Millimeter Wave Massive MIMO Channel Estimation and Tracking

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Abstract. With the rapid development of 5G, massive MIMO technology has become one of the most important technologies in 5G. However, while massive MIMO technology could provide reliable performance guarantee, with the number of antennas increases, the problem of channel estimation has also become more complicated. To solve this problem, we propose a channel state estimation and tracking algorithm based on particle filter, when considering the temporal correlation between the millimeter wave narrowband block fading channels. A channel state model including channel gain, the angle of arrival, and the angle of departure has been established. A performance comparison is carried out, in terms of normalized mean square error, considering massive MIMO channel estimation, for different algorithms. We also take into account the performance affected by signal to noise ratio and the number of antennas. Numerical results show that the performance can be considerably improved in the case of a large number of antennas over the conventional scheme. Furthermore, this algorithm also has better performance under traditional MIMO conditions.

Keywords: Millimeter wave · Massive MIMO · Channel tracking · Particle filter

1 Introduction

With the rapid development of 5G technology, millimeter wave technology has become a research hotspot in 5G related technologies, because millimeter wave can provide lower delay and higher transmission rate for the applications in 5G [10]. However, millimeter wave has higher frequency band compared with traditional microwave communication. In this case, millimeter wave has a shorter wavelength, which is much smaller than the size of obstacles. Therefore millimeter has limited scatterings and higher path loss during transmission, which also makes the millimeter wave channel sparse [6]. Compared with the traditional communication technology with a lower carrier frequency range, millimeter wave technology has higher cost and power consumption.

In order to overcome the above shortcomings of millimeter wave, massive multiple-input multiple-output (MIMO) technology can be used to provide huge

S. Han et al. (Eds.): AICON 2019, LNICST 286, pp. 57–69, 2019.
https://doi.org/10.1007/978-3-030-22968-9_6

gain for communication systems [8]. Setting tens or even hundreds of antenna elements at the base station can provide sufficient array gain and path gain for the communication system, with high spectral efficiency and energy efficiency [5]. Since the wavelength of the millimeter wave is short, and the antenna unit spacing of the uniform linear array (ULA) is half the wavelength, the transmitter and the receiver can use a large-scale antenna array for information transmission under the condition of the limited antenna array size. The great path gain of massive MIMO technology can meet the stringent performance requirements in 5G [9].

Beamforming technology is one of the most important technologies of massive MIMO. Because beamforming technology can provide huge beam gain for communication systems to overcome the severe path loss problems caused by millimeter wave and the hardware limitations in massive MIMO systems [5]. Whether analog beamforming, digital beamforming, or hybrid analog beamforming, the transmit beamforming vector and the receive combiner vector need to be designed based on channel state information (CSI). Therefore, the design of a good beamforming structure needs to be obtained through effective channel estimation.

In general, channel estimation can be divided into two ways: implicit channel estimation and explicit channel estimation. In the implicit channel estimation process, the transmitter and the receiver do not need to know the complete channel state information, and the beamforming can be performed iteratively through the pre-designed beamforming codebook, under certain standard conditions [1]. However, in this way, the massive MIMO communication system needs to bear higher complexity costs, which is not able to meet the stringent performance requirements of the millimeter wave communication system. For explicit channel estimation, Alkhateeb proposed an adaptive algorithm for estimating the channel, which uses a beamforming vector with different beamwidths at different stages to search for the paths [2]. The authors of [11] proposed an algorithm to estimate the angles of arrival (AoA) and angles of departure (AoD) of the beam by using extended Kalman filter, and achieved good results. The algorithm is still worth continuing to improve when the number of antennas is large.

The contributions of this paper are mainly in two aspects. First, we propose a channel estimation and tracking algorithm that can be applied to multi-antenna MIMO, where the number of antennas of massive MIMO can sometimes be as many as hundreds. The traditional MIMO channel estimation algorithm can only work when the number of antennas is small. Second, our proposed algorithm also has a good performance when the number of antenna is small, especially in the case of high signal to noise ratio (SNR).

The rest part of the paper is organized as follows. Section 2 illustrates the channel model and the temporal correlated of millimeter wave MIMO channel. Section 3 gives the expression of the observation and presents our algorithm for channel estimation and tracking. In Sect. 4, performance analysis is presented comparing our method with traditional one. Then, Sect. 5 concludes this paper.

2 System Structure and Channel Model

This section introduces the millimeter wave MIMO channel model. A channel model of the millimeter wave can be established in the angle domain, the expression of the channel matrix $\mathbf{H}$ is

$$\mathbf{H} = \sum_{n=1}^{N} \alpha_n \mathbf{a}_r\left(\phi_{n,A}\right) \mathbf{a}_t^H\left(\phi_{n,D}\right), \tag{1}$$

where N is the number of paths, α_n is the paths gain, $\mathbf{a}_r\left(\phi_{n,A}\right)$ and $\mathbf{a}_t\left(\phi_{n,D}\right)$ represent the array response vectors of AoA and AoD, respectively. The $\phi_{n,A}$ and $\phi_{n,D}$ denote AoAs and AoDs of N independent paths. Assume that the ULA model is deployed at the transmitter and the receiver. Then, the expressions of the array response vectors in the AoA and AoD are

$$\mathbf{a}_r\left(\phi_{n,A}\right) = \frac{1}{\sqrt{N_r}}\left[1, e^{j\frac{2\pi}{\lambda}d\cos(\phi_{n,A})}, \cdots, e^{j(N_r-1)\frac{2\pi}{\lambda}d\cos(\phi_{n,A})}\right]^T \tag{2}$$

and

$$\mathbf{a}_t\left(\phi_{n,D}\right) = \frac{1}{\sqrt{N_t}}\left[1, e^{j\frac{2\pi}{\lambda}d\cos(\phi_{n,D})}, \cdots, e^{j(N_t-1)\frac{2\pi}{\lambda}d\cos(\phi_{n,D})}\right]^T, \tag{3}$$

where λ is the wavelength of the carrier and d is the distance between transmit antennas.

The following is a signal model for a millimeter wave massive MIMO system. For simplicity, assume that the pilot $\mathbf{x}$ to be transmitted is $\mathbf{1}$. The received signal expression can be expressed as

$$\begin{aligned}\mathbf{y} &= \mathbf{w}^H\mathbf{H}\mathbf{f} + \mathbf{v}\\ &= \sum_{n=1}^{N} \alpha_n \mathbf{w}^H \mathbf{a}_r\left(\phi_{n,A}\right) \mathbf{a}_t^H\left(\phi_{n,D}\right)\mathbf{f} + \mathbf{v},\end{aligned} \tag{4}$$

where the vectors $\mathbf{w}$ and $\mathbf{f}$ represent the combiner vector and the beamforming vector at the receiver and the transmitter, respectively. The received signal expression for a specific path will be further introduced in the Sect. 3, and its form will be further simplified.

2.1 Temporal Correlated Millimeter Wave MIMO Channel Model

Now we consider narrowband block-fading channel state model in the millimeter wave massive MIMO system [4]. In this channel model, parameters are stable in the same block and have the temporal correlation in adjacent blocks. First, we define a channel state vector at block k as

$$\mathbf{x}[k] = \left[\alpha_R[k], \alpha_I[k], \phi_A[k], \phi_D[k]\right]^T, \tag{5}$$

where $\alpha_R[k]$ and $\alpha_I[k]$ represent the real and imaginary parts of the corresponding path gain of the same path respectively, $\phi_A[k]$ and $\phi_D[k]$ are AoA and AoD of the corresponding path. Assume that the path gain in the channel matrix at different blocks is subject to the first-order Gauss Markov model, as shown below

$$\alpha_i [k+1] = \rho\alpha_i [k] + \varsigma [k], \tag{6}$$

where $\rho \in [0,1]$ is the time correlation coefficient, which follows Jakes' model [7] according to $\rho = J_0 (2\pi f_D T)$. The $J_0 (\cdot)$ is the zeroth order Bessel function of first kind, and the f_D and T denote the maximum Doppler frequency and channel block length, respectively. The subscript i indicates the real or imaginary part of the path gain, $\varsigma [k] \sim \mathcal{N}\left(0, \frac{1-\rho^2}{2}\right)$ represents excitation noise between adjacent states. In the adjacent channel blocks, the corresponding AoA and AoD state update models are

$$\phi [k+1] = \phi [k] + n. \tag{7}$$

$\phi [k] \in (-\pi, \pi]$ represents the AoA or AoD of the path in the channel, and n represents state excitation noise.

Through the path gain between the adjacent channel blocks and the state update model of AoA and AoD, we can conclude that the corresponding channel state vector equation of the millimeter wave massive MIMO system can be expressed as follows

$$\mathbf{x} [k+1] = \mathbf{F}\mathbf{x} [k-1] + \mathbf{u} [k-1], \tag{8}$$

where $\mathbf{F} = diag ([\rho, \rho, 1, 1])$ and $\mathbf{u} [k] \sim \mathcal{CN} (0, \Sigma_u)$, $\Sigma_u = diag([\frac{1-\rho^2}{2}, \frac{1-\rho^2}{2}, \sigma_A^2, \sigma_D^2])$ represents the noise during the update of different state variables.

3 Observation and Channel Tracking

3.1 Observation

In order to complete the implementation of dynamic channel estimation and tracking algorithm, the corresponding observation equation is needed. We define $\Phi_A = \cos\phi_A - \cos\overline{\phi_A}$, $\Phi_D = \cos\phi_D - \cos\overline{\phi_D}$, where $\overline{\phi_A}$ represents the beam direction controlled by the combiner, and $\cos\overline{\phi_D}$ represents the direction of the beamforming vector at the transmitter. A simplified expression can be get as follows

$$\mathbf{w}^H (\overline{\phi_A})\mathbf{a}_r(\phi_A) = \frac{1}{N_r}\frac{1-e^{jN_r kd\Phi_A}}{1-e^{jkd\Phi_A}}. \tag{9}$$

Similarly, we can get a simplified expression of $\mathbf{a}_t^H(\phi_D)\mathbf{f}(\overline{\phi_D})$. Then received signal can be simplified as

$$\mathbf{y} = \sum_{i=1}^{I} \frac{\alpha_i}{N_r N_t}\frac{1-e^{jN_r kd\Phi_{A,i}}}{1-e^{jkd\Phi_{A,i}}}\frac{1-e^{-jN_t kd\Phi_{D,i}}}{1-e^{-jkd\Phi_{D,i}}} + \mathbf{v}. \tag{10}$$

The transmitter transmit the known training symbols through the beamforming vector at the transmitter, and the final received signal can be obtained by using the combiner vector at the receiver. Section 2 has introduced the millimeter wave dynamic channel state model. Therefore, in the training phase, we pre-designed the corresponding beamforming matrix $\mathbf{F}$ and the combiner matrix $\mathbf{W}$, which are expressed as

$$\begin{aligned}\mathbf{W} &= [\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_N],\\ \mathbf{F} &= [\mathbf{f}_1, \mathbf{f}_2, \cdots, \mathbf{f}_N],\end{aligned} \tag{11}$$

where each column of $\mathbf{F}$ and $\mathbf{W}$ represents a beamforming control vector in a specified direction. The final expression of the received signal that can be obtained is

$$\begin{aligned}\mathbf{Y} &= \mathbf{W}^H\mathbf{H}\mathbf{F} + \mathbf{V}\\ &= \begin{bmatrix} \mathbf{w_1^H H f_1} & \mathbf{w_1^H H f_2} & \cdots & \mathbf{w_1^H H f_N} \\ \mathbf{w_2^H H f_1} & \mathbf{w_2^H H f_2} & \cdots & \mathbf{w_2^H H f_N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{w_N^H H f_1} & \mathbf{w_N^H H f_2} & \cdots & \mathbf{w_N^H H f_N} \end{bmatrix} + \mathbf{V}.\end{aligned} \tag{12}$$

We select the elements on the diagonal we want to get

$$\begin{aligned} y_n &= \mathbf{w}_n^H \mathbf{H} \mathbf{f}_n + v \\ &= \sum_{i=1}^{I} \mathbf{w}_n^H \alpha_i \mathbf{a}_r(\phi_{i,A}) \mathbf{a}_t^H(\phi_{i,D}) \mathbf{f}_n + v \\ &= \alpha_n \mathbf{w}_n^H \mathbf{a}_r(\phi_{n,A}) \mathbf{a}_t^H(\phi_{n,D}) \mathbf{f}_n + \\ &\sum_{i \neq n} \alpha_i \mathbf{w}_n^H \mathbf{a}_r(\phi_{i,A}) \mathbf{a}_t^H(\phi_{i,D}) \mathbf{f}_n + v. \end{aligned} \tag{13}$$

For (13), due to the determinism of the main lobe direction of beamforming, consider the latter two parameters as a new noise v_n. Let $\mathbf{f}_n = \mathbf{a}(\bar{\phi}_{n,D})$, $\mathbf{w}_n = \mathbf{a}(\bar{\phi}_{n,A})$, by using the conclusion of (10), we can get

$$\begin{aligned} y_n &= \frac{\alpha_n}{N_r N_t} \frac{1 - e^{jN_r kd\Phi_A}}{1 - e^{jkd\Phi_A}} \frac{1 - e^{-jN_t kd\Phi_D}}{1 - e^{-jkd\Phi_D}} + v_n \\ &= h(\mathbf{x}[k]) + v_n. \end{aligned} \tag{14}$$

In this way, the millimeter wave dynamic channel state model and the observation model of the system have been established. Next, we are going to present a algorithm to estimation and tracking the channel state under the state-space model.

3.2 Particle Filter Based Channel Tracking

Now, the state-space model of the channel through the state model (8) and the observation model (14) have been established. Here we can try to obtain the estimation of the channel state parameters by using the Bayesian estimation idea. The idea of Bayesian estimation can be expressed as estimating the hidden channel state information by prediction and correction. Prediction refers to estimating the channel state of the current block by using the channel state estimation of the previous block in consideration of the temporal correlation of the adjacent blocks. Correction means that the estimated current channel state is corrected based on the observation value of the current block, so that it is more likely to close to the true channel state. The idea of Bayesian estimates is shown in Fig. 1.

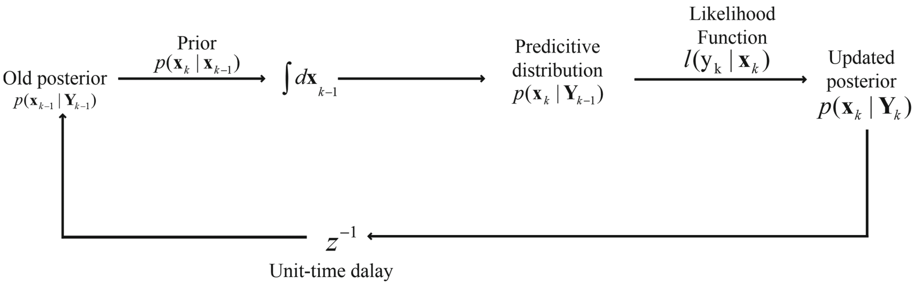


Fig. 1. The idea of Bayesian estimation.

In order to use Bayesian estimation, the state model (8) and the observation model (14) is needed. (8) expresses the changing relationship of channel parameters between adjacent blocks, and (14) expresses the relationship between the channel state and the received signal at a certain block. Bayesian estimates have a variety of specific forms, such as Kalman filter, extended Kalman filter, unscented Kalman filter, particle filter, etc., and their application scenarios are different. The algorithm in [11] tracking the AoA and AoD of the channel by extended Kalman filter. Extended Kalman filter can be used to estimate parameters for slightly nonlinear systems with Gaussian noise, therefore authors of [11] use it to estimate the AoA and AoD of channel. With the number of MIMO antennas increases, the nonlinearity of the system will be further improved. It will be difficult to solve the channel state estimation problem of massive MIMO by using extended Kalman filter. At the same time, since the channel characteristics of massive MIMO do not have an ideal model, the actual noise may not be described by simple Gaussian noise. Therefore, it is considered to estimate the channel of massive MIMO with particle filter [3]. Particle filter has a good performance when processing nonlinear systems with non-Gaussian noise.

We give the specific content of our algorithm in Algorithm 1, and then give some definitions of symbols. $x_{k,pre}$ is the predicted value for the $\mathbf{x}$ state at k block, and $y_{k,pre}$ is the observed value for the $\mathbf{x}$ state predictor at k block. It is worth noting that the $y_{k,pre}$ here is a plural. $x_k^{(i)} \sim p(x_k)$ represents the particle $x_k^{(i)}$ extracted from the posterior distribution $p(x_k)$ at k block. $w_k^{(i)}$ is the weight of the corresponding particle.

Algorithm 1. Particle Filter based Channel Tracking.

Require: Observation = $\{y_1, y_2, \cdots, y_n\}$. Initial value of channel state x_0.
Ensure: x_k as the estimation of channel state.

1: Initialization: $w_0^{(i)} = \frac{1}{N}$, $x_0^{(i)} \sim p(x_0)$
2: **for** each blocks $k = 1, 2, 3, \cdots$ **do**
3: **for** each particle $i = 1, 2, 3, \cdots, N$ **do**
4: State predict: $x_{k+1,pre}^{(i)} = \mathbf{F}x_k^{(i)} + u_k$
5: Observation predict: $y_{k+1,pre}^{(i)} = h\left(x_{k+1}^{(i)}\right) + v_{k+1}$
6: Weights update: $e_{k+1}^{(i)} = |y_{k+1,pre} - y_{k+1}|^2$,
7: $w_{k+1}^{(i)} = \frac{1}{2\pi N_0} e^{-\frac{e_{k+1}^{(i)}}{2N_0}}$
8: **end for**
9: $x_{k+1} = \sum_{i=1}^{N} w_{k+1}^{(i)} x_{k+1,pre}^{(i)}$
10: Resample: $x_{k+1}^{(i)} \sim p(x_{k+1})$, $w_{k+1}^{(i)} = \frac{1}{N}$
11: **end for**

When the channel is estimated by particle filter, the diversity of particles may gradually disappear over time. Therefore, we try to maintain the diversity of the particles by resampling after the estimation is completed at each blocks. The specific method is drawing a set of N discrete random variables $\left\{I^{(1)}, I^{(2)}, \cdots, I^{(N)}\right\}$ that take values in the corresponding set $\{1, 2, \cdots, N\}$ with probabilities $P\left(I^{(s)} = i\right) = w_k^{(i)}$. Then let $w_k^{(i)} = \frac{1}{N}$. When using particle filtering, the selection of the importance distribution $q\left(\mathbf{x}_k|y_k\right)$ is critical. In this algorithm, the left side of the particles of the previous moment are multiplied by the state transition matrix $\mathbf{F}$ and then the disturbance according to the statistics of the state transition noise $\mathbf{u}[k]$ is added.

By observing the algorithm table, It is easy to know that the complexity of particle filter increases linearly with the increase of the number of particles, and the requirement of storage also increases linearly with the increase of the number of particles. Therefore, particle filter's complexity and storage requirements are improved compared to extended Kalman filter.

4 Performance Analysis

In order to further verify the reliability of channel estimation and tracking of the algorithm in millimeter wave massive MIMO system, we carried out a series of simulation experiments and obtained the conclusion based on the corresponding simulation results.

The number of ULA antennas used at the transmitter and receiver are N_t, N_r respectively. ULA at the transmitter and receiver both are based on an analog beamforming structure. The number of RF chain is one. Pilots is transmitted under the channel model of millimeter wave narrowband block-fading. Since the channel of millimeter wave MIMO is sparse, different paths under the channel can be more easily distinguished. Assume that there is only one path falls within the main lobe, and the weak signals in the side lobes are treated as noise. The time correlation coefficient between adjacent channel blocks is $\rho = 0.995$, $\sigma_A^2 = \sigma_D^2 = \frac{0.5}{180}\pi^2$. The proposed algorithm is been simulated with using $N = 1000$ as the number of particles. For the sake of easy, assume only consider one path in the channel model. For each point in the figures, we simulate 100 blocks.

Compare our proposed algorithm (Abbreviated as PF) with the extended Kalman filter (Abbreviated as EKF) described in [11], and use the normalized mean square error (NMSE) $\frac{\mathbb{E}\left(\|\hat{\mathbf{x}}-\mathbf{x}\|_F^2\right)}{\mathbb{E}\left(\|\mathbf{x}\|_F^2\right)}$ (where $\hat{\mathbf{x}}$ is the estimation of $\mathbf{x}$) of the channel state as the evaluation criterion.

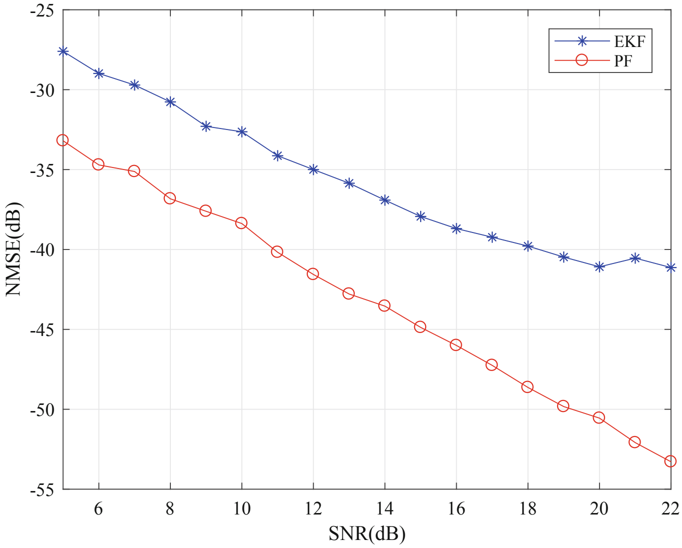


Fig. 2. NMSE of $\mathbf{x}$ versus SNR, $N_t = N_r = 16$.

First, consider the case where the NMSE of both algorithm changes with SNR from 5 dB to 22 dB when the number of transmitting antennas $N_t = 16$ and the receiving antennas $N_r = 16$. From Fig. 2, it can be seen that the system has strong linearity due to the small number of transmitting and receiving antennas. Therefore the EKF has achieved good results in estimating and tracking the channel in this case. However, it is easy to find that the performance of the proposed algorithm is still better than EKF at all range of SNR. The difference between the two algorithms' NMSE is further widened with the increase of SNR. The difference is only about 5 dB when SNR = 5 dB, but the difference is about 12 dB when SNR = 22 dB. Moreover, EKF reaches the performance limit when SNR = 20 dB, and the NMSE curve tends to be gentle. And the estimated value of both algorithm is more closer to the real situation with the SNR increases.

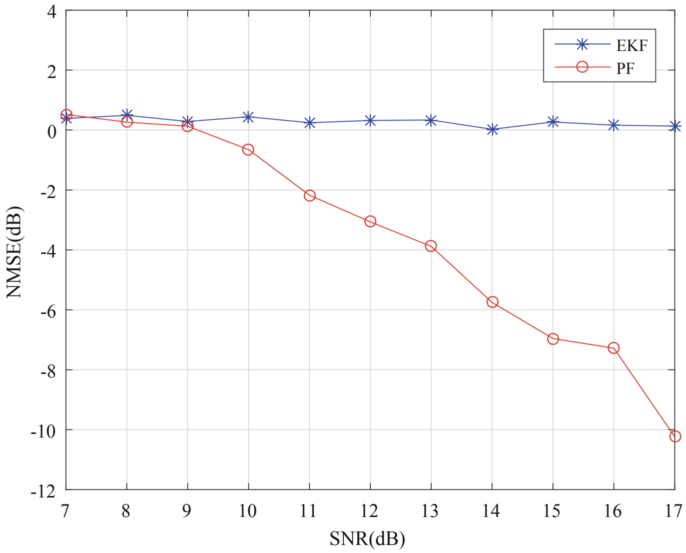
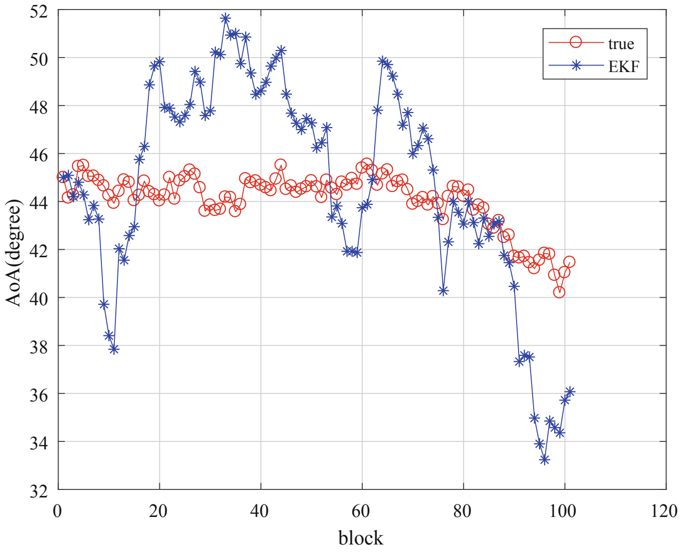


Fig. 3. NMSE of $\mathbf{x}$ versus SNR, $N_t = N_r = 64$.

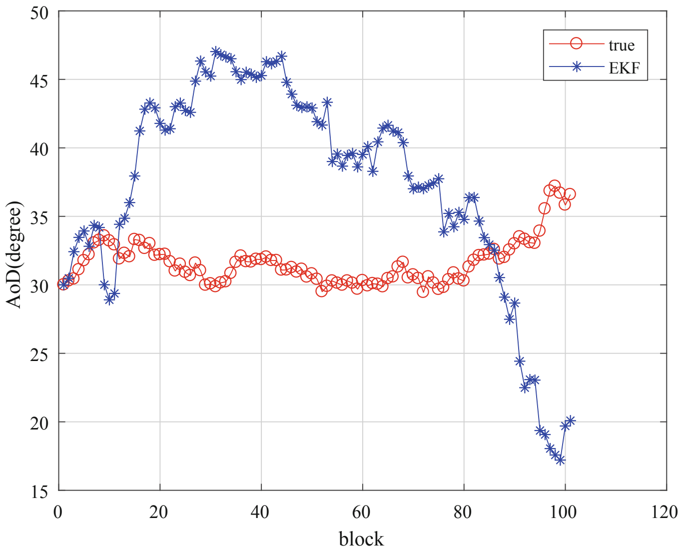
Considering that the number of antennas for massive MIMO can be as high as hundreds, we have the simulation with $N_t = N_r = 64$. Figure 3 shows the NMSE of both algorithm changes with SNR from 7 dB to 17 dB. It can be seen from Fig. 3 that the EKF is almost completely unable to estimate the state of the channel in this case, especially the estimation of AoA and AoD. We present simulation results of the tracking of AoA and AoD over time by EKF and PF under $N_t = N_r = 64$ and SNR = 20 dB, which are shown in Figs. 4 and 5, respectively.

We also consider the case where the NMSE of both algorithm changes with number of transmitting antennas N_t and the receiving antennas N_r from 4 to 64 when the SNR = 20 dB. It can be seen from the Fig. 6 that the processing capability of the EKF for the channel state is limited to the case where the number of antennas is less than 32. When the number of antennas exceeds 32, the estimation and tracking ability of the channel state cannot meet the basic requirements. The PF algorithm still has good ability to estimate and track channels when the number of antennas is more than 32.

The reason why EKF does not perform well under these conditions is extended Kalman filter can only handle a certain degree of nonlinear system. From the basic principle of EKF, due to the nonlinear tracking of EKF is mainly approximated to the true value by first-order partial derivative of the Jacobian matrix. Therefore, when the degree of nonlinearity is high, the EKF cannot track the dynamically changing parameter information very well. While particle filter can handle any nonlinear problem. With the number of transmitting antennas

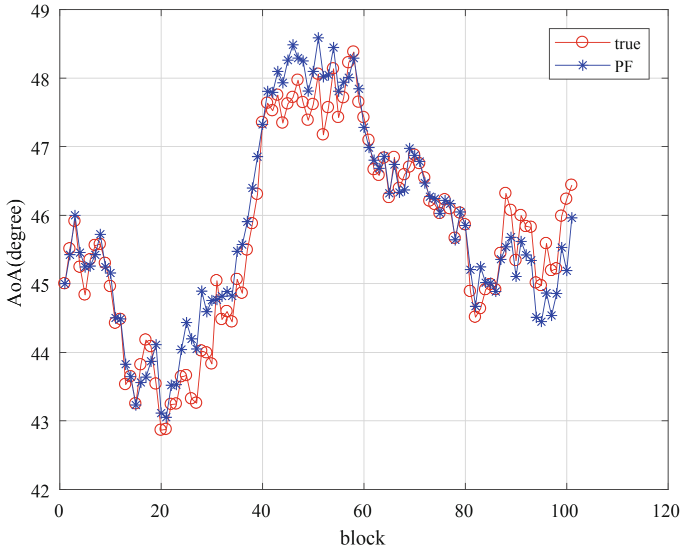


(a)

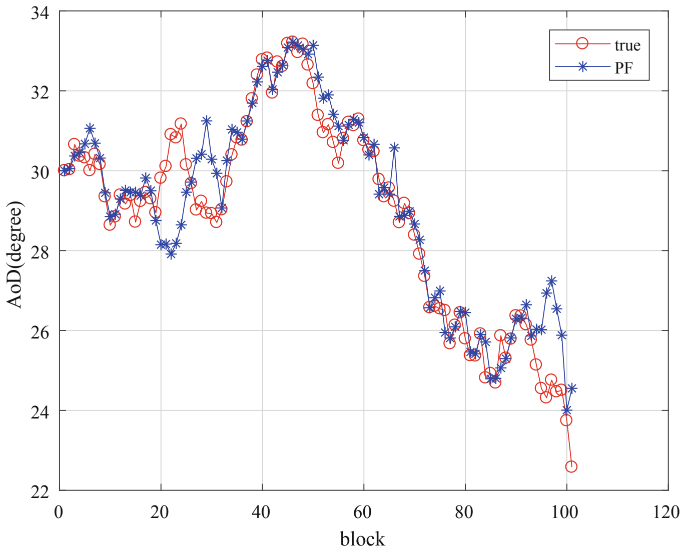


(b)

Fig. 4. EKF, AoA and AoD changes with blocks, SNR = 20 dB, $N_t = N_r = 64$.



(a)



(b)

Fig. 5. PF, AoA and AoD changes with blocks, SNR = 20 dB, $N_t = N_r = 64$.

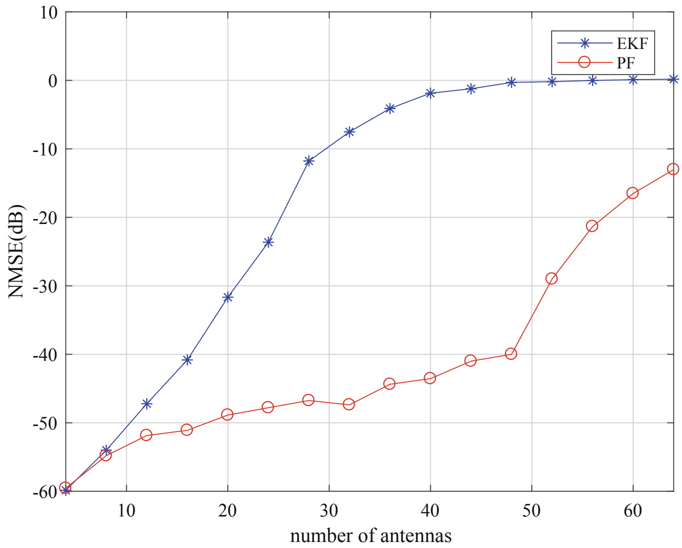


Fig. 6. NMSE of $\mathbf{x}$ versus the number of antennas, SNR = 20 dB.

and receiving antennas increases, the degree of nonlinearity of the system also increases, so PF will achieve better performance.

When the number of antennas is small and the signal to noise ratio is low, the PF has only a limited gain compared to the EKF. However, the performance improvement of PF comes at the cost of increased complexity and storage. Therefore, in practical applications, it is necessary to make a suitable choice among the two algorithms according to specific requirements.

5 Conclusions

In this paper, we propose a estimation and tracking algorithm based on particle filter, which can solve the problem of estimating and tracking the channel state of massive MIMO including channel gain, AoA and AoD. Although the algorithm has higher complexity than the traditional algorithm, the algorithm still has these advantages: This algorithm has better performance under traditional MIMO conditions, especially when the SNR is limited. This algorithm can be extended with the number of antennas increases, and have a great performance improved by tracking the AoA and AoD of channel.

Acknowledgment. This research was supported by the National Science Foundation of China (Under Grant: 61671173).

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