2D DOA Estimation of PR-WSF Algorithm Based on Modified Fireworks Algorithm

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Abstract. Two-dimensional direction of arrival (DOA) estimation has more application significance than one-dimensional estimation. However, the increase of computation scale causes serious problems of slow speed of solution and poor real-time performance. Among the common algorithms of two-dimensional direction of arrival (DOA) estimation, the weighted subspace fitting (WSF) algorithm possesses high accuracy, but its complexity in solving process weakens its performance advantage. In addition, the accuracy of WSF is poor under the condition of low signal-to-noise ratio (SNR) and insufficient snapshot number (i.e. threshold). Hence, this paper proposes a PR-WSF algorithm based on modified fireworks algorithm: the radius and number of explosions in fireworks algorithm are initially improved, then the ESPRIT algorithm combined with cramer-rao bound (CRB) is adopted to create a smaller searching space, and finally the pseudo-random noise resampling (PR) algorithm is introduced to improve the "threshold performance. The experimental results show that this algorithm balances the relationship between global search and local search, reduces unnecessary computation, and has better estimation performance at the threshold.

Keywords: 2-D DOA estimation · Weighed subspace fitting · Fireworks algorithm · Pseudo-random noise resampling

1 Introduction

Early algorithms of DOA estimation mostly studied one-dimensional parameter performance estimation [1], which mainly estimate the azimuth or pitch angle of the incident signal source in a space plane. Considering the actual needs, however, the information in three-dimensional space can reflect the actual situation more comprehensively. For example, in the actual mobile communication system, users often send and receive signals in multi-dimensional space, thus the multi-dimensional spatial spectrum direction finding can be better applied to the actual environment. Compared with one-dimensional DOA estimation, the two-dimensional one provides more accurate source spatial location information for base stations. The more reliable the information, the stronger the directivity of beamforming, and the more effective the interference is suppressed, so that the signal can be delivered to the target user more accurately. In massive MIMO technology [2], one of the key technologies of the future 5G wireless communication system, the base station terminal needs to estimate the

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two-dimensional direction of arrival of the source to achieve better space division multiplexing and improve link reliability [3]. In addition, multidimensional spatial spectrum direction finding can make full use of the spatial redundancy of channel and make the angle direction finding performance better. Multidimensional spatial spectrum direction finding can take full advantage of the spatial redundancy of channel and achieve better angle direction finding performance. Therefore, two-dimensional, even multi-dimensional, direction finding algorithm has been widely studied. The joint estimation of two-dimensional parametric azimuth and elevation angles is a key research topic.

As an important algorithm in DOA estimation, weighted signal subspace fitting algorithm possesses the advantage of more accurate estimation of signal direction [4]. However, due to its non-linear estimation function, it cannot solve the incident direction of signal efficiently, which makes the solving speed a disadvantage of WSF algorithm. Herein multi-dimensional search is normally adopted, which is quite complex, and two parameters – elevation angle and azimuth angle – need to be estimated in two-dimensional DOA estimation. With the increase of the number of signal sources, the dimensions of the matrix increase correspondingly, and the computation needed in the search process increases exponentially, which increases the difficulty of real-time application and cannot meet the real-time requirements in the actual environment. Even with the rapid development of computer hardware, it is still difficult to search DOA of multiple signal sources simultaneously. With the continuous development of intelligent optimization algorithms, they have been applied to the solution of WSF algorithm. Genetic algorithm is adopted to optimize WSF algorithm, which limits the genetic search space and reduces the complexity of WSF algorithm by shortening the genetic length [5]. However, this method may cause poor convergence effect of genetic algorithm and unpredictable poor mutation in the subsequent iteration process. Another algorithm adopts particle swarm optimization (PSO) to optimize WSF [6], which joints ESPRIT algorithm of low complexity and accuracy and particle swarm optimization algorithm, greatly reduces the initialization space of particles and the required particles, and obtained improved computational speed and accuracy compared with the traditional particle swarm optimization. Nevertheless, neither of the above two algorithms can solve the problem of poor threshold performance of WSF algorithm.

With a different search mechanism from the above optimization algorithms, the fireworks algorithm is mainly used to solve optimization problems in continuous space. In an iteration process, the fireworks population will retain more than one individual, while the above algorithms usually retain one only. This explosion mechanism of fireworks algorithm enables a more thorough search in nearby area, which is conducive to improving the convergence speed.

In order to solve the threshold effect of traditional DOA estimation, Gershman proposed a pseudo-random resampling (PR) algorithm [7]. PR technique repeatedly sample the same set of data by artificially generated pseudo-random noise, which can redistribute the noise in the original data and eliminate the unreliable data, thus creating a good operating environment for restoring the performance of DOA estimation algorithm.

Therefore, a PR-WSF algorithm based on improved fireworks algorithm is proposed to solve the problem of large computation, slow solution and poor accuracy in the case of low SNR and insufficient snapshots.

2 Problem Formulation

Suppose the receiving array is uniform square array in the dimension of $M \times N$, where M and N respectively represent number of antennas configured on two adjacent edges, and d stands for spacing of adjacent elements. Q narrow band far-field signals are incident from different azimuth angles $\{\theta_1, \cdots \theta_Q\}$ and pitch angles $\{\varphi_1, \cdots \varphi_Q\}$ at the uniform square array. Therefore, at the moment t, the signal received by URA can be expressed as

$$x_l(t) = \sum_{i=1}^{Q} s_i(t - \tau_{li}) + n_l(t) \quad l = 1, \ldots, MN \tag{1}$$

In this equation, τ_{li} represents the time difference between arrival of signal i' to array element l and its arrival to the reference element, and $n_l(t)$ stands for the noise value received by element l at the moment t.

Since $s_i(t)$ are narrow band far-field incident signals, the signals $x_l(t)$ received by the l-th array element are

$$x_l(t) = \sum_{i=1}^{Q} s_i(t) e^{(-j\omega_0 \tau_{li})} + n_l(t) \quad l = 1, \ldots, MN \tag{2}$$

Write the Eq. (2) in matrix form as

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_{MN}(t) \end{bmatrix} = \begin{bmatrix} 1 & \cdots & 1 \\ e^{-j\omega_0 \tau_{21}} & \cdots & e^{-j\omega_0 \tau_{2Q}} \\ \vdots & \ddots & \vdots \\ e^{-j\omega_0 \tau_{MN1}} & \cdots & e^{-j\omega_0 \tau_{MNQ}} \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_Q(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_{MN}(t) \end{bmatrix} \tag{3}$$

Therefore, its vector expression can be written as

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{4}$$

where the steering matrix $\boldsymbol{A}$ is a $MN \times Q$ matrix, $\boldsymbol{x}(t) = [x_1(t), \cdots, x_{MN}(t)]^T$ is the vector that receives data of $MN \times 1$, $\mathbf{s}(t) = [s_1(t), \cdots, s_Q(t)]^T$ is the vector of narrow band far-field incident signals of $Q \times 1$, and $\mathbf{n}(t) = [n_1(t), \cdots, n_{MN}(t)]^T$ is the vector of zero-mean additive Gaussian white noise of $MN \times 1$.

For arbitrary antennas arrays, the phase delay τ_{li} of i-th signal source in l-th array element channel can be expressed as

$$\tau_{li} = \frac{1}{c}(x_l \cos\theta_i \sin\varphi_i + y_l \sin\theta_i \sin\varphi_i + z_l \cos\varphi_i) \tag{5}$$

where θ_i and φ_i respectively represent the azimuth angle and the pitch angle of l-th signal source, (x_l, y_l, z_l) is the space coordinate of l-th array element, and c is speed of light. So, the $M \times N$ steering matrix $\boldsymbol{A}$ can be written as

$$\boldsymbol{A} = \left[\boldsymbol{a}(\theta_1, \varphi_1), \cdots, \boldsymbol{a}\left(\theta_Q, \varphi_Q\right)\right] \tag{6}$$

where the steering vector $\boldsymbol{a}(\theta_i, \varphi_i) = \left[1, e^{-j\omega_0\tau_{2i}}, \cdots, e^{-j\omega_0\tau_{MNi}}\right]^T$.

According to the vector representation of received signals, the data covariance expression under ideal state can be obtained as

$$\boldsymbol{R} = E\left[\boldsymbol{x}(t)\boldsymbol{x}^H(t)\right] = \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I} \tag{7}$$

where $\boldsymbol{R}_s = E[\boldsymbol{s}(t)\boldsymbol{s}^H(t)]$ is the $Q \times Q$ signal data covariance matrix, σ_n^2 is the power value of zero-mean additive Gaussian white noise, and $\boldsymbol{I}$ is the $MN \times MN$ unit diagonal matrix.

In concrete implementation, the result of eigen-decomposition operation on $\boldsymbol{R}$ is

$$\boldsymbol{R} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H = \sum_{i=1}^{MN} \lambda_i \boldsymbol{u}_i \boldsymbol{u}_i^H = \boldsymbol{U}_s\boldsymbol{\Lambda}_s\boldsymbol{U}_s^H + \boldsymbol{U}_n\boldsymbol{\Lambda}_n\boldsymbol{U}_n^H \tag{8}$$

where $\boldsymbol{U}_s$ is the signal subspace, and $\boldsymbol{U}_n$ is the noise subspace.

Because the space $\boldsymbol{A}$ of is the same as that of $\boldsymbol{U}_s$, there is a fitting relationship between them:

$$\boldsymbol{U}_s = \boldsymbol{A}(\theta, \varphi)\boldsymbol{T} \tag{9}$$

where $\boldsymbol{T}$ is a full rank matrix.

Subspace fitting algorithm estimates the DOA of signals by reconstructing the fitting relationship in noisy environment: an estimated value of matrix $\boldsymbol{T}$ is obtained from the equation below, and achieve the best fitting effect with $\boldsymbol{U}_s$ in the least squares sense.

$$\theta, \varphi, \hat{\boldsymbol{T}} = \min\left\|\boldsymbol{U}_s - \boldsymbol{A}\hat{\boldsymbol{T}}\right\|_F^2 \tag{10}$$

where the DOA of signals can be obtained

$$(\theta, \varphi) = \min \operatorname{tr}\left\{\boldsymbol{P}_{\boldsymbol{A}}^{\perp}\hat{\boldsymbol{U}}_s\hat{\boldsymbol{U}}_s^H\right\} = \max \operatorname{tr}\left\{\boldsymbol{P}_{\boldsymbol{A}}\hat{\boldsymbol{U}}_s\hat{\boldsymbol{U}}_s^H\right\} \tag{11}$$

Generally, the weight matrix $\boldsymbol{W}$ can be introduced in the Eq. (11) to reach a better fitting result

$$(\theta, \varphi) = \min \operatorname{tr}\left\{\boldsymbol{P}_{\boldsymbol{A}}^{\perp}\hat{\boldsymbol{U}}_s\boldsymbol{W}\hat{\boldsymbol{U}}_s^H\right\} = \max \operatorname{tr}\left\{\boldsymbol{P}_{\boldsymbol{A}}\hat{\boldsymbol{U}}_s\boldsymbol{W}\hat{\boldsymbol{U}}_s^H\right\} \tag{12}$$

The performance of WSF will achieve the best when the weight matrix satisfies

$$\boldsymbol{W} = \boldsymbol{W}_{\text{opt}} = \left(\hat{\boldsymbol{\Lambda}}_s - \sigma_n^2\boldsymbol{I}\right)^2\hat{\boldsymbol{\Lambda}}_s^{-1} \tag{13}$$

where the diagonal matrix $\hat{\boldsymbol{\Lambda}}_s$ is composed by $\hat{\boldsymbol{U}}_s$'s eigenvalue, σ_n^2 represents the noise power, and $\boldsymbol{I}$ is the $Q \times Q$ unit matrix.

3 PR-WSF Algorithm Based on Modified Fireworks Algorithm

3.1 Fireworks Algorithm

Fireworks algorithm, inspired by fireworks exploding in the night sky, is used to solve global optimization problems of complex functions [8]. The explosion process of fireworks can be viewed as a local search around a certain point from which fireworks explode and fly off sparks. In this algorithm, the goal is to find a point x_i that satisfies $f(x_i) = y$, and fireworks continue to explode in this potential space until a target spark appears near the point x_i.

3.2 The Improvement of Fireworks Algorithm

The Improvement of Explosion Number. The principle of the number of sparks means the closer the particles are to the best fireworks from last explosion, the more sparks will be generated. And best fireworks from the last explosion will produce most sparks. Considering that the number of fireworks explosion should be integer, sparks generated by fireworks explosion can be expressed as

$$\begin{gathered} s_i = m \cdot \frac{f(\mathrm{X}_i) - y_{\min} + \xi}{\sum_{i=1}^{n} f(\mathrm{X}_i) - y_{\min} + \xi} \\ s_i = \begin{cases} s_{\max}, & if \quad s_i > s_{\max} \\ s_{\min}, & if \quad s_i < s_{\min} \\ round(s_i), & others \end{cases} \end{gathered} \tag{14}$$

where $s_{\max}$ and $s_{\min}$ are the maximum and minimum spark number respectively.

Since the number of explosions is an integer, the differences in fitness value of the algorithm is not obvious due to the selection of the objective function when solving the specific problem of two-dimensional WSF. That is to say, when the values of $y_{\max}$ and $y_{\min}$ are close, the difference between the maximum and the minimum of s_i is probably less than 1, and the value tends to be the same after rounding, which means all fireworks have the same number of explosions. Therefore, the above equation cannot realize the principle of higher fitness values together with more sparks. It weakens the searching ability of fireworks algorithm near the optimal solution and increases the number of sparks generated near the inferior solution, which is not conducive to the fast convergence of fireworks algorithm. In view of the above problems, the calculation rules for the number of sparks are improved here, and the improved algorithm for number of sparks is

$$s_i = M \cdot (f(\mathrm{x}_i) - y_{\min}) \tag{15}$$

$$M = \frac{s_{\max} - s_{\min}}{y_{\max} - y_{\min}} \tag{16}$$

The Eq. (15) compares the fitness values of all fireworks to the minimum, no longer dividing them from $\sum_{i=1}^{n} f(\mathrm{x}_i) - y_{\min} + \xi$, and amplifies the tiny fitness values. At the same time, in order to guarantee the limit of the number of explosions, M is no longer a fixed value in the Eq. (16). M' value will be determined by the difference between the optimal solution and the worst inferior solution produced by this explosion. The smaller the difference is, the denser the solution will be, so it will take a larger value. On the contrary, the smaller the difference value is, the more dispersed the solution is, so its value should be reduced at this time. According to the above equation, the number of sparks calculated varies greatly, and sparks generated by each iteration are more near the better individuals, which is conducive to the search throughout the space and careful search.

The Improvement of Blast Radius. Consider the process of fireworks explosion as the process of searching for the optimal solution, and the i-th blast radius of firework is

$$\mathrm{A}_i = A \cdot \frac{y_{\max} - f(\mathrm{x}_i) + \xi}{\sum_{i=1}^{n} y_{\max} - f(\mathrm{x}_i) + \xi} \tag{17}$$

where A represents the preset maximum blast radius, and $y_{\max} = \max(f(\mathrm{x}_i))$ stands for the maximum value of target function for n fireworks.

The main principle of the Eq. (17) is that when the value of the target function corresponding to fireworks is larger, i.e., the closer to the optimal solution, the smaller the blast radius is. Since there is a greater possibility of the global optimal solution around the particle, it is necessary to give a smaller blast radius, strengthen the search near the particle, and improve the possibility of finding the global optimal solution. But there is a problem that when the firework is the best from the last explosion, $f(\mathrm{x}_i) = y_{\max}$, the result can be substituted into the Eq. (17)

$$\mathrm{A}_i = A \cdot \frac{\xi}{\sum_{i=1}^{n} y_{\max} - f(\mathrm{x}_i) + \xi} \tag{18}$$

Since ξ is a minimal constant, it is not difficult to find out that when this fireworks is the best from the last explosion, the result of the above equation is close to 0. However, the better the fitness value of fireworks is, the more fireworks will be produced, so the optimal fireworks should generate the maximum number of sparks. It means that the best fireworks from the last explosion will generate the maximum number of sparks within the radius of zero in this explosion, which is equivalent to the repetition of the previous optimal solution in this explosion and find no other solutions around the fireworks. In the next explosion, for the same fireworks $d(x_i - x_j) = 0$, therefore, the probability of these sparks being selected is zero, and all the repeated solutions will be discarded, which not only increases the calculation of algorithm, but also reduces search opportunities. In view of the above disadvantages of the basic fireworks algorithm, the blast radius will be perfected. The improved equation for calculating the blast radius is shown in (19).

$$\begin{aligned} A_i &= A_t - (A_t - A_{\min}) \times f(x_i)/y_{\max} \\ A_t &= A \times (T - t + 1)/T \end{aligned} \tag{19}$$

In Eq. (19), t represents the iteration number of explosion search, T is the iteration number of preset total search, A_t is the maximum blast radius under the current iteration number, and $A_{\min}$ is the preset minimum blast radius. Obviously, with the increase of t, A_t will gradually decrease and A_i will also decrease, which makes the algorithm focus on global search with a large radius at the beginning, and reduce the radius when the optimal value is found in the later stage. This is conductive to fine search in local areas. Also, it guarantees that under the same search iteration number, the better the particle is, the smaller the explosion radius is, which is conducive to strengthening the search near the current optimal solution and improving the ability of global search.

The Improvement of Search Space. In the solution space of WSF algorithm, ESPRIT algorithm combined with Cramer-Rao bound are used to determine a smaller search space containing the global optimal solution, and then the modified fireworks algorithm is used to solve in this small space to reduce the computational complexity of the algorithm.

Since the ESPRIT algorithm can directly obtain the DOA through calculation without the multi-dimensional search of global extremum [9], the complexity is much lower than that of WSF. Although the distinguishability of ESPRIT algorithm is lower than that of WSF algorithm, it is undeniable that the solutions of ESPRIT algorithm and WSF algorithm are all angle estimates of the DOA, so their solutions are similar. Therefore, ESPRIT is used to solve the coarse estimated value as a center to select the search space. Under the condition of ensuring certain accuracy, if the selection of search space is too large, the fireworks algorithm needs to select more individuals in the initial population, the required blast radius is increased, and more iterations will be needed to realize the global search. However, if the selection of search space is too small, it will not contain the global optimal value. Hence, a suitable search space is crucial for the selection of primary fireworks and blast radius.

CRB is the lower bound of the variance of unbiased estimator in parameter estimation, a parameter related to SNR and the number of snapshots. With the decrease of SNR or number of snapshots, the deviation between coarse estimated value and real value of ESPRIT algorithm increases, and CRB also becomes larger. Therefore, CRB, which is magnified μ times, is taken as the half boundary of the search space, and the search space can be expressed as $[\theta_e - \mu CRB, \theta_e + \mu CRB]$ and $[\varphi_e - \mu CRB, \varphi_e + \mu CRB]$.

3.3 Pseudo-Random Noise Re-sampling Technique

Pseudo-random noise re-sampling technology adopts artificial pseudo-random noise to repeatedly sample the same set of data. This process can reallocate the noise in the original data and eliminate the unreliable data, and then, thus creating a good operating condition for DOA estimation algorithm. The operation procedures can be summarized as follows:

The same group of received data is sampled repeatedly by pseudo-random noise to recreated data matrixes.

Conduct DOA estimation on the newly generated data in parallel, and save the results.

Set the reliability test conditions, screen the estimated results of the previous step to remove the unreliable data, and the remaining reliable data is the final result.

Let the receiving data of original array be $\boldsymbol{X}$ and the number of snapshots be K, then the data matrix after re-sampling can be defined as

$$\boldsymbol{Y} = \boldsymbol{X} + \boldsymbol{Z} \tag{20}$$

where $\mathbf{Z}$ represents pseudo-random noise $MN \times K$ matrix, and it has the following features:

$$E[\boldsymbol{Z}] = 0,\quad E\left[\boldsymbol{Z}\boldsymbol{Z}^H\right] = \sigma_z^2 N\boldsymbol{I},\quad E\left[\boldsymbol{Z}\boldsymbol{Z}^T\right] = 0 \tag{21}$$

where $\boldsymbol{I}$ represents $MN \times MN$ unit matrix, and σ_z^2 stands for pseudo-random noise power.

Reliability test plays an important role in PR algorithm, and its accuracy in harsh environment directly determines the performance of PR algorithm. DOA estimation is carried out on the data from pseudo-random noise re-sampling, and the results need to be filtrated by reliability tests, so as to eliminate the unreliable data and obtain the reliable data.

In PR algorithm, the essence of reliability tests is the union of azimuth and pitch angle regions of each incident signal DOA. If their sets are set as Θ, Φ respectively, then the condition for reliability test will be that the DOA estimation results of the re-sampled data are all within Θ, Φ. In the PR algorithm, Bartlett algorithm can be used to solve Θ, Φ and can be expressed as [10]

$$\begin{aligned} \Theta &= \bigcup_{p=1}^{P} \left[\theta_p^{\max} - \theta_p^{left}, \theta_p^{\max} + \theta_p^{right}\right] \\ \Phi &= \bigcup_{p=1}^{P} \left[\varphi_p^{\max} - \varphi_p^{left}, \varphi_p^{\max} + \varphi_p^{right}\right] \end{aligned} \tag{22}$$

where $\theta_p^{\max}(p = 1, \cdots P)$ is the azimuth angle corresponding to the p-th power peak in the algorithm output results, and $\varphi_p^{\max}(p = 1, \cdots P)$ is the pitch angle corresponding to the p-th power peak in the algorithm output results. θ_p^{left} and φ_p^{right} are left and right boundaries of the p-th subinterval in Θ. Similarly, θ_p^{left} and φ_p^{right} are left and right boundaries of the p-th subinterval in Φ, and both are consistent with the corresponding angle of 3 dB bandwidth of the peak power in the interval.

The modified fireworks algorithm and pseudo-random noise re-sampling algorithm are adopted to solve the equation of weighted-signal subspace fitting. The global optimal solution of the optimization problem is the DOA estimated value of the signal source. Therefore, in this maximum optimization problem, the corresponding fitness function can be defined as

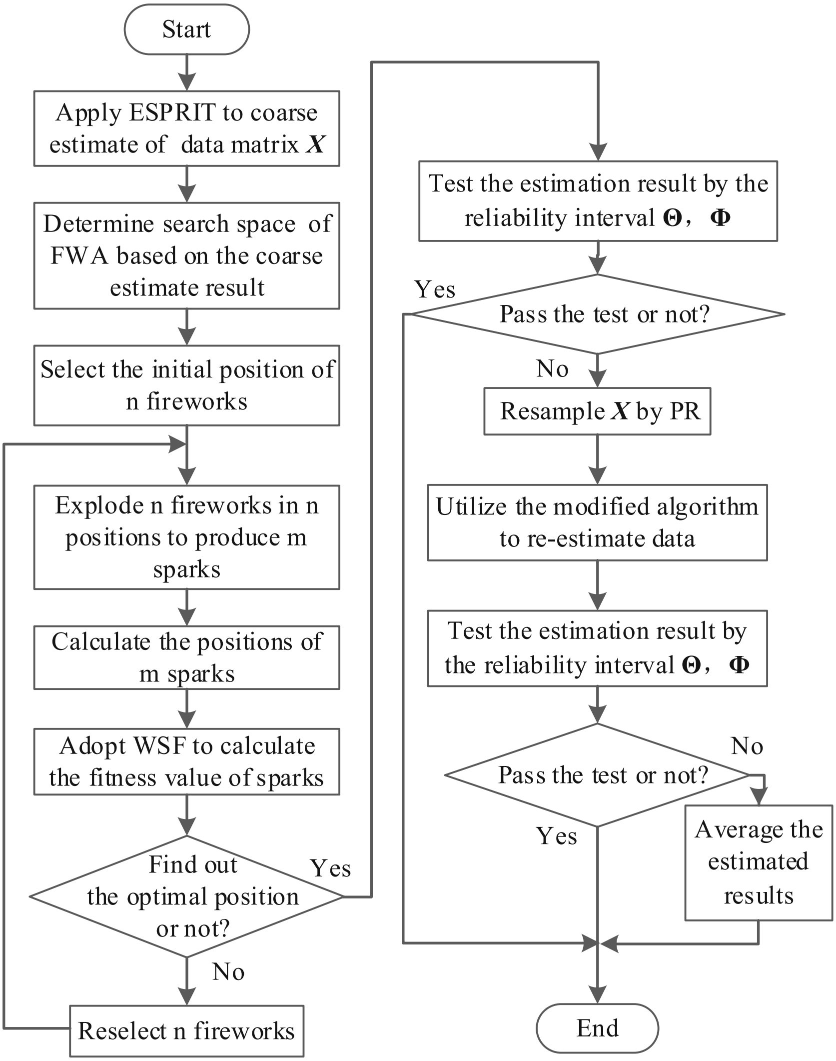


Fig. 1. Algorithm flow chart

$$F(\theta, \varphi) = \mathrm{tr}\left\{ \boldsymbol{P_A} \hat{\boldsymbol{U}}_s \boldsymbol{W} \hat{\boldsymbol{U}}_s^H \right\} \tag{23}$$

The position of the fireworks and sparks (θ_i, φ_i) correspond to a set of angular estimates.

According to the above algorithms, the flowchart of PR-WSF algorithm based on the modified fireworks algorithm is shown in the Fig. 1.

4 Performance Study

4.1 Modified FWA

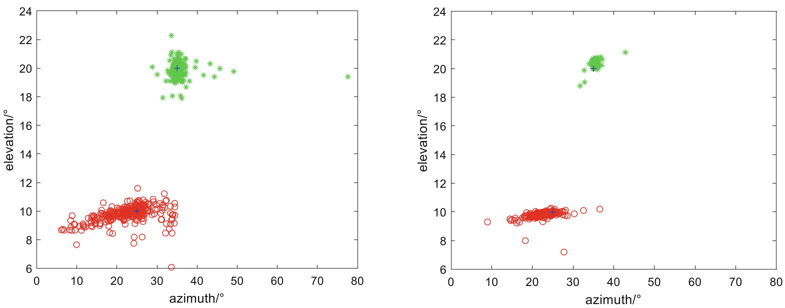
The simulation model is a 8×8 matrix URA with the spacing between elements $d = \lambda/2$, the two narrow-band signals are incident on the array at azimuth $25°, 35°$ and elevation $10°, 20°$, and the noise is additive Gaussian white noise. To verify the effectiveness of the modified fireworks algorithm, it is compared with FWA WSF, PSO WSF, limited GA WSF [5] and Joint-PSO WSF [6].

The root-mean-square error is defined as

$$\begin{aligned} RMSE_\theta &= \sqrt{\frac{1}{RQ} \sum_{r=1}^{R} \sum_{i=1}^{Q} \left(\hat{\theta}_{i,r} - \theta_i \right)^2} \\ RMSE_\varphi &= \sqrt{\frac{1}{RQ} \sum_{r=1}^{R} \sum_{i=1}^{Q} \left(\hat{\varphi}_{i,r} - \varphi_i \right)^2} \end{aligned} \tag{24}$$

where R is the number of Monte Carlo independent experiments, $\hat{\theta}_{i,r}, \hat{\varphi}_{i,r}$ is the estimate of azimuth and elevation of the i-th incident signal obtained from the r-th Monte Carlo experiment, and θ_i, φ_i is their accurate value.

Figure 2 reveals the comparison of estimation results between FWA WSF and the modified FWA WSF in 300 times of Monte Carlo independent experiments when SNR



(a) FWA (b) Modified FWA

Fig. 2. Results of different methods

is −5 dB, and snapshots number is 100, with "+" for the real position of incident signals. Simulation results show that under the same iteration times, the modified fireworks algorithm has a smaller deviation from the real value when solving the WSF algorithm, thus obtains better effects. The modified fireworks algorithm has smaller search space, which makes the results inevitably fall within the range closer to the real value. Combined with the improvement of explosion radius, the modified fireworks algorithm can search accurately in a small range, and the iterative solution process is less likely to fall into local optimum.

Figure 3 shows the performance comparison of different algorithms with different SNRs and snapshot numbers in 500 times of Monte Carlo independent experiments, where SNR goes from −15 dB to 10 dB, and the snapshots from 10 to 1000.

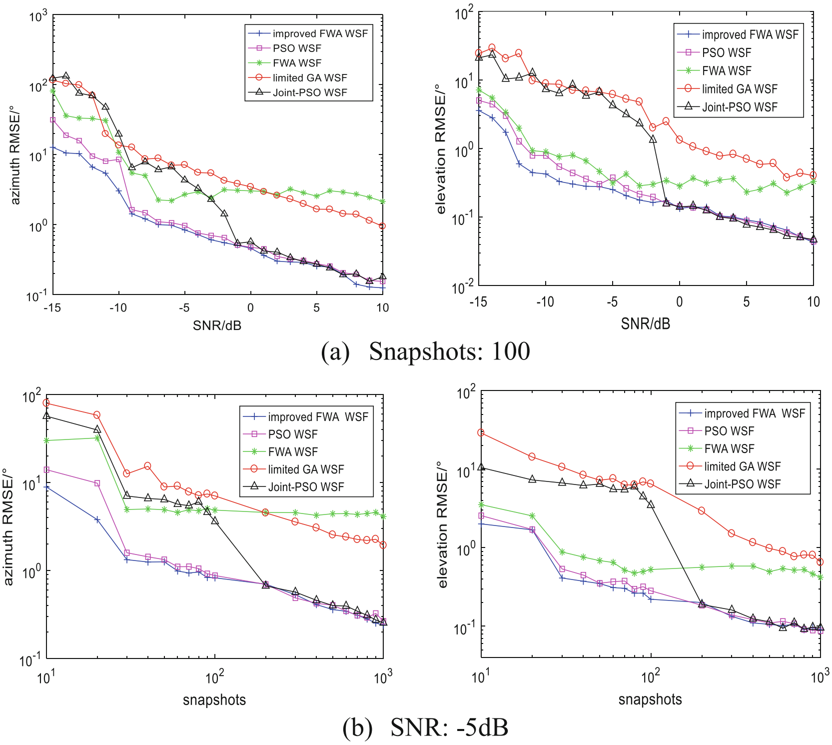


Fig. 3. RMSE of different methods

Table 1 reveals the comparison of solution time taken by different algorithms when SNR is −5 dB, and snapshots number is 100. To avoid errors in a single experiment, 500 independent Monte Carlo experiments are conducted for each algorithm, and the running time is averaged. The simulation environment is MATLAB (2016b), and the computer is configured as AMD Ryzen 3 2200G processor, main frequency 3.50 GHz, memory 8 GB, 64-bit Windows operating system.

Table 1. Comparison of solution time

Algorithm	PSO WSF	FWA WSF	Modified FWA WSF	Joint-PSO WSF	Limited GA WSF
Time for a single solution/s	8.84	5.62	5.70	7.81	9.61
Time for a single iteration/s	0.022	0.056	0.057	0.026	0.032

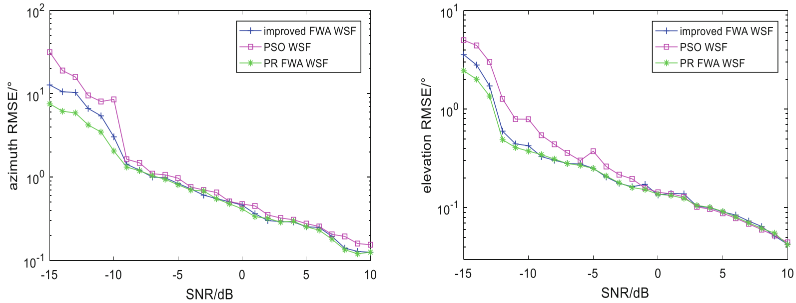
It can be seen from the above two groups of experiments that the original fireworks algorithm has poor ability to optimize the target function of WSF algorithm, and the incomplete search results lead to great errors in estimation results. The estimation error of limited GA WSF algorithm is apparently higher than that of other algorithms due to the poor convergence performance of the genetic algorithm and the tendency of getting into the local optimum in the iteration process, but the modified algorithm improves the calculation speed obviously. In the case of high SNR and number of snapshots, the mean-square error curve basically overlaps with the PSO WSF algorithm. However, with the decrease of SNR and number of snapshots, the deviation between the initial population and the real value of this algorithm becomes larger, and the same iterative solution process improves the operation speed at the cost of the breakdown accuracy. The accuracy of the modified fireworks algorithm is similar to that of the particle swarm optimization, but the its performance is improved slightly in the region where the threshold effect occurs due to the assistance of coarse estimation of ESPRIT algorithm. The modified explosion strategy improves the search performance of the algorithm, and the overall estimation performance is better than other methods.

When the solution results are similar, the particle swarm optimization algorithm needs 8.84 s, and the modified fireworks algorithm needs 5.70 s. It should be noted that in this experiment, algorithm of particle swarm optimization needs 400 iterations for a single solution process, while the improved fireworks algorithm only needs 100. It can be concluded that algorithm of particle swarm optimization has small amount of calculation and fast single iteration. It takes a long time for the improved fireworks algorithm to calculate a large amount of data in a single iteration in parallel, but its fast convergence speed makes the final solution time less than that of particle swarm optimization algorithm. In conclusion, the modified fireworks algorithm has higher accuracy and faster single solution speed for WSF algorithm.

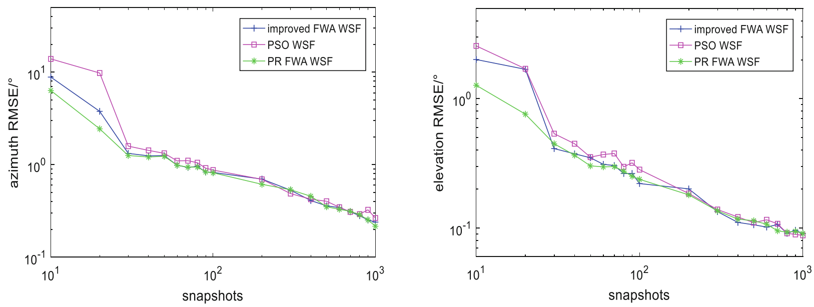
4.2 PR-WSF Algorithm Based on Improved Fireworks Algorithm

In order to prove the effectiveness of this algorithm, it is compared with POS WSF and WSF algorithm based on improved FWA. The simulation conditions are the same as Sect. 4.1.

Figure 4 shows the performance comparison of each algorithm at different SNRs and snapshot numbers. Simulation results reveals that the algorithm modified by pseudo-random noise re-sampling has better performance than other algorithms when the SNR is lower than −9 db or the number of snapshots is less than 30. However, as the cost of performance improvement, its computational complexity increases significantly, which leads to a decrease in calculation speed. When the SNR and the number of snapshots increase to a certain amount, the algorithm will not take the re-sampling step. At this time, the complexity and speed of the algorithm are approximately equal to the WSF algorithm based on improved FWA.



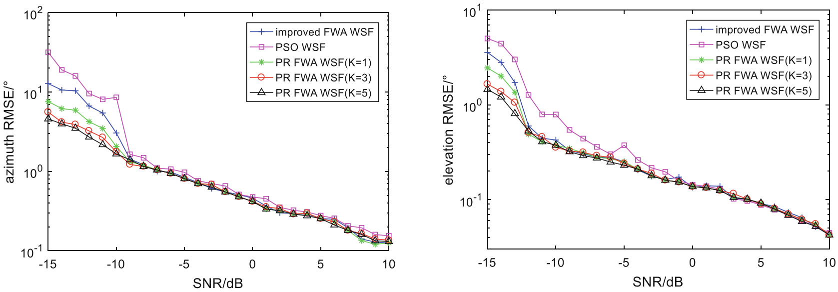
(a) Snapshots: 100



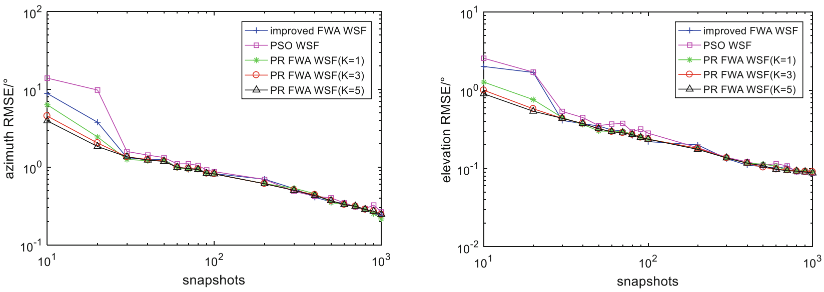
(b) SNR: -5dB

Fig. 4. RMSE of different methods

Figure 5 shows the influence of different re-sampling times on algorithm performance. The re-sampling times are set as 1, 3 and 5 respectively. Simulation results reveals that pseudo-random noise re-sampling can effectively improve the DOA estimation performance under the condition of low SNR and insufficient number of snapshots, and multiple re-samplings can explore the useful information in data more thoroughly and make the results more accurate. However, as re-sampling times increases, the mean-square error decreases gradually. The reason is that in the re-sampling operation, the dynamic pseudo-random noise introduced changes the noise distribution in the original data, and the re-sampled data contains more complex noise information, and the influences on the original data gradually reach the limit. In addition, re-sampling times are directly related to the computational complexity. The more re-sampling times are, the longer the required operation time is needed, and the corresponding solution accuracy is also improved.



(a) Snapshots: 100



(b) SNR: -5dB

Fig. 5. RMSE of different methods

5 Concluding Remarks

In view of the complexity of WSF algorithm in solving process, which does not meet the real-time, this paper introduces the fireworks algorithm into the solving process, and improves the parameters design of the fireworks algorithm; meanwhile, inspired by literature, a fireworks algorithm with limited search space is proposed. In order to improve the convergence rate of intelligent algorithm, this algorithm adopts limited search space to make fireworks algorithm solve data in parallel while reducing the total amount of data to accelerate the solving process. In addition, the DOA estimation algorithm under the influence of threshold effect is studied, and a PR-WSF algorithm based on improved fireworks algorithm is proposed. With the purpose of improving data utilization, this PR-WSF algorithm combines PR algorithm to resample data, and improves the performance of the algorithm under low SNR and small number of snapshots condition. The simulation study reveals that the proposed PR-WSF algorithm based on improved fireworks algorithm has relatively faster solving speed and higher threshold performance.

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