Sparse Decomposition Algorithm Based on Joint Sparse Model

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Abstract. Orthogonal Matching Pursuit (OMP) algorithm is the most classical signal recovery algorithm in compressed sensing. It is also applicable to the Joint Sparse Model (JSM) of distributed compressive sensing. However, OMP algorithm suffers from high computational complexity and poor anti-noise ability without considering the correlation between signals. Therefore, by combining the characteristics of the JSM-1 and JSM-2 models, we propose the corresponding joint sparse decomposition algorithms, named JSM1-OMP and JSM2-OMP. The JSM2-OMP algorithm can be viewed as improvement of the JSM1-OMP algorithm. Furthermore, a better JSM-OMP algorithm is proposed by modifying the JSM2-OMP algorithm. The simulation experiments demonstrate the effectiveness of the proposed algorithms.

Keywords: OMP · JSM · JSM1-OMP algorithm · JSM2-OMP algorithm · JSM-OMP algorithm

1 Introduction

Compressed sensing (CS) [1, 2] is a new signal sampling theory for finding sparse solutions of underdetermined linear systems. On the premise that the signal is sparse or sparsely represented, a signal is sampled at a sampling rate much lower than the Nyquist theorem, and then the original signal is accurately reproduced by a nonlinear recovery algorithm. Sparse decomposition is initially used for compressed sensing because the signal is sparse or sparsely decomposable.

Now that there are many recovery algorithms, and they can be summed up into three major categories. The first type of algorithm is the well-known convex relaxation algorithm. The traditional convex relaxation algorithm is to convert the l_0 norm into a l_1 norm, commonly used Lasso algorithm [3], the Basis Pursuit (BP) algorithm [4]. Greedy iterative algorithm is the second most commonly used sparse decomposition algorithm. In [5], Mallat first introduced the concept of redundant dictionary and applied its flexibility instead of a relatively fixed orthogonal basis to propose an algorithm that is still widely used today, namely the Matching Pursuit (MP) algorithm. Orthogonal Matching Pursuit (OMP) was first proposed by Mallat. Gilbert and Tropp later further demonstrated this algorithm and proved its theoretical convergence [6].

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The representation results obtained by the above two algorithms are single form, and the representation form affects the effect of sparse decomposition. Therefore, one begans to exploit a variety of sparse representations to represent a signal. Elad and Yavneh pointed out that the joint sparse representation is superior to a single sparse representation, and proposed a random orthogonal matching pursuit (RandOMP) algorithm. In [7], Zhang proposed a combined algorithm based on the forward greedy algorithm, but the backward steps adaptively point to the direction with the smallest residual. The last kind of recovery algorithm is a sparse decomposition algorithm based on sparse Bayesian models, such as relevance vector machine (RVM) [8]. In [9], supposing that the hyperparameters in the prior distribution of estimators are La-place prior distributions, a hierarchical Bayesian model is proposed. In this work, sparse coefficients are treated as implicit parameters in the hyperparameter estimation process. And then, the minimum mean-squared error estimate is also used as the optimal metric to obtain the optimal solution under the Bayesian framework [10].

Distributed Compressed Sensing (DCS) extends the compressive sensing of a single signal to a multi-signal model [11], making full use of the correlation between multiple signals and creating joint recovery conditions further reduce the number of measurements required for successful recovery. Baron proposed three signal models for different scenarios, called JSM-1, JSM-2, and JSM-3 [12]. Corresponding signal recovery methods are also different for different joint sparse models. How to study the exact and appropriate recovery method based on the specific joint sparse model is the focus of the distributed compressive sensing.

The rapid development of 5G has gradually turned the massive antenna system (Massive MIMO) into the mainstream of wireless communication. At the same time, distributed processing of multi-antenna signals has gradually become a research hot-spot in the field of communications. In this paper, we extend the single measurement model to a multiple measurement model. When the transmitted signal reaches different receiving antennas through different channels, the signals will come with different amplitudes due to multipath fading, but the position of the spectrum remains unchanged, and sparse coefficient positions remain unchanged. Therefore, the signal received by different receiving antennas in a Massive MIMO system shares the same frequency spectrum, but the non-zero amplitude is different. This multiple measurement frame signal structure is basically the same as the JSM-2 model [13]. In addition, we often use the JSM-1 model to represent the actual corresponding signal when we consider the correlation between signal locations. Both of these models are commonly used for modelling multi-antenna sparse signals.

To deal with the mentioned deficiencies, we fully consider the correlation of signals and propose an improved sparse decomposition algorithm for each model. We will introduce the relevant system model in the second section, and describe the three improved algorithms proposed in the third section. Finally, simulation experiments are performed to verify the effectiveness of our proposed algorithms.

2 System Mode

Compressive sensing can be expressed as $Y = \Phi x$, where $x \in R^{N\times 1}$ is the original signal to be observed. Sparsity or compressibility is one of the two preconditions for the application of compressed sensing theory. Consider a real-valued signal X of length N, denoted as $X(n), n \in [1,2,\ldots,N]$. Combined with the signal theory, X can be represented by a set of base $\Psi \in [\psi_1, \psi_2, \ldots, \psi_n, \ldots, \psi_N]$ linear combinations

$$X = \Psi\Theta = \sum_{i=1}^{N} \psi_i \theta_i \tag{1}$$

where $\theta_i = \langle X, \psi_i \rangle$, $\Psi \in R^{N\times N}$. When signal X has only K non-zero values on a certain transform basis, it is said to be K-sparse on this basis. The basis Ψ is the sparse basis of signal X. Then, $Y = \Phi x = \Phi\Psi\Theta = A^{CS}\Theta$, $A^{CS} = \Phi\Psi \in R^{M\times N}$. The fundamental task of compressive sensing is to solve the sparse coefficient vector from the representation

$$\Theta = \arg\min_{\theta} \| \theta \|_0 \ s.t. y = A^{CS}\Theta \tag{2}$$

Distributed compressed sensing originates from the simultaneous sparse compression sensing of multiple signals. The model expression is written as

$$\mathrm{y}_j = \varphi_j x_j\,, \quad j \in \{1,2,\ldots,J\} \tag{3}$$

Multiple signals have the following expression

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_J \end{bmatrix}, \; Y = \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_J \end{bmatrix}, \; \varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \cdots \\ \varphi_J \end{bmatrix} \tag{4}$$

where, $X \in R^{JN}, Y \in R^{\bar{M}\cdot JN}, \varphi \in R^{\bar{M}\cdot JN}, \overline{M} = \sum_{j\in\wedge} M_j$, $Y = \varphi X^T$. Both the JSM-1 and JSM-2 models can be represented as

$$x_j = z_c + z_j \quad j \in \{1,2,\cdots,J\} \tag{5}$$

where $z_c = \Psi \cdot \Theta_c$, $||\Theta_c||_0 = K_c$, $z_j = \Psi \cdot \Theta_j$, $||\Theta_j||_0 = K_j$.

For all original signals $x_j, j \in \{1,2,\cdots,J\}$, z_c is the sparse common part, and the sparsity of z_c over the sparse base Ψ is k_c; z_j is the unique part of each original signal, and the sparsity on the same sparse base Ψ is k_j. In JSM-1, each signal contains two parts. The signal constructed by this model is mostly affected by the global environment, and the local environment affects the local signal. For example, a sensor network is distributed over a forest to measure temperature, where sunlight is the global influence, while shades of leaves, water flow, and animal activity can all be considered as local influences. In contrast, signals under the JSM-2 model only contains the second part.

The joint sparse decomposition optimization algorithm proposed in this paper is based on the two models to solve the corresponding sparse coefficient vector.

3 Description of the Proposed Method

3.1 JSM1-OMP Algorithm

First define the signal to be decomposed is $X = [x_1, x_2, \cdots, x_J]$, J is the number of antennas, $x_j \in \mathbb{R}^M$ is the signal received by the antenna j, $x_j(m)$ is the m*th* sample of x_j, dictionary is $D \in \mathbb{R}^{M \times N}$ and X can be sparsely represented on the dictionary D.

Considering characteristics of JSM-1 model, we propose the JSM1-OMP algorithm by exploiting the correlation of signals. We estimate the common part of certain signal as the common part of other signals, and then calculate the unique part for each signal. Finally, we sum over the common part and the unique part to recovery the signals. The specific process is summarized as follows.

(1) Parameter initialization: The number of antennas is J, the antenna receiving sample is $X = [x_1, x_2, \cdots, x_J]$, the dictionary is D, the common sparsity is K_c, and the specific part of sparsity is K_j.

(2) The OMP algorithm is used to sparsely decomposes a certain signal. Here, we select the first signal, and the sparsity is set as the common sparsity K_c so that the common sparse coefficients of all signals are obtained, i.e.

$$\theta_c = OMP(x_1, D, K_c) \tag{6}$$

where θ_c is the sparsity of the common coefficient, then the common sparse signal is computed by $z_c = D\theta_c$.

(3) To find the unique part of each signal, we subtract the common sparse portion obtained by Step (2) from each signal

$$x'_j = x_j - D\theta_c \tag{7}$$

(4) Calculate the unique sparse coefficients of each signal, and use the OMP algorithm to sparsely decompose the unique sparse signal of each channel. The sparsity is $K_j, j \in \{1, 2, \cdots, J\}$. The dictionary D is still used and the solution process is

$$\theta_j = OMP(x'_j, D, K_j) \tag{8}$$

(5) Calculate the sparse coefficient of each signal and add the common sparse coefficient and the unique sparse coefficient

$$\theta'_j = \theta_c + \theta_j \quad j \in \{1, 2, \cdots, J\} \tag{9}$$

(6) Recovery multiple antenna samples

$$\hat{x}_j = D\theta'_j \quad j \in \{1, 2, \cdots, J\} \tag{10}$$

A simple explanation is given for the complexity of the proposed method. First, for the JSM1-OMP algorithm, the sparse decomposition of the first step exploits the OMP algorithm, and the complexity is the same as the case in which an OMP algorithm is used for an antenna with the same sparsity. Then, when the common sparse coefficient is used as a prior condition to solve the special sparse coefficients, the complexity is same as the case in which all the antenna signals use the OMP algorithm. Therefore, the JSM1-OMP possesses lower computational complexity. We use three receiving antennas as an example and assume that the sparsity of the same component is K_c, the sparsity of different components is K_1, K_2, K_3, and the size of the dictionary is $M \times N$. The algorithm complexity of using OMP is $O((4K_c + K_1 + K_2 + K_3)MN)$, The complexity of JSM1-OMP is $O((K_c + K_1 + K_2 + K_3)MN)$.

3.2 JSM2-OMP Algorithm

The JSM2-OMP algorithm is an extension of the OMP algorithm under the JSM-2 model. The OMP algorithm under this model is optimized based on the structural characteristics. The core idea of the algorithm is to require the participation of all signals to find the index of the coefficient position, rather than just a single signal at a time. The signal residuals are respectively summed with the dictionary and then summed as a condition for finding the sparse position index. Specific steps are offered as follows.

(1) Parameter initialization: count $l = 1$, Sub-dictionaries $T_0 = []$, Residual $r_{j0} = x_j (j = 1, 2, \cdots, J)$, atomic index set $t_0 = []$, The maximum number of iterations $interNum = K$.
(2) Select the position index of the atom that most closely matches the signal

$$i_l = \arg \max_{i=1,2,\cdots,N} \left(\sum_{j=1}^{J} \left| d_i^H r_{j,l-1} \right| \right) \tag{11}$$

(3) Combine the atomic index found in step 2 with the previous index set

$$t_l = t_{l-1} \cup i_l \tag{12}$$

(4) Merge the atom corresponding to the index found in step 2 with the previously selected atom set

$$T_l = T_{l-1} \cup d_l \tag{13}$$

(5) Solve residuals

$$r_{j,l} = y_j - T_l((T_l^H T_l)^{-1} T_l^H y_j) \tag{14}$$

(6) The end of the algorithm is judged: if $l > \mathrm{int}erNum$ is satisfied, then step 7 is performed, if not, then return to step 2.
(7) Find sparse coefficients

$$\theta_j' = (T_l^H T_l)^{-1} T_l^H x_j \tag{15}$$

(8) Recovery multiple antenna receive signals

$$x_j' = D\theta_j' \tag{16}$$

The complexity of the JSM2-OMP algorithm is mainly reflected in the steps of summing residuals and the inner product of atoms which coincides with the OMP algorithm. Therefore, the two algorithms are the same complexity. Taking the sparse decomposition of J antenna signals as an example, we assume that the sparsity of each antenna signal is K, and the size of the dictionary is $M \times N$. The complexity of the two algorithms is both $O(JKMN)$

3.3 JSM-OMP Algorithm

The core of this algorithm is to add JSM2-OMP algorithm in the framework of JSM1 model. Considering the characteristics of these two models, in the JSM1-OMP algorithm, the first step of solving the common coefficient sparseness can be replaced with the JSM2-OMP algorithm. By doing so, the sparse locations can be found more accurately and errors can be reduced more effectively. We call the improved algorithm JSM-OMP algorithm. The basic flow of this algorithm is basically the same as that of JSM1-OMP. It differs only when the second step is to solve common sparse coefficients

$$\theta_c = \mathrm{JSM2\_}OMP(X, D, K_c) \tag{17}$$

We have already analyzed that the complexity of the JSM2-OMP is the same as that of the OMP. In the first step, the JSM-OMP is used. The complexity of the algorithm is the same as the OMP algorithm. When estimating the sparse coefficients of different compositions, all OMP algorithms are used, and their complexity is also the same. Therefore, the total complexity of the JSM-OMP algorithm is the same as that of the OMP algorithm. Taking the sparse decomposition of four antenna signals as an example, the total complexity of these two algorithms can be expressed as $O((4K_c + K_1 + K_2 + K_3 + K_4)MN)$.

4 Numerical Simulations

To illustrate the performance of the proposed algorithm, we first compare it with the traditional OMP algorithm, and then analyze the performance of the JSM-OMP algorithm and the JSM1-OMP algorithm. The simulation experiments use the actual signal model and the signal RMSE to measure the performance of the algorithm. RMSE can be defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}[x(n) - \hat{x}(n)]} \tag{18}$$

4.1 Sparse Decomposition Effect Simulation of JSM-1 Model Signal

Specific simulation parameters are set as follows. Dictionary is Fourier basis. The number of rows is $M = 256$, columns is $N = 256$. The number of antennas is 6, the common sparsity is 6, the unique sparsity is set to be the same as 2, each sample is a different frequency sample combination, and all sample frequencies have the same part and different parts. Signal-to-noise ratio is set to $(-15 \sim 25)$ dB. The results are shown in Fig. 1. It is observed that their sparse decomposition performance is basically similar for different SNR. Especially when the SNR is large, it is not difficult to understand because the essence of the JSM1-OMP algorithm is the OMP algorithm. Therefore, the algorithm's sparse decomposition performance should be similar to the OMP algorithm, so its advantages are mainly reflected in the reduced complexity.

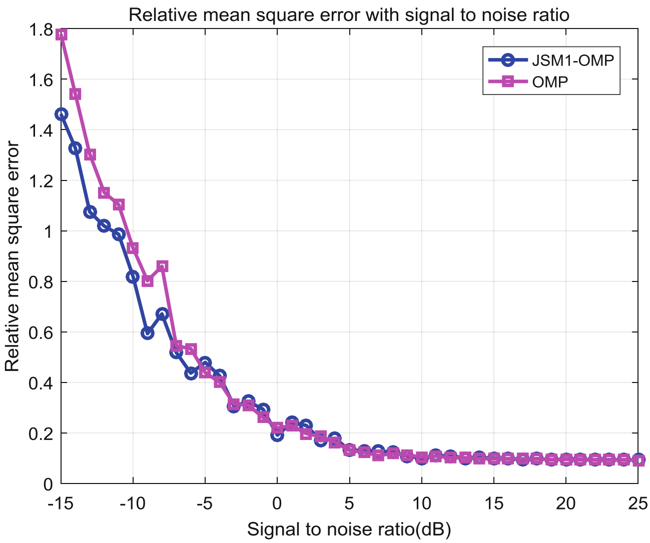

Fig. 1. Performance comparison between JSM1-OMP algorithm and OMP algorithm

4.2 Sparse Decomposition Effect Simulation of JSM-2 Model Signal

Specific simulation parameters are set as follows. Dictionary is Fourier basis. The number of rows is $M = 256$, Columns is $N = 256$.The number of antennas is 10, the sparsity is $K = 7$. Each signal is a combination of signals of different frequencies, and the same signal frequency of different antennas has different coefficients. The SNR is set to $(-15 \sim 25)$ dB.

The simulation results are illustrated in Fig. 2. It can be seen that the recovery performance of JSM2-OMP is better than that of OMP. This advantage is especially obvious at low signal-to-noise ratios because the JSM2-OMP algorithm makes full use of the correlation between signals.

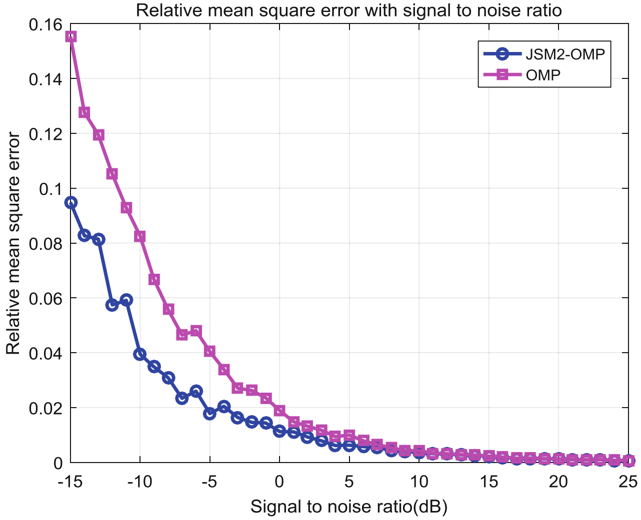

Fig. 2. Performance comparison between JSM2-OMP algorithm and OMP algorithm

4.3 Sparse Decomposition Effect Simulation of JSM-1 Model Signal

The specific simulation parameter setting is the same as the parameter setting in the JSM1-OMP algorithm simulation. The simulation results are shown in Fig. 3. Simulation results show that our improved algorithm has obvious performance advantages over the previously proposed JSM1-OMP algorithm.

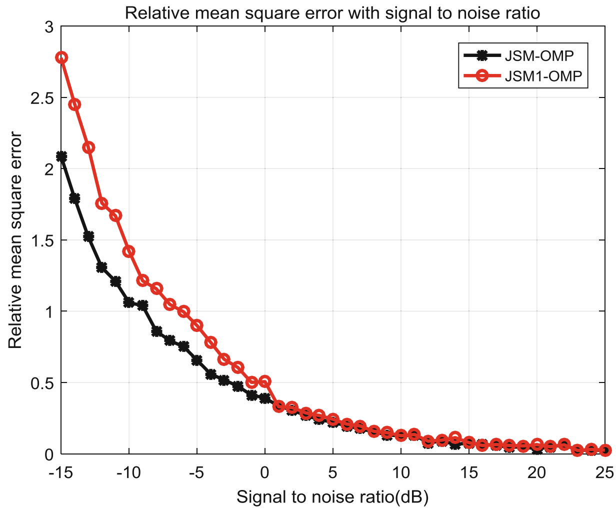

Fig. 3. Performance comparison between JSM-OMP algorithm and JSM1-OMP algorithm

5 Conclusions

We first introduce JSM-1 and JSM-2. Then, based on the structural characteristics of these two models, we propose JSM1-OMP and JSM2-OMP algorithm. The two algorithms are compared with the traditional sparse decomposition algorithm OMP in terms of complexity and sparse decomposition performance under multi-antenna conditions. In terms of complexity, we find that JSM1-OMP is superior to OMP, and JSM2-OMP is the same as OMP. When it comes to sparse decomposition performance, JSM1-OMP is similar to OMP, while JSM2-OMP is much better than OMP, especially in low SNR cases. Then, combining JSM-1 model with JSM2-OMP algorithm, we propose another joint sparse decomposition algorithm, namely JSM-OMP. This algorithm compensates for the defects of OMP in finding common sparse coefficients and makes the decomposition performance of the algorithm much improved compared to JSM1-OMP.

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