A Resistance Frequency Offset Synchronization Scheme Based on the Zadoff-Chu Conjugate Sequence

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Abstract. Zadoff-Chu (ZC) sequences have been used as synchronization sequences in many wireless communication systems because of their perfect correlation properties. However, almost all of these ideal characteristics are based on the assumption of zero carrier frequency offset (CFO). Under large frequency offset circumstances, the perfect autocorrelation property of ZC sequence is destroyed, where the main correlation peak is decreasing while the vice peak is increasing, consequently degrading the timing performance. In this paper, the autocorrelation of the ZC sequence and its conjugate sequence are investigated, and the symmetry between the modulus values of their autocorrelation functions is developed as well. Taking advantage of this symmetry, a novel training sequence composed of ZC sequence and ZC conjugate sequence is proposed. Also proposed is a corresponding synchronization scheme enabling robust timing synchronization based on the ZC sequence and ZC conjugate sequence at the receiver in the presence of large CFO.

Keywords: Zadoff-Chu sequences · Resistance frequency offset · Synchronization method

1 Introduction

In 4G systems, ZC sequences are used as the downlink primary synchronization signals (PSSs) [1,2] by means of replacing the PN sequences which are used in 2G and 3G systems. From a mathematical point of view, ZC sequences indeed have perfect autocorrelation properties [3], but almost all of these characteristics are based on the assumption of zero carrier frequency offset (CFO). However, for a practical wireless communication system, the frequency offset is almost

This work is supported partially by the fund of Science and Technology on Communication Networks Laboratory under grand (No. SXX18641X027) and the National Natural Science Foundation General Program of China (No.61671179).

S. Han et al. (Eds.): AICON 2019, LNICST 286, pp. 270–276, 2019.
https://doi.org/10.1007/978-3-030-22968-9_24

unevitable because of the mismatch between the transmitter and receiver oscillators as well as the impact of Doppler shift [4]. In the cases where the CFO is small, ZC sequence can overcome the effect of CFO by its self-robustness, however, the autocorrelation characteristics of ZC sequence could be destroyed with the gradual increase of CFO [5,6]. Under large frequency offset circumstances, the perfect autocorrelation property of ZC sequence is lost because of the decreasing main correlation peak and the increasing vice peak as well, degrading the time performance consequently as a result. As the carrier frequency increases, the CFO between the transceiver and the receiver becomes increasingly larger. Taking the system with 6 GHz carrier frequency as an example, even if the crystal oscillator with 3 mmp accuracy is used, the CFO of the system is up to 18 kHz. In this case, the traditional ZC sequence can not achieve accurate synchronization as a result.

In order to reduce the impact of CFO, the ZC sequence is replaced by m sequence in 5G system [7], which will, however, lead to the deterioration of peak to average power ratio (PAPR). [8] designs a training sequence with two OFDM symbols each of which is with a cyclic suffix (CS) in addition to the CP. Although this approach can improve the robustness of time synchronization, the addition of cyclic suffix reduces the spectral efficiency of the system. [9] adopts hybrid carrier to combat CFO which can only enhance the resistance frequency offset ability of the signal, but can not solve the timing problem brought by the CFO.

In this paper, a novel training sequence is proposed on the basis of the symmetry characteristics of the autocorrelations between the ZC and its conjugate sequences. Moreover, a novel resistance frequency offset synchronization approach is developed correspondingly, which enables robust time synchronization in the presence of CFO.

2 System Model

In this paper, a base-band equivalent system model where the training sequence is composed of a ZC sequence and its conjugate sequence is considered. Assuming that the training sequence length in the conventional frame structure is N, as shown in Fig. 1(a), in order to ensure that the proposed training sequence does not generate additional overhead, the ZC sequence length N_{zc} used in the proposed scheme is

$$N_{zc} = \lfloor N/2 \rfloor \tag{1}$$

where $\lfloor . \rfloor$ represents the rounding down. According to the frame structure shown in Fig. 1(b), the training sequence can be given by the formula below:

$$X(n) = \begin{cases} ZC(n) & 0 \leq n < N_{zc} \\ ZC^*(n) & N_{zc} \leq n < 2N_{zc} \end{cases} \tag{2}$$

For a communication system adopting the training sequence shown in (2) above, in the cases where frequency offset exists between the transmitter and the receiver, the received signal can be expressed as

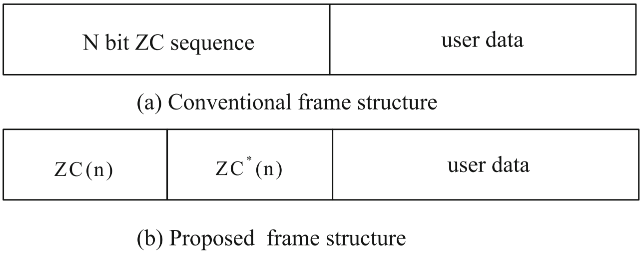

Fig. 1. Frame structure for different systems.

$$R(n) = X(n)\exp[-j2\pi\frac{\Delta f}{f_s}n] + w(n) \tag{3}$$

where Δf is the frequency offset between the transmitter and receiver, f_s is the baseband sampling rate, and $w(n)$ is the n^{th} additive white gaussian noise (AWGN) sample, respectively.

3 Proposed Synchronization Method

The original definition of a N_{zc}-length Zadoff-Chu sequence is as follows [10]:

$$a_u(n) = \exp[-j2\pi u\frac{n(n+1)/2 + ln}{N_{zc}}] \tag{4}$$

where $u \in \{1, ..., N_{zc}-1\}$ is the root index of ZC sequence, $n = 0, 1, ..., N_{zc}-1$, l can be any integer. For the sake of simplicity, l is typically set to 0 in most actual system (such as LTE system). Thus, in the actual system, the ZC sequence used for synchronization is given by

$$ZC_u(n) = \exp[-j\frac{\pi un(n+1)}{N_{zc}}] \tag{5}$$

Assuming that the transmitter only transmits the ZC sequence, and the receiver correlates the received signal with the sliding local ZC sequence, where the correlation function is:

$$r_{zc}(\tau) = \sum_{n=0}^{N_{zc}-1} R(n-\tau)ZC_u^*(n) \tag{6}$$

where τ is the time offset. Replacing $R(n)$ and $ZC_u(n)$ in (6) without considering the effect of noise, it yields

$$\begin{aligned} r_{zc}(\tau) = &\exp\left[-j\frac{\pi u(\tau^2-\tau)}{N_{zc}} + j\frac{2\pi\Delta f\tau}{f_s}\right] \\ &* \sum_{n=0}^{N_{zc}-1} \exp\left[j2\pi n\left(\frac{u\tau}{N_{zc}} - \frac{\Delta f}{f_s}\right)\right] \end{aligned} \tag{7}$$

As can be seen from formula (7), the correlation function of the ZC sequence can be expressed as the product of the two separate parts in the presence of the frequency offset. The former part only impacts the phase of the correlation function without affecting the amplitude while the latter one is just the sum of geometric series. In the cases where the time offset is τ, only $N_{zc} - |\tau|$ elements of the summation items could be nonzero. Through the use of geometric series summation formula, the modulus of the ZC sequence correlation function can be written as

$$|r_{zc}(\tau)| = \begin{cases} N_{zc} - |\tau| & \frac{u\tau}{N_{zc}} - \frac{\Delta f}{f_s} \in Z \\ \left| \frac{1-\exp\left[j2\pi(N_{zc}-|\tau|)\left(\frac{u\tau}{N_{zc}} - \frac{\Delta f}{f_s}\right)\right]}{1-\exp\left[j2\pi\left(\frac{u\tau}{N_{zc}} - \frac{\Delta f}{f_s}\right)\right]} \right| & \text{others} \end{cases} \tag{8}$$

where Z represents the set of integers. Similarly, when the transmitter only transmits the conjugate ZC sequence, the modulus of its correlation function can be given by

$$|r_{zc^*}(\tau)| = \begin{cases} N_{zc} - |\tau| & \frac{-u\tau}{N_{zc}} - \frac{\Delta f}{f_s} \in Z \\ \left| \frac{1-\exp\left[j2\pi(N_{zc}-|\tau|)\left(\frac{-u\tau}{N_{zc}} - \frac{\Delta f}{f_s}\right)\right]}{1-\exp\left[j2\pi\left(\frac{-u\tau}{N_{zc}} - \frac{\Delta f}{f_s}\right)\right]} \right| & \text{others} \end{cases} \tag{9}$$

As indicated by (8) and (9), with the gradual increase of the Δf, the modulus of the main correlation peak $|r_{zc}(0)|$(or $|r_{zc^*}(0)|$) will decrease, even as low as to 0 (when $\Delta f = f_s/N_{zc}$). In contrast, some vice peaks will increase for whatever ZC sequence or its conjugate sequence. These changes will destroy the autocorrelation characteristic of the ZC sequence, and degrade its timing synchronization performance. However, by comparing the (8) and (9), it is easy to find that

$$|r_{zc}(\tau)| = |r_{zc^*}(-\tau)| \tag{10}$$

That is to say, for a particular Δf, if the correlation value of ZC sequence has a peak at the position of τ ahead, the correlation value of the ZC conjugate sequence would inevitably have an equal peak at the position of τ lag.

For ZC sequence and ZC conjugate sequence, due to the symmetry between their autocorrelation peaks, the advantage of whose character could be taken in order to achieve the correct synchronization under frequency offset conditions. First and foremost, for the proposed training sequence, the ZC sequence and ZC conjugate sequence are adopted by the receiver for the purpose of sliding correlation. Secondly, each peak position exceeding the threshold respectively is recorded and all of these positions are averaged subsequently. Last but not least, $N_{zc}/2 + 1$ is added to the average position to get the starting position of the user data. In this way, the starting position of the user data can be calculated as follows:

$$P = \frac{\sum_{k=0}^{M_1} I_{zc}(k) + \sum_{k=0}^{M_2} I_{zc^*}(k)}{M_1 + M_2} + N_{zc}/2 + 1 \tag{11}$$

where M_1 and M_2 are the number of over threshold peaks in the two groups of sliding correlation, I_{zc} and I_{zc^*} are the position indexes of the corresponding correlation peaks.

Moreover, whether the 2 sequences have good cross-correlation or not needs to be taken into consideration since the proposed training sequence is composed of ZC sequence and its conjugate sequence as well. With the result of $\exp[-j\pi n(n+1)] = 1$, the ZC conjugate sequence is expressed as

$$\begin{aligned} ZC_u^*(n) &= \exp\left[j\frac{\pi un(n+1)}{N_{zc}}\right] \\ &= \exp\left[-j\frac{\pi(-u)n(n+1)}{N_{zc}}\right]\exp[-j\pi n(n+1)] \\ &= ZC_{N_{zc}-u}(n) \end{aligned} \tag{12}$$

Thus, it is shown that ZC conjugate sequences with root exponent u are exactly the ZC sequences with the root exponent $N_{zc} - u$. Given that the cross-correlation between two ZC sequences of different root exponents is quite small [11], the interaction of ZC sequence and ZC conjugate sequence in the sliding correlation could be therefore ignored.

4 Simulation Results

In order to evaluate the performance of the proposed scheme in resistance frequency offset, two groups of sliding correlation values are simulated with $\Delta f = 0$ kHz and $\Delta f = 8$ kHz, respectively. The simulation parameters are as follows: $N = 127$, $N_{zc} = 63$, $u = 29$, $f_s = 1$ MHz, and the threshold is set to 60% of the maximum autocorrelation value. In the cases where the frequency offset is zero, the maximum of the correlation peaks occurs at the location where the local sequence is exactly aligned with the training sequence, as shown in Fig. 2. In this case, both the conventional method and the proposed method can achieve correct synchronization. Additionally, in the cases where a 8 kHz frequency offset exists between the transmitter and receiver, the autocorrelation value at the correct synchronization position is made no longer the largest. Under such conditions, the traditional single-sequence sliding correlation will place the system synchronization in the wrong position, as shown in Fig. 3, on the contrary, the proposed method can still achieve the correct synchronization.

Figure 4 shows the Monte Carlo simulation results of the relationship between the correct synchronization probability and the frequency offset under the conditions of SNR = 10 dB. It can be seen from Fig. 4 that the proposed method can achieve the correct synchronization in the larger frequency offset range in comparison with the traditional method as a result. It should be noted that the correct synchronization probability of the proposed scheme decreases to a certain extent near the 8 kHz CFO, which is due to the close relationship between the two correlation peaks and the threshold in this case. Nevertheless, compared with the traditional scheme, the proposed scheme is still very significant for improving the correct synchronization probability in the case of large CFO.

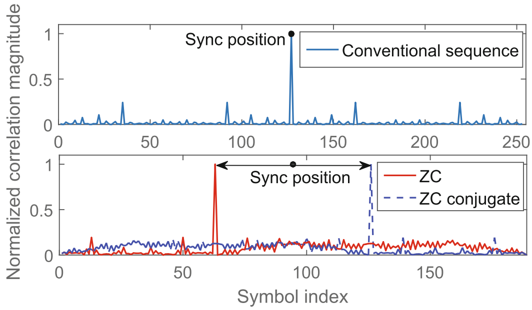

Fig. 2. Training sequence correlation without CFO

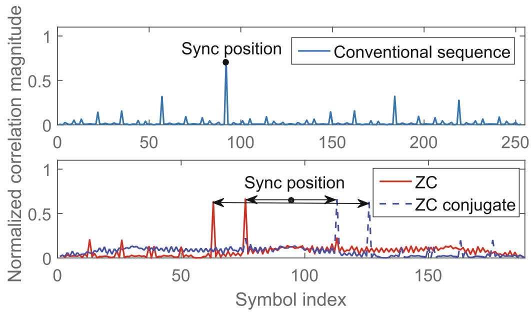

Fig. 3. Training sequence correlation with 8 kHz CFO

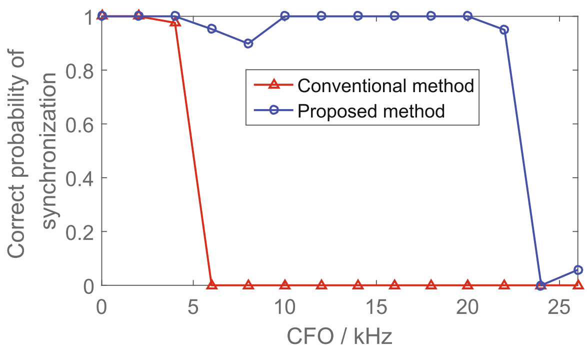

Fig. 4. Correct probability of synchronization when SNR = 10 dB

5 Conclusion

In this paper, a novel training sequence composed of ZC sequence and ZC conjugate sequence is proposed by taking advantage of the symmetry between their autocorrelation peaks. In addition, a corresponding synchronization method is developed by adopting ZC sequence and ZC conjugate sequence respectively for the purpose of sliding correlation, which enables robust time synchronization in the presence of CFO. Simulation results show that the proposed method has better robustness under large frequency offset in comparison with the traditional method as a result.

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