



# Application of Wavelet Analysis Method in Radar Echo Signal Detection

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**Abstract.** In this paper, we focus on several signal detection method and wavelet analysis method for radar echo signal detection. According to the characteristics of signal detection and modern signal processing theory, we have deduced and analyzed the principles of these algorithms in mathematics, which involves more profound knowledge such as higher-order statistics and wavelet, and of course. It is important that we perform wavelet analysis on the echo signals of the HF ground wave radar to remove the weak ionospheric clutter and the method performs well. Wavelet is an important mathematical application method in signal detection.

**Keywords:** Signal detection · Radar echo · Wavelet

## 1 Introduction

Signal detection is an important branch of signal processing. It has been widely used in radar, communication, sonar and automatic fault detection. With the advancement of modernization, electronic equipment is becoming more and more diversified, and the electromagnetic environment in communication is becoming more and more complex, which puts forward higher requirements for signal detection algorithm. With the emergence of many theories and methods, such as high-order statistics, adaptive filtering, time-frequency analysis and neural networks, new vitality has been injected into the field of signal detection. The theoretical level and application level of signal detection have been greatly improved, and the performance of signal detection has been greatly improved. In this paper, we analyze the principle of correlation detection algorithm, and study the source of higher-order statistics and the principle of signal detection based on bispectrum analysis. At the same time, we also perform wavelet processing on the echo signals of high-frequency ground wave radar, which shows the effect of wavelet on signal processing.

## 2 Signal Detection Method

### 2.1 Correlation Detection

Correlation detection is a technology developed in the 1960s. The earliest practical correlation detection system was realized by using tape recorder technology such as Bennett of Bell Laboratory in 1953. In 1961, Weinreb's article described the use of autocorrelation to extract periodic signals from random noise. Since then, a lot of work has been done and this technology has been widely used.

The correlation detection mainly carries on the correlation analysis to the signal and the noise, and the correlation function  $R(t)$  is the main physical quantity of the correlation analysis. The values of deterministic signals at different times are generally highly correlated. As for the interference noise, because of its strong randomness, the correlation of the values at different times is generally poor. Using this difference, the deterministic signal is distinguished from the interference noise.

Correlation detection includes autocorrelation method and cross correlation method. autocorrelation method measures the correlation before and after a random process by autocorrelation function, while cross correlation method measures the correlation between two random processes by cross correlation function. Compared with autocorrelation method, the stronger the ability of cross-correlation method to extract signals, the more thorough the noise suppression. Generally, cross-correlation is based on the repetition period or known frequency of the received signal, which sends out the same reference signal as the frequency of the signal to be measured at the receiving end, and correlates the reference signal with the input signal mixed with noise. The cross-correlation function is expressed as formula (2.1).

$$R_{xy}(\tau) = \lim_{T \rightarrow 0} \frac{1}{T} \int_0^T x(\tau)y(t - \tau)dt \quad (2.1)$$

Let the signal to be measured be  $x(t) = S(t) + n(t)$ , where  $S(t)$  is the characteristic signal and  $n(t)$  is the noise. If  $y(t)$  is the reference signal and  $R_{xy}(\tau)$  is the cross-correlation function of  $x(t)$  and  $y(t)$ , then the cross-correlation function is formula (2.2):

$$\begin{aligned} R_{xy}(\tau) &= E[x(t)y(t - \tau)] \\ &= E[S(t)y(t - \tau)] + E[n(t)y(t - \tau)] \\ &= R_{Sy}(\tau) + R_{ny}(\tau) \end{aligned} \quad (2.2)$$

If  $n(t)$  is not related to  $y(t)$ , then  $R_{ny}(\tau) = 0$ . Therefore,  $R_{xy}(\tau) = R_{Sy}(\tau)$ , in which  $R_{Sy}(\tau)$  is the cross-correlation function of  $S(t)$  and reference signal  $y(t)$ .

### 2.2 Signal Detection Based on Higher Order Statistics

High-order statistics contain a lot of rich information that second-order statistics do not have, so the application of high-order statistics in signal detection can achieve higher performance than second-order statistics in signal detection.

Let the random variable  $X$  have a probability density function  $f(x)$ , the characteristic function is defined as formula (2.3):

$$\Phi(\omega) = \int_{-\infty}^{\infty} f(x)e^{j\omega x} dx = E\{e^{j\omega x}\} \tag{2.3}$$

The eigenfunction is also called the first eigenfunction. The second eigenfunction is defined as formula (2.4):

$$\Psi(\omega) = \ln[\Phi(\omega)] \tag{2.4}$$

$K$ -order moment  $m_k$  of random variable  $x$ , such as  $k$ -order derivative of first characteristic function of random variable at origin, as formula (2.5):

$$m_k = \Phi^k(\omega)|_{\omega=0} \tag{2.5}$$

The  $k$ -order derivative of the second characteristic function of a random variable at the origin is equal to the  $k$ -order cumulant  $c_k$  of the random variable  $x$ , as formula (2.6):

$$c_k = \Psi^k(\omega)|_{\omega=0} \tag{2.6}$$

Let  $\{x(n)\}$  be a  $k$ -order stationary process with zero mean, then the  $k$ -order moments of the process are defined as formula (2.7):

$$m_{kx}(\tau_1, \tau_2, \dots, \tau_{k-1}) = mom\{x(n), x(n + \tau_1), \dots, x(n + \tau_{k-1})\} \tag{2.7}$$

The  $k$ -order cumulant is defined as formula (2.8):

$$c_{kx}(\tau_1, \tau_2, \dots, \tau_{k-1}) = cum\{x(n), x(n + \tau_1), \dots, x(n + \tau_{k-1})\} \tag{2.8}$$

The most common high-order spectrum is the third-order spectrum (bispectrum) as formula (2.9):

$$B_x(\omega_1, \omega_2) = \sum_{\tau_1=-\infty}^{\infty} \sum_{\tau_2=-\infty}^{\infty} c_{3x}(\tau_1, \tau_2) e^{-j(\omega_1\tau_1 + \omega_2\tau_2)} \tag{2.9}$$

When the detected signal is detected, the given data is divided into  $K$  segments, each segment contains  $M$  observation samples, namely  $N = KM$ , and the average value of each segment is subtracted from the data. If necessary, add zero to each segment of data to meet the requirement of the general length  $M$  of FFT. Next, calculate the DFT coefficient, as formula (2.10).

$$Y^{(i)}(\omega) = \frac{1}{M} \sum_{n=0}^{M-1} y^{(i)}(n) \exp(-j2\pi n\omega/M) \tag{2.10}$$

$$\omega = 0, 1, \dots, M/2, i = 1, 2, \dots, K$$

Continue to calculate the triple correlation of DFT coefficients as formula (2.11):

$$\begin{aligned}
 b_i(\omega_1, \omega_2) = \frac{1}{\Delta_0^2} \sum_{k_1=-L_1}^{L_1} \sum_{k_2=-L_2}^{L_2} Y^{(i)}(\omega_1 + k_1) Y^{(i)}(\omega_2 + k_2) Y^{(i)}(-\omega_1 - k_1 - \omega_2 - k_2) \\
 \begin{aligned}
 & i = 1, 2, \dots, K, \\
 & 0 \leq \omega_2 \leq \omega_1, \\
 & \omega_1 + \omega_2 \leq f_s/2, \\
 & M = (2L_1 + 1)N
 \end{aligned}
 \end{aligned} \tag{2.11}$$

From this, we can get the bispectrum estimation of the measured data, which is showed as formula (2.12):

$$\begin{aligned}
 B_D(\overline{\omega}_1, \overline{\omega}_2) &= \frac{1}{K} \sum_{i=1}^K b_i(\overline{\omega}_1, \overline{\omega}_2) \\
 \overline{\omega}_1 &= \left(\frac{2\pi f_s}{N_0}\right) \omega_1, \overline{\omega}_2 = \left(\frac{2\pi f_s}{N_0}\right) \omega_2
 \end{aligned} \tag{2.12}$$

### 2.3 Signal Detection Based on Wavelet Transform

Wavelet analysis is a kind of time-frequency analysis. It is the development of Fourier analysis, but it is better than Fourier analysis. Although Fourier analysis is widely used as a classical method, it has its own shortcomings, that is, it can not express the most critical time-frequency localization properties of signals. In order to analyze and process non-stationary signals, short-time Fourier transform and wavelet transform are generated. The window width of the wavelet transform is adjustable. It has the ability to characterize the local characteristics of signals in both time and frequency domains, and has the characteristics of multi-resolution analysis.

Wavelet transform is defined as follows: Let  $\Psi(t) \in L^2(\mathbb{R})$ , whose Fourier transformation satisfies admissible condition (2.13):

$$\int_{\mathbb{R}} \frac{|\Psi(\omega)|^2}{\omega} d\omega < \infty \tag{2.13}$$

$\Psi(t)$  is called the wavelet generating function. The scaling and translation of the wavelet generating function are carried out. The scaling factor is  $a$  and the translation factor is  $\tau$ . If the scaling function is  $\Psi_{a,\tau}(t)$ , the scaling factor of the wavelet generating function is as formula (2.14):

$$\Psi_{a,\tau}(t) = a^{-\frac{1}{2}} \Psi\left(\frac{t-\tau}{a}\right), a > 0, \tau \in \mathbb{R} \tag{2.14}$$

$\Psi_{a,\tau}(t)$  is called a wavelet basis function. Expansion of any function  $f(t) \in L^2(\mathbb{R})$  on wavelet basis is called continuous wavelet transform of function  $f(t)$ . Its expression is as formula (2.15):

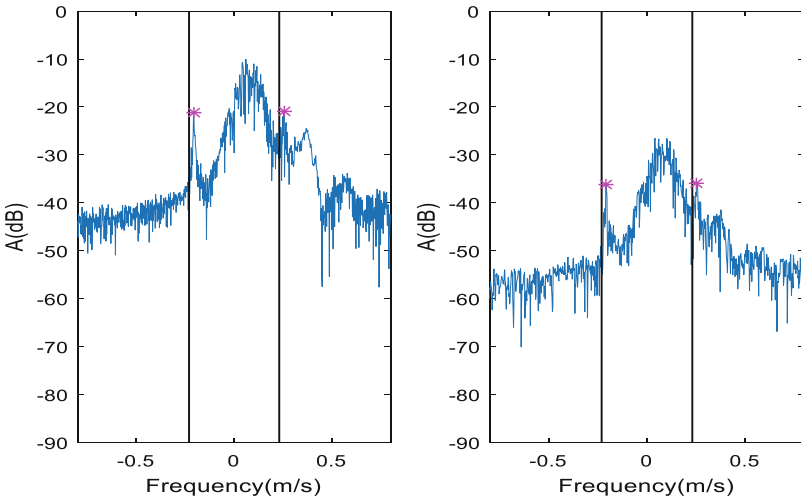
$$WT_f(a, \tau) = \frac{1}{\sqrt{a}} \int_R f(t) \Psi^* \left( \frac{t - \tau}{a} \right) dt \quad (2.15)$$

Reference formula (2.14), formula (2.15) can be written in the form of inner product as formula (2.16).

$$WT_f(a, \tau) = \int_{-\infty}^{+\infty} f(t) \Psi_{a,\tau}^*(t) = \langle f(t), \Psi_{a,\tau}(t) \rangle \quad (2.16)$$

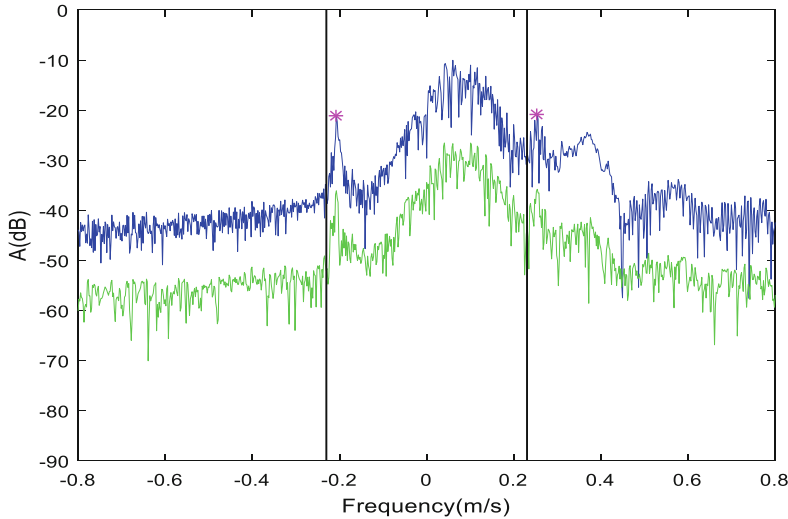
It can be seen from the formula that when scale  $a$  increases, the whole  $f(t)$  is observed with extended  $\Psi(t)$  waveform, whereas when scale  $a$  decreases, the local  $f(t)$  is measured with compressed  $\Psi(t)$  waveform. In a certain scale or within a certain scale, the intensity of the wavelet coefficients of the signal is larger. Therefore, the wavelet coefficients can be used as the object of signal detection.

Next we take the high-frequency ground wave radar echo signal with a large number of ionospheric clutter as an example, and use the processing of different wavelet bases to compare the wavelet to help the data detection processing (Figs. 1 and 2).



**Fig. 1.** Comparison before and after wavelet processing

Observing and comparing the radar echo spectrum and the spectrogram obtained by processing the data 1 and data 2 using two wavelet bases, the signal-to-noise ratio of the unprocessed signal is calculated to be 10.1. After processing the data 1 by the wavelet method, the haar wavelet base improves the signal miscellaneous The ratio is 2.06, and the signal-to-noise ratio improved by sym8 wavelet is 0.12. After processing data 2 by wavelet method, the haar wavelet base improves the signal-to-noise ratio to 1.97, and the sym8 wavelet improves the signal-to-noise ratio to 1.09. Then continue to



**Fig. 2.** Visual comparison before and after wavelet processing

select db9, db8 and sym5 three wavelet bases, in the case of 5 layers decomposition, select the same coefficient, compare data 1 and data 2 after different wavelet basis wavelet method to improve the signal-to-noise ratio.

### 3 Conclusion

Among many weak signal detection methods, correlation detection is one of the most common and effective methods. The accuracy of system identification using correlation detection technology will be affected by integration time and signal bandwidth. The wider the signal bandwidth, the longer the integration time, the higher the accuracy. Up to now, the theory of high-order spectral analysis has been perfected. The signal processing using high-order spectrum (statistics) can suppress the unknown Gauss noise in signal detection, parameter estimation and classification. It can detect the nonlinearity of time series, and reconstruct the amplitude and phase response of signal or system. The third-order spectrum (bispectrum) technology of high-order spectrum is used to detect signals in strong noise background, and good results are obtained. As for the detection method of wavelet coefficients, we can choose the wavelet coefficients in a single scale as the object of study, or we can enhance the strength of the useful signals by averaging the wavelet coefficients in several scales in a certain scale range, that is, the method of coefficient accumulation.

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