Trajectory Optimization Under Constrained UAV-Aided Wireless Communications with Ground Terminals

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Abstract. Using the unmanned aerial vehicles (UAV) to form a communication platform is of great practical significance in future wireless networks. This article investigates the flight trajectory optimization problem with minimum energy consumption when the UAVs are mobile servers and communicate with the ground terminals (GT). The proposed trajectory considers the features of conventional paths as well, i.e., the channel quality and energy saving. Numerical results show that our approach outperforms the other schemes in terms of the throughput of data and the features of the UAV.

Keywords: Unmanned aerial vehicle · Energy optimization · Wireless networks · Mobile server · Trajectory optimization

1 Introduction

The use of unmanned aerial vehicles (UAVs) as communication platforms is of great practical significance in future wireless networks. For example, UAVs can be utilized as mobile relays to help information exchange between far-apart ground users [15]. UAV can be viewed as mobile base stations (BSs) where a UAV is dispatched as a BS to serve a group of users on the ground [2,3,9,12,13]. UAV technology is becoming more and more sophisticated where the weight of drones is getting lighter and the UAV can fly longer and longer. However,

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the energy consumption of UAV is still a challenge. For the UAV-aided wireless communication system, UAVs cannot guarantee long-term data transmission with the limited energy. The UAVs need return to the depot for battery charging or exchanging which will cause service interruptions. This severely hinders the practical implementation of UAV-enabled communications.

For the UAV-enabled multiuser communication networks, a novel cyclical multiple access scheme is proposed in [10] where the UAV periodically serves each of the ground users along its cyclical trajectory via TDMA. In [13], a joint user scheduling, power control, and trajectory optimization problem is investigated for a multi-UAV enabled multiuser system. The problem of joint caching and resource allocation is investigated in [5] for a network of cache-enabled unmanned aerial vehicles (UAVs) that service wireless ground users over the LTE licensed and unlicensed (LTE-U) bands. The problem of proactive deployment of cache-enabled UAVs for optimizing the quality-of-experience (QoE) of wireless devices in a cloud radio access network is studied in [4].

Via energy-efficient trajectory designs [14], the UAV endurance problem remains improvable. For the endurance issue, we mainly optimize the UAV trajectory to save the UAV's energy consumption. In this article, we focus on the fixed-wing UAVs. Even though the reduction of the link distance saves the communication power, the UAV systems are subject to additional propulsion power consumption for maintaining the UAV aloft and supporting its mobility (if necessary), which is usually much higher than the communication power consumption. The purpose of this article is minimizing the propulsion power consumption of the UAV by optimizing the flight path. The proposed trajectory considers the features of conventional paths, when the UAV flies straight over the GT's center. UAV starts to communicate with the GT after entering the ground terminal communication range. The UAV can also fly beside over the GT's center instead of flying directly above the GT's center.

The main contributions of this article include:

(1) We derive a system model of the data transfer rate. At the same time, we derive a theoretical model for the propulsion energy consumption of fixed-wing UAVs as a function of flying velocity and acceleration.
(2) We optimize the trajectory with minimum energy consumption, which allows the UAV to move away from GT's center point.

The rest of the paper is organized as follows. Section 2 introduces the system model, and defines the UAV's propulsion energy consumption based on a theoretical model. Section 3 describes how to find the path that minimizes the energy and the path planning algorithm is presented for energy minimization. Section 4 reports a set of experimental results to validate the proposed approach. Finally, we conclude the paper and state our future work in Sect. 5.

2 System Model

2.1 Data Rate Model

We aim to optimize the UAV's trajectory so as to minimize its energy consumption. Without loss of generality, we consider a three-dimensional (3D) Cartesian coordinate system where the GT is located at the origin (0, 0, 0). Furthermore, for simplicity, we assume that the UAV flies horizontally at a fixed altitude H. In practice, H could correspond to the minimum altitude required for safety considerations (e.g., terrain or building avoidance) without frequent aircraft ascending and descending. The extensions on varing H will be left as a future work.

Denote the UAV trajectory projected on the horizontal plane as $\mathbf{q}(t) = [x(t), y(t)]^\top \in \mathbb{R}^{2\times 1}$. Thus, the time-varying distance from the UAV to the GT can be expressed as:

$$\begin{aligned} d(t) &= \sqrt{H^2 + \|\mathbf{q}(t)\|^2} \\ &= \sqrt{H^2 + x(t)^2 + y(t)^2}. \end{aligned} \tag{1}$$

The Doppler effect due to the UAV's mobility is assumed to be perfectly compensated. Therefore, the time-varying channel power gain from the UAV to each GT follows the free-space path loss model [11], which can be given by:

$$h(t) = \beta_0 d^{-2}(t) = \frac{\beta_0}{H^2 + \|\mathbf{q}(t)\|^2}, \tag{2}$$

where β_0 denotes the channel power gain at the reference distance $d_0 = 1$ meter (m), whose value depends on the carrier frequency, antenna gain, etc. and $d(t)$ is the link distance between the UAV and the GT at time t. The instantaneous channel capacity from the UAV to the GT in bits/second can be expressed as:

$$\begin{aligned} R(t) &= Blog_2\left(1 + \frac{Ph(t)}{\sigma^2}\right) \\ &= Blog_2\left(1 + \frac{\gamma_0}{H^2 + \|\mathbf{q}(t)\|^2}\right), \end{aligned} \tag{3}$$

where B denotes the channel bandwidth and σ^2 is the white Gaussian noise power at the GT receiver. In the following, we use the unit of bps/Hz to measure the throughput per unit bandwidth, also known as the spectrum efficiency. $\gamma_0 = \beta_0 P/\sigma^2$ is the reference received signal-to-noise ratio (SNR) at $d_0 = 1\,\text{m}$.

For each GT, the rate $R(t)$ is symmetric and unimodal, which achieves its maximum when the UAV flies closest to the ground terminal (e.g., $x = 0$ and $y = 0$). As an illustration, Fig. 1 plots the instantaneous rate of each GT versus the UAV position x, with $P = 10\,\text{dBm}$, $\gamma_0 = 80\,\text{dB}$, $H = 100\,\text{m}$.

The total amount of information bits that can be transmitted from the UAV to the GT over the duration T is a function of the UAV trajectory $\mathbf{q}(t)$, expressed as:

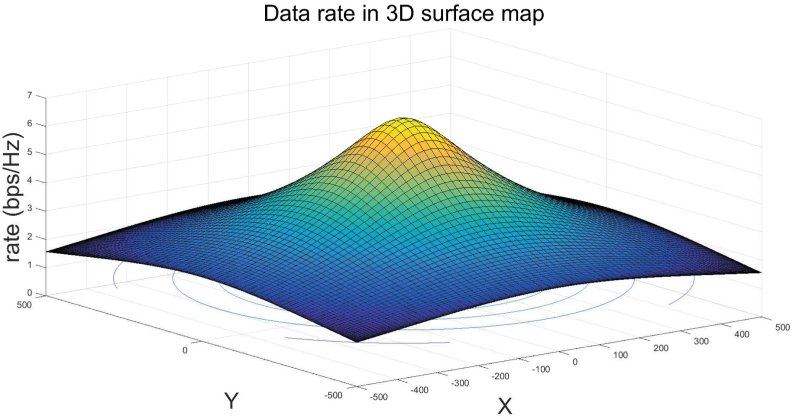

Fig. 1. The illustration of the throughput versus UAV's position.

$$\bar{R}(\mathbf{q}(t)) = \int_0^T Blog_2\left(1 + \frac{\gamma_0}{H^2 + \|\mathbf{q}(t)\|^2}\right) dt. \tag{4}$$

The issue is the packet loss [1] due to the highly dynamic wireless channels between the GT and the moving UAV. Thus, the trajectory of the UAV should be properly designed.

2.2 UAV Energy Consumption Model

The total energy consumption of the UAV includes two components. The first one is the communication-related energy and the other is the propulsion energy. Note that in practice, the communication-related energy is usually much smaller than the UAV's propulsion energy, e.g., a few watts [6] versus hundreds of watts [7], and thus is less considered in this paper.

Furthermore, for fixed-wing UAVs, the total propulsion energy required is a function of the trajectory $\mathbf{q}$(t), which is corresponded to the classic aircraft power consumption model known in aerodynamics theory [8]. The function is expressed as:

$$\begin{aligned} E(\mathbf{q}(t)) = \int_0^T &\left[c_1\|\mathbf{v}(t)^3\| + \frac{c_2}{\|\mathbf{v}(t)\|}(1 + \frac{\|\mathbf{a}(t)\|^2 - \dfrac{\mathbf{a}^T(t)\mathbf{v}(t)^2}{\|\mathbf{v}(t)\|^2}}{\mathbf{g}^2}) \right] dt \\ &+ \frac{1}{2}m(\|\mathbf{v}(T)\|^2 - \|\mathbf{v}(0)\|^2). \end{aligned} \tag{5}$$

where

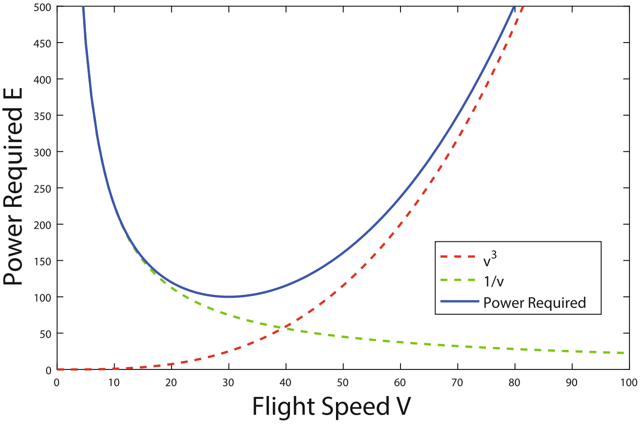

Fig. 2. Typical power required curve versus speed V for a UAV in straight-and-level flight.

$$\mathbf{v}(t) \triangleq \dot{\mathbf{q}}(t), \quad \mathbf{a}(t) \triangleq \ddot{\mathbf{q}}(t), \tag{6}$$

denote the instantaneous UAV velocity and acceleration vectors, respectively. c_1 and c_2 are two parameters related to the aircraft's weight, wing area, air density, etc., g is the gravitational acceleration with nominal value $9.8\,\mathrm{m/s^2}$ and m is the mass of the UAV including its all payload.

We first consider the special case of steady straight-and-level flight (SLF) with constant speed V, i.e., $\|\mathbf{v}(t)\| = \mathrm{V}$ and $\|\mathbf{a}(t)\| = 0$, $\forall$t. In this case, (5) reduces to:

$$\bar{E}_{SLF}(V) = T\left(c_1 V^3 + \frac{c_2}{V}\right). \tag{7}$$

The power consumption of (7) as a function of V is illustrated in Fig. 2, which consists of two terms. The first term, which is proportional to the cube of the speed V, is known as the parasitic power for overcoming the parasitic drag due to the aircraft's skin friction, form drag, etc. The second term, which is inversely proportional to V, is known as the induced power for overcoming the lift-induced drag, i.e., the resulting drag force due to wings redirecting air to generate the lift for compensating the aircraft's weight.

Next, we consider another special trajectory where the UAV flies at a constant speed V but with possibly time-varying headings. In this case, we have $\|\mathbf{v}(t)\| = \mathrm{V}$ and $\mathbf{a}^\top(t)\mathbf{v}(t) = 0$, $\forall$t. Thus, (5) reduces to:

$$\bar{E}(V, a(t)) = \bar{E}_{SLF}(V) + \frac{c_2}{V\mathsf{g}^2}\int_0^T a^2(t)\,dt. \tag{8}$$

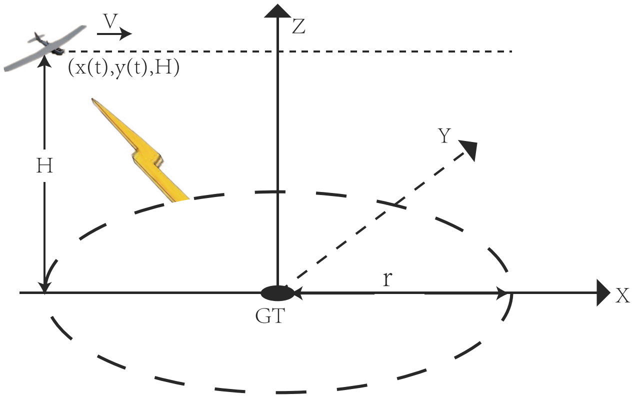

Fig. 3. The illustration of UAV severs the ground terminal.

As we aim to cost the least power of the UAV to satisfy data transmission, the problem is formulated as:

$$\begin{aligned} &\text{minimize} \;\; \bar{E} \\ &\text{subject to} \; V_{min} \leq \|\mathbf{v}(t)\| \leq V_{max} \\ &\qquad\qquad\; \|\mathbf{a}(t)\| \leq a_{max} \\ &\qquad\qquad\; \bar{R} = R_{req} \\ &\text{variables}: \mathbf{q}(t), \mathbf{v}(t), \mathbf{a}(t). \end{aligned} \tag{9}$$

where V_{min}, V_{max} denote the UAV's slowest speed and fastest speed respectively, a_{max} denotes the UAV's maximum acceleration and R_{req} denotes the volume of the requirement data. This problem is difficult to be directly solved. Firstly, it requires the optimization of the continuous function $\mathbf{q}$(t), as well as its first- and second-order derivatives $\mathbf{v}$(t) and $\mathbf{a}$(t) and the objective function lacks closed-form expressions. In the following, a path planning algorithm is proposed.

3 UAV Trajectory Design

As far as we know, the current conventional approach is the UAV flies straight over the GT's center while UAV starts to communicate with the GT after entering the ground terminal communication range, as shown in Fig. 3. In practice, each GT could correspond to a cluster head that serves as a gateway for a cluster of nearby nodes communicating with the UAV. Conventionally, in this scenario the trajectory of UAV would be designed as straight path.

We consider that when the UAV flies directly above the GT's center. Generally, the channel quality is relatively better at this time. Therefore, the communication time can be reduced in some degree. Although the UAV has long distance trajectory, the overall duration of data transmission is shorter with reliable channel connections. At this time, we term this situation as scenario I.

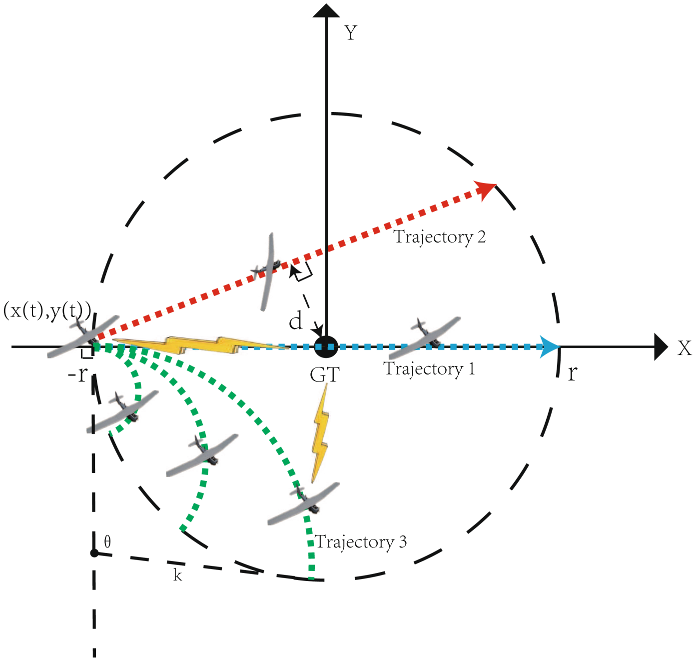

Fig. 4. Three trajectories of the UAV that severs one GT with straight or curve trajectories. The blue trajectory 1 is classic path that cross the GT's center. The red trajectory 2 is the path generated by Algorithm 1. The green curve trajectory 3 is the path generated by Algorithm 2. (Color figure online)

Since there are multiple GTs need to be served, the origin and the destination of the UAV are often related to the position of the GT, so when we design single UAV path, it is not necessary to strictly consider the origin and destination and this research will be discussed in future work.

Intuitively, Scenario I is more suitable for this situation that the UAV and GT need a reliable channel. In other words, Scenario I can be selected to ensure reliable data transmission. But the flight speed at this time is relatively large, so the energy consumption is uncertain which needs to be discussed in detail in the experimental part.

In addition to this, the UAV can fly beside over the GT's center instead of flying directly above the GT's center. At this time, we term this situation as Scenario II. In this situation, UAV can choose straight flight or curve flight, as shown in Fig. 4.

For simplicity, we first study the steady straight-and-level flight. When the UAV flies into the ground terminal communication range, the data transmission rate is not as good as the former Scenario I, leading to a longer communication time between the UAV and the GT. In addition, the UAV's flight distance is shorter at this time. Both of these situations lead the speed of the UAV to be slower. At this point, the energy consumption of the UAV can be calculated by the corresponding energy consumption formula in Sect. 2 with the speed and time variables. Intuitively, the energy consumption of the Scenario II will be less than the former Scenario I and the results will be discussed in the experimental section based on the data transmission model and energy consumption model in Sect. 2. By referring the red trajectory 2 in Fig. 4, the straight trajectory design algorithm is presented as follows:

Algorithm 1. (Straight Tragectory Beside GT's center)

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1. Initialization:
    • Initialize the volume of data $\bar{R}$, height $H$ and communication range $r$.

2. Calculate the energy consumption of the trajectory:
    • Let $d = 0\,m$ and $l = 0$ then update the trajectory $\mathbf{q}(t)$ of UAV, where
        $x(t) = -\sqrt{r^2 - d^2} + Vt, \quad y(t) = d.$    (10)
    • Calculate the communication time $T$ with the function (4) and variable $\mathbf{q}(t)$.
    • Update the speed $V$ of the UAV under the constraints (9):
        $V = \frac{\|\mathbf{q}(t) - \mathbf{q}(0)\|}{T}$    (11)
    • Update the energy consumption $\bar{E}_l(V)$ of the trajectory with the equation (7).

3. Update the variable $d$ and $l$:
        $d = d + l * n, \quad l = l + 1$    (12)

4. If a maximum number of iterations has been reached or $d = r$, find the optimal $d$ satisfying:
        $d = \arg \min_{l \geq 0, l \in \mathbb{Z}} \bar{E}_l(V)$    (13)
   else go to Step 2
   end
```

At this point, the algorithm of the curve flight trajectory can be obtained by the same reason, by reference the green trajectory 3 in Fig. 4 as shown below:

Algorithm 2. (Curve Tragectory Beside GT's center)

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1. ***Run Algorithm 1, replace the Eqs.*** (10)***,*** (11)***,*** (12) ***and*** (13) ***by the following computations:***
    - *Let flight radius $k = 0$, $l = 0$ and update the trajectory $\mathbf{q}(t)$ of UAV, where*

$$x(t) = k * cos(\frac{\pi}{2} - \theta(t)), \quad y(t) = k * sin(\frac{\pi}{2} - \theta(t)). \tag{14}$$

    where θ denotes the flight angular of the circle trajectory.

    - *Update the speed V and acceleration $a(t)$ of the UAV:*

$$V = \frac{k\theta(t)}{T}, \quad a(t) = \frac{V^2}{k} \tag{15}$$

    - *Update the energy consumption $\bar{E}_l(V, a(t))$ of the trajectory with the equation* (8)
    - *Update flight radius k*

$$k = k + l * n, \quad l = l + 1 \tag{16}$$

2. ***Find the optimal k satisfying:***

$$k = \arg \min_{l \geq 0, l \in \mathbb{Z}} \bar{E}_l(V, a(t)) \tag{17}$$

The variables d and k are very complicated, so these variables are calculated by brute-force method. It is interesting to note that there is a bond of the volume of data, when the curve trajectory performs relatively ideal.

Note that since each iteration of Algorithm the variables are complicated, the time complexity of the proposed algorithm presents an exponential explosion. Furthermore, the trajectory optimization problem can be solved off-line before the UAV dispatch at the ground control station with a high computational capability.

4 Numerical Results

In this section, numerical results are provided to validate the proposed design. The UAV altitude is fixed at $H = 100\,\text{m}$. The communication bandwidth is $B = 1\,\text{MHz}$ and the noise power spectrum density at the GT receiver is assumed to be $N_0 = -170\,\text{dBm/Hz}$. Thus, the corresponding noise power is $\sigma^2 = N_0 B = -110\,\text{dBm}$. We assume that the UAV transmission power is $P = 10\,\text{dBm}$ (i.e., 0.01W), and the reference channel power is $\beta_0 = -50\,\text{dB}$. As a result, the maximum SNR achieved when the UAV is just above the GT can be obtained as 30 dB. Furthermore, we assume that $c_1 = 9.26 \times 10^{-4}$ and

Table 1. System parameters

Parameter	Value	Parameter	Value	Parameter	Value
H	100 m	B	1 MHZ	σ^2	−110 dBm
P	10 dBm	β_0	−50 dB	V_{em}	30 m/s
c_1	9.26×10^{-4}	c_2	2250	P_{em}	100 W
V_{max}	100 m/s	V_{min}	3 m/s	a_{max}	5 m/s

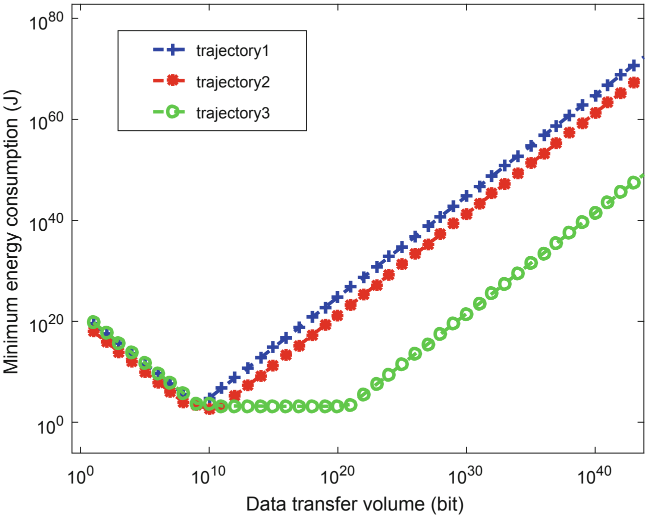

Fig. 5. The figure of minimum energy consumption versus data transfer volume for different trajectories. The blue line is the energy performance of the trajectory 1. The red line is the energy performance of the trajectory 2 generated by Algorithm 1. The green line is the energy performance of the trajectory 3 generated by Algorithm 2. (Color figure online)

$c_2 = 2250$, where these parameters are subject to the flight properties of fixed-wing drones in [8], such that the UAV's energy-minimum speed is $V_{em} = 30$ m/s as shown in Fig. 2 and the corresponding minimum propulsion power consumption is $P_{em} = 100$ W. Note that we have $P \ll P_{em}$, thus the UAV transmission power can be less considered.

Based on this situation, when $\bar{R} = 10^9$ bit, $V_{max} = 100$ m/s, $V_{min} = 3$ m/s and $a_{max} = 5$ m/s^2, the energy consumption of the trajectory 1, trajectory 2 and trajectory 3 are 4.0786×10^3 J, 2.8364×10^3 J and 5.7332×10^3 J respectively. The parameters are shown in the Table 1. From this we can find that the energy consumption performance of the trajectory 2 is better than the trajectory 1 and trajectory 3.

At this time, when the amount of data transmitted is relatively small, e.g., $\bar{R} \leq 10^{10}$ bit, the data volume is termed as transmitting a document file, the

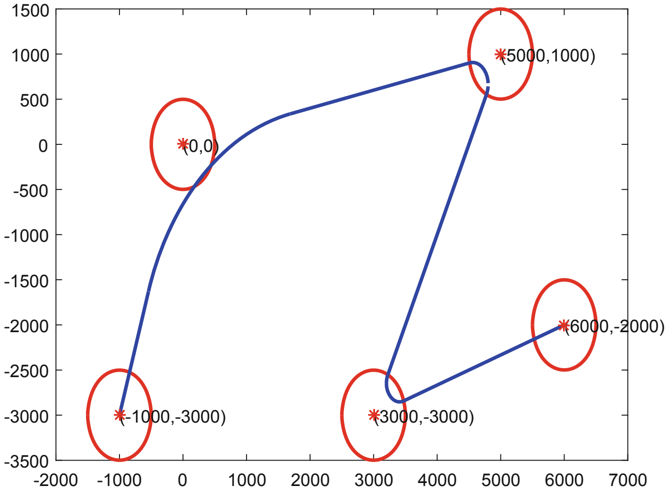

Fig. 6. The trajectory of the UAV which severs 5 GTs.

energy consumption performance is shown in Fig. 5. From this figure, we can find that the energy consumption of red trajectory 2 is the least, and the energy consumption of curve trajectory 3 is the largest.

But when the amount of data transmitted is relatively large, e.g., $\bar{R} \geq 10^{10}$ bit, for example, when real-time high-definition video transmission is required, the data volume is relatively large. At this point, we can find that the energy consumption generated by curve trajectory 3 is the lowest, as showed in Fig. 5, so we can conclude that when there is a large amount of data need to be transmitted, we can choose the curve flight path without passing over the GT's center. The reason is when there is a great deal of data need to be transmitted, the curve trajectory 3 ensures a better channel quality that is the feature of trajectory 1 and also ensures the optimal energy consumption that is the feature of the trajectory 2. Therefore, when these two features are combined, energy consumption performance of the curve trajectory 3 is relatively better especially for massive data. As we mentioned before, when the curve trajectory performs relatively ideal, there is a bond of the volume of data and the bond of the volume is related to the channel quality, e.g. $\bar{R} = 10^{10} bit$ when the transmission power is 0.01W. When the channel quality is reliable, the bond becomes larger, which means the curve trajectory is suitable for massive data transmission especially for reliable channel quality.

Based on the previous scenario, when there are multiple GTs which are severed by the UAV, e.g., 5 GTs are generated randomly and there is a lot of data to be transmitted with reliable channel quality. At this time, the trajectory of the UAV can refer the curve trajectory 3 for minimum energy consumption, as shown in Fig. 6. At this time, compared with the conventional trajectory, this curve trajectory performs better. Since fixed-wing UAV can't hover, so when there are multiple GTs need to be severed, the drone which flies along the conventional trajectory 1 needs to spend extra energy to change direction, and the energy consumption of this hovering steering is very large may even be infinite. But the UAV flies around the GT along the trajectory 3 with smooth turning angles, which achieves a good balance between rate maximization and energy minimization.

5 Conclusion

This paper studies a new trajectory of the UAV which can save more energy for long endurance. By exploiting the conventional trajectory that cross the GT's center and a straight flight path which is beside the GT's center, we design a new trajectory that combines both features of the former trajectories. Specifically, this designed trajectory not only ensures an ideal channel quality but also ensure less energy consumption. Based on these results, a trajectory design algorithm is proposed to jointly optimize the channel quality and energy consumption. Numerical results show that when there is little data to be transmitted, the performance of the curve trajectory is not as ideal as the conventional straight path, but when there is a large amount of data, the curve trajectory performs better, which shows the great potential of this new trajectory. The result in this paper can be further extended by considering the energy efficiency of the UAV.

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