therefore, the MGF equation based on PDF is used to transform the performance analysis of the hybrid-fading system into the performance analysis of the LogN-LogN system. Furthermore, the exact closed expressions of system outage probability and BER are obtained by synthesizing the approximation of logarithmic normal distribution variable sum and Mehta algorithm. As can be seen from the numerical figure, the fading parameters of each link affect the system performance to varying degrees, so that the LogN link plays a dominant role in the whole system. In addition, for the purpose of green energy-saving communication, there exists an optimal ratio of source transmission power to relay transmission power, which makes the system performance optimal.

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