DOA Estimation for Coherent and Incoherent Targets with Co-prime MIMO Array

Yong Jia[(✉)], Chao Yan, Chuan Chen, Bin Duo, Xiaoling Zhong, Yong Guo, and Shiying Yin

College of Information Science and Technology, Chengdu University of Technology, Chengdu, China
jiayong2014@cdut.edu.cn

Abstract. In this paper, we consider the problem of DOA estimation for a mix of incoherent and coherent targets by using the monostatic co-prime MIMO array with N sparse transmitting sensors and $2M-1$ sparse receiving sensors. The co-prime MIMO array generates a non-redundant and uniform sub sum co-array with $\mathrm{O}(MN)$ contiguous sensors using only $\mathrm{O}(M+N)$ physical sensors. Based on the concept of sum co-array equivalence, we can obtain different configurations of virtual MIMO arrays with $\mathrm{O}(MN)$ contiguous virtual sensors, and then construct the corresponding virtual data matrices, which provides different tradeoffs between the number of resolvable targets and the maximum number of mutually coherent targets that can be resolved. On the basis of the virtual data matrix and the conventional DOA estimation approaches such as MUSIC, $\mathrm{O}(MN)$ mixed coherent and incoherent targets can be resolved only with $\mathrm{O}(M+N)$ physical sensors, namely the number of resovable targets exceeds the limitation of the number of physical sensors. Finally, simulation results demonstrate the effectiveness of the proposed DOA estimation method with the monostatic co-prime MIMO array in the presence of both the coherent and incoherent targets.

Keywords: DOA estimation · Coherent and incoherent targets · Monostatic MIMO array · Co-prime array

1 Introduction

The co-prime geometry consists of a combination of two sparse uniform linear subarrays whose sensor numbers and inter-sensor spacings depend on two co-prime integers and which was presented firstly to be used in the only-receiving array, implementing DOA estimation for far-field distributed incoherent targets

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[2–4,8–10,13–16]. The most attracted point of the receiving co-prime array is a uniform sub-coarray in the associated difference co-array with much more contiguous co-array sensors than physical sensors and the fundamental spacing of half a wavelength, which is capable of providing a dramatic promotion in the degrees-of-freedom and hence can resolve significantly more incoherent targets than the number of physical sensors. That is because the number of the identifiable incoherent targets is determined directly by the sensor number of the difference co-array instead of the co-prime passive array [6].

However, in the case of coherent targets as encountered in multi-path propagation, the DOA estimation methods in [2–4,8–10,13–16] are not feasible because of non-zero cross correlation between two coherent targets. Based on the researches in [7,17], the active transmitting and receiving array, namely monostatic MIMO array can be used to identify a mix of coherent and incoherent targets, and the numbers of receiving sensors and transmitting sensors determine respectively the numbers of the total targets and coherent targets which can be identified. Recently, the co-prime geometry was introduced to the monostatic MIMO array with three different transmitting and receiving configurations for the active DOA estimation of coherent and incoherent targets [1,5]. Specifically, a sparse reconstruction method was proposed for the vectorized covariance matrix of the emulated sum co-array observations from the original data measurements of co-prime MIMO array. Moreover, a new scheme to implement active DOA estimation of coherent and incoherent targets was designed for the co-prime MIMO array in [11], where the DOAs of the incoherent targets were first estimated using subspace-based methods, whereas those of the coherent targets were resolved using Bayesian compressive sensing.

In this paper, we consider the most basic configuration of co-prime MIMO array, namely the first subarray is employed to transmit and the other is for receiving. Obviously, the original data matrix is unable to be used directly for the active DOA estimation because the uniform and sparse spatial sampling from the receiving subarray results in the spectrum aliasing. To address the aliasing problem, inspired by [10], we need to find a desired virtual MIMO array with the receiving inter-sensor spacing of half a wavelength which has the same sum co-array with the co-prime MIMO array. As the sum co-array determines the collected original data matrix of the co-prime array, it can be rearranged to construct the original data matrix of the virtual MIMO array named virtual data matrix which can be utilized to estimate the DOAs of the coherent and incoherent targets.

For a monostatic co-prime MIMO array including N transmitters and $2M-1$ receivers (M and N is two co-prime numbers and M is assumed to be less than N), there is a non-redundant and uniform sub sum co-array with $MN+M-1$ contiguous sensors and the spacing of half a wavelength, which makes it easy to search for a desired virtual MIMO array with the receiving inter-sensor spacing of half a wavelength. In light of $MN+M-1$ co-array sensors, the virtual MIMO array consists of $MN+M$ virtual sensors at most, which is capable of obtaining the DOAs of $\mathrm{O}(MN)$ targets from only $\mathrm{O}(M+N)$ physical sensors of the co-prime MIMO array. Moreover, we can get multiple different configurations of virtual MIMO arrays with the identical sum co-array, which provide different numbers of tradeoffs between the total number of the resolvable targets and the largest number of mutually coherent targets that can be identified.

2 Methodology

2.1 Signal Model of Monostatic Co-prime MIMO Array

Assume M and N are coprime numbers and M is less than N. A monostatic co-prime MIMO array is shown in Fig. 1, including N transmitters and $2M-1$ receivers whose positions are given by the set

$$\begin{aligned} P_t &= \{Mnd;\ n = 0, 1, \cdots, N-1\} \\ P_r &= \{Nmd;\ m = 1, 2, \cdots, 2M-1\} \end{aligned} \tag{1}$$

where d is a fundamental spacing defined as half one wavelength of transmitting narrowband signal to avoid spatial aliasing. It is clear that the co-prime MIMO array is subjected to uniformly sparse transmitting and sparse sampling.

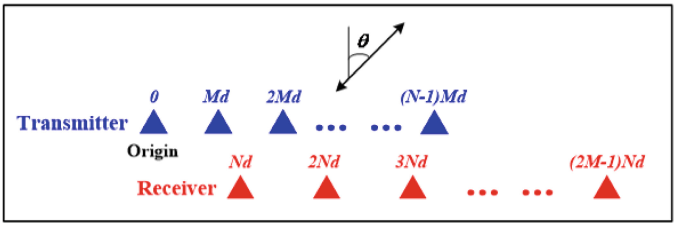


Fig. 1. Geometry of monostatic co-prime MIMO array.

Assume that a mix of targets with L incoherent targets and K coherent targets are located at far-field of the array. For simplification, the incoherent targets are ordered from 1 to L and the coherent targets are ordered from $L+1$ to $L+K$. A narrowband signal is individually transmitted by each of the N transmitters to illuminate these targets, and the reflectivity of all targets is collected by all $2M-1$ receivers to form an $(2M-1)$-by-N natural data matrix modeled as

$$\begin{aligned} \mathbf{X} &= \mathbf{S} + \mathbf{N} \\ &= \left[\sum_{k=1}^{L+K} \bar{s}_{k,n} \cdot \exp\left[jk_0(r_m + t_n)\sin\theta_k\right] + \bar{n}_{m,n} \right]_{m,n} \\ &= \left[\sum_{k=1}^{L+K} \bar{s}_{k,n} \cdot \exp\left\{jk_0\left[mNd + (n-1)Md\right]\sin\theta_k\right\} + \bar{n}_{m,n} \right]_{m,n} \\ & m = 1, 2, \cdots, 2M-1;\ \ n = 1, 2, \cdots, N \end{aligned} \tag{2}$$

where the (m,n)th entry is the complex reflectivity amplitude measured by the mth receiver when the nth transmitter is applied to illuminate. Moreover, $k_0 = 2\pi/\lambda$ is the wavenumber of the emitted narrowband signal. t_n and r_m are the positions of nth transmitter and mth receiver. θ_k is the DOA of the kth targets. The noise $\bar{n}_{m,n}$ is assumed to be zero mean, temporally and spatially white, and uncorrelated from the targets. The complex reflectivity amplitude $\bar{s}_{k,n}$ can be defined as

$$\bar{s}_{k,n} = \begin{cases} \alpha_k e^{j\beta_{k,n}}, & k = 1, 2, \cdots, L \\ \alpha_k e^{j\beta_n}, & k = L+1, L+2, \cdots, L+K \end{cases} \tag{3}$$

where α_k is a constant complex amplitude and $\beta_{k,n}$ is a random phase for L incoherent targets and N illuminations. However, the random phase β_n is common to all K coherent targets on the nth illumination.

In this case, we express the correlation of target reflectivity as follows.

$$E\left[\bar{s}_{k,n}\bar{s}^*_{k',n'}\right] = \begin{cases} \alpha_k\alpha^*_{k'}E\left[e^{j(\beta_n-\beta_{n'})}\right], & (k, k' = L+1, L+2, \cdots, L+K \\ \alpha_k^2 E\left[e^{j(\beta_{k,n}-\beta_{k,n'})}\right], & (k, k' = 1, 2, \cdots, L \text{ and } k = k') \\ 0, & (\text{otherwise}) \end{cases} \tag{4}$$

In this paper, we consider a simplified scene where the reflectivity of each target is completely coherent during N transmissions, namely $E\left[e^{j(\beta_n-\beta_{n'})}\right] = 1$ and $E\left[e^{j(\beta_{k,n}-\beta_{k,n'})}\right] = 1$.

2.2 Sum Coarray of Co-prime MIMO Array

Based on the $(2M-1)$-by-N natural data matrix of the co-prime MIMO array, we can obtain an $(2M-1)$-by-$(2M-1)$ covariance matrix, but which cannot be used to resolve the targets due to the spectrum aliasing caused by the sparse sampling.

Clearly, the phases of the original data in (2) are determined by the sum of the transceiver positions, namely the sensor position of the sum co-array of the co-prime MIMO array. That is to say, the original data matrix is dependent on the sum co-array directly. Herein, we can search for a desired virtual MIMO array whose sum co-array is identical to that of the co-prime MIMO array, and then construct the associated virtual data matrix by rearranging the original data matrix. Finally, we can use the virtual data matrix to estimate the DOAs.

In order to search for a desired virtual MIMO array being sum-coarray equivalent with the co-prime MIMO array, we first need to exploit the characteristic of the sum co-array of the co-prime MIMO array. The sum co-array of the co-prime array in Fig. 1 is given by Fig. 2.

Due to the co-prime relationship, each transceiver pair forms a unique sensor of the sum co-array. In other words, the co-prime MIMO array is non-redundant from the point of view of the sum co-array. More interesting is that as shown in Fig. 2, the sum co-array can be divided into three sub-coarrays, namely two non-uniform sub-coarrays and a uniform sub-coarray with the fundamental inter-sensor spacing of d. For the uniform sub-coarray, $MN+M-1$ sensors are located

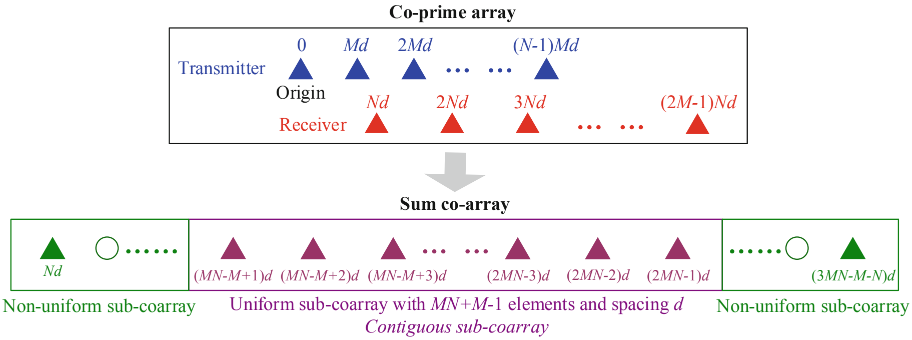


Fig. 2. Geometry of sum co-array of the monostatic co-prime MIMO array.

contiguously from the position $(MN-M+1)d$ to $(2MN-1)d$ with the spacing of d. Therefore, we refer to the uniform sub-coarray with the fundamental inter-sensor spacing as the contiguous sub-coarray as well.

2.3 Searching for Virtual MIMO Array

First the contiguous sub-coarray with $MN+M-1$ sensors and spacing d is chosen as the reference sum co-array due to the convenience in determining a desired virtual MIMO array with the receiver spacing of d avoiding the problem of spectrum aliasing.

The locations of $MN+M-1$ contiguous sensors can be written as the set

$$P_c = \{(MN-M+q)\,d;\ \ q = 1, 2, \cdots, MN+M-1\} \tag{5}$$

Assume a desired virtual MIMO array with A transmitters and B receivers is given by

$$\begin{aligned} P_{v,t} &= \{(x_t + \alpha_i i)\,d;\ \ i = 1, 2, \cdots, A\} \\ P_{v,r} &= \{(x_r + j)\,d;\ \ j = 1, 2, \cdots, B\} \end{aligned} \tag{6}$$

where the spacing of receiving array is fundamental spacing d to avoid spectrum aliasing, while the transmitter spacing is a variable positive integer α_i times d.

The virtual MIMO array is narrowly sum co-array equivalent with the co-prime MIMO array if

$$\begin{aligned} &x_t + \alpha_i i + x_r + j = MN - M + Q, \\ &\forall i \in [1, 2, \cdots, A],\ j \in [1, 2, \cdots, B],\ \ Q \in [1, 2, \cdots, MN+M-1] \end{aligned} \tag{7}$$

Therefore, the sum co-array of the virtual MIMO array is identical to the contiguous sub-coarray of the co-prime MIMO array.

In order to use the definition of sum co-array equivalence, the numbers of virtual transmitters and receivers should be determined firstly by the number of targets to be identified. This is based on the point that there must be at least as many transmitters as coherent targets and the numbers of receivers must be less than the total number of incoherent and coherent targets. For example, for M_1 targets with M_2 coherent targets ($M_1 \geq M_2$), the virtual MIMO array has to consist of at least M_2 transmitters and M_1 receivers. Then a desired virtual MIMO array can be acquired readily.

Now we give a special example of finding the virtual MIMO array under the condition of $\alpha_i = 1$ and sum coarray equivalence. In this case, a virtual MIMO array including the maximum sensors can be obtained, meaning the maximum number of identifiable targets. The virtual MIMO array can be found by solving the simple optimization problem

$$\begin{array}{l} \begin{cases} x_t + A + x_r + B = 2MN - 1 \\ x_t = b \end{cases} \\ \text{subject to } A + B = MN + M \end{array} \tag{8}$$

where b can be set as an arbitrary integer. As a result, the positions of the virtual transmit/receive sensors can be expressed as

$$\begin{array}{l} P'_{v,t} = \{(b+i)\,d;\ i = 1, 2, \cdots, MN + M - B\} \\ P'_{v,r} = \{(MN - M - b - 1 + j)\,d;\ j = 1, 2, \cdots, B\} \end{array} \tag{9}$$

As A and B are variables, we can obtain multiple virtual MIMO arrays with different numbers of transmitters and receivers, achieving great flexibility to select a desired virtual MIMO array to satisfy the identification for a different numbers of targets.

2.4 Construction of Virtual Data Matrix

For each desired virtual MIMO array, the virtual data matrix can be constructed by rearranging the original data matrix of the co-prime MIMO array through the following two steps.

First, $MN + M - 1$ distinct data units corresponding to the uniform sub-coarray are extracted from the original data matrix to constitute co-array data vector as

$$\begin{aligned} X_c &= \left[x_{c1}, x_{c2}, \cdots, x_{c(MN+M-1)}\right]^T \\ &= \left[\sum_{k=1}^{L+K} \bar{s}_k \cdot \exp\left\{jk_0(MN - M + q)d \sin\theta_k\right\} + \bar{n}_q\right]_q^T \\ & q = 1, 2, \cdots, MN + M - 1 \end{aligned} \tag{10}$$

which is identical to the standard model for passive array processing, with sensors located on the $MN + M - 1$ positions of contiguous sub-coarray.

Then one or multiple sensor positions of the virtual data matrix are filled with the corresponding sensor of the co-array data vector. These sensors are coupled with the same sum co-array position. After repeating this operation for all other sensors, a filled virtual data matrix is obtained. For example, the B-by-($MN + M - B$) virtual data matrix of the virtual MIMO array in (10) is constructed as

$$\begin{aligned}\mathbf{X}_v &= [X_c(1), X_c(2), \cdots, X_c(MN+M-B)] \\ &= \left[x_{ck}, x_{c(1+k)}, \cdots, x_{c(B-1+k)}\right]_k^T, \\ & k = 1, 2, \cdots, MN+M-B \end{aligned} \tag{11}$$

2.5 Calculation of MUSIC Spectrum

From a desired B-by-A virtual data matrix, we can obtain a B-by-B virtual covariance matrix, which has been proved to be of the same rank with the number of targets when the number of receivers is more than the number of targets and the transmitters is at least as many as the coherent targets.

Consequently, the MUSIC approach in [12] is performed on the virtual covariance to acquire the DOA spectrum lines of the incoherent and coherent targets, given by

$$CP_{music} = \frac{1}{\bar{v}(\theta)^H \mathbf{U_N} \mathbf{U_N}^H \bar{v}(\theta)} \tag{12}$$

where $\mathbf{U_N}$ is noise subspace composed of the eigenvectors from eigenvalue decomposition of virtual data matrix, and $\bar{v}(\theta) = [1, e^{j\pi \sin\theta}, e^{j2\pi \sin\theta}, \cdots, e^{j(B-1)\pi \sin\theta}]^T$ is the same with the manifold vector of uniform linear receiver array with B sensors. As a result, the DOA spectrum lines of at most B-1 targets including A coherent targets are acquired in the case of $A < B$, while B-1 targets weather coherent or incoherent can be identified at most as $A \geq B$. Considering A and B are associated with the $MN + M - 1$ coarray sensors, in other words, we can estimate the DOAs of O(MN) targets based on the co-prime MIMO array only including O$(M + N)$ physical sensors.

It is worthy that in the case of $A \geq B$, the number of total resolvable coherent and incoherent targets is $B - 1$ that is less than A. That is to say, the A degree-of-freedom for coherent targets is not utilized completely. Therefore, in the searching for a desired virtual MIMO array, the number of virtual transmitters is demanded to be less than the number of virtual receivers in general, making the total degree-of-freedom from the virtual transceivers serve for the DOA estimation of coherent and incoherent targets.

3 MATLAB Simulation Results and Analyses

A monostatic co-prime MIMO array with $N = 4$ transmitters and $2M - 1 = 5$ receivers is chosen in the MATLAB simulations as shown in the subfigure (a) of Fig. 3. For simplification, the fundamental spacing d of half one wavelength is set up as 1. White Gaussian noise with 10 dB signal-to-noise ratio (SNR) and 2000 snapshots is applied for the array noise and the covariance estimation. A 5-by-4 original data matrix is generated and then a 5-by-5 covariance matrix can be calculated to perform MUSIC directly. As shown in Fig. 4, the problem of spectrum aliasing appears in the direct MUSIC spectrum due to the sparse

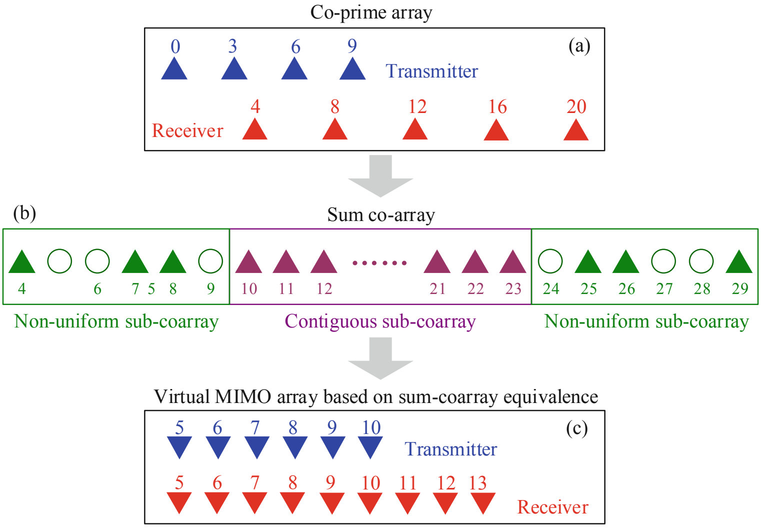


Fig. 3. Geometries of co-prime MIMO array with $M = 3$ and $N = 4$, the corresponding sum co-array and virtual MIMO array based on sum co-array equivalence.

sampling of 2 times one wavelength, which makes it impossible to identify the deployed 4 four targets including 3 coherent targets.

For this co-prime MIMO array, as shown in the subfigure (b) of Fig. 3, a contiguous sub sum co-array is composed of 14 sensors located from $MN - M + 1 = 10$ to $2MN - 1 = 23$ with the inter-sensor spacing of half one wavelength. Therefore, based on the defined sum co-array equivalence, we seek out a virtual MIMO array shown in the subfigure (c) of Fig. 3, consisting of 6 transmitters at [5, 6, 7, 8, 9, 10] and 9 receivers at [5, 6, 7, 8, 9, 10, 11, 12, 13]. Accordingly, 9-by-6 virtual data matrix and 9-by-9 virtual covariance matrix are acquired. Thus 8 targets with 6 coherent targets can be identified at most. As shown in Fig. 5, the spectrum aliasing is disappeared due to the sampling space of half one wavelength. Under the conditions of 10 dB SNR and 2000 snapshots, 8 targets including 6 coherent targets are identified accurately in Fig. 5. Obviously, the numbers of identifiable coherent targets and all targets exceed the number limitations of physical transmitting and receiving sensors respectively.

Finally, we evaluate the flexibility of DOA estimation by searching for different configurations of virtual MIMO array based on the sum co-array equivalence. Figure 6 reveals another two virtual MIMO arrays whose sum co-arrays are the same with the contiguous sub sum co-array. On the one hand, the virtual MIMO array 3 consists of the most seven transmitters when the transmitter number is smaller than the receiver number, which means the maximum identifiable coherent targets. In this case, all seven coherent targets without incoherent targets

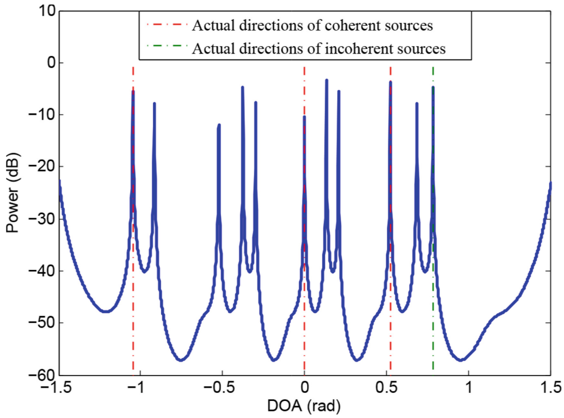


Fig. 4. MUSIC spectrums with 2000 snapshots and 10 dB SNR based on the co-prime MIMO array.

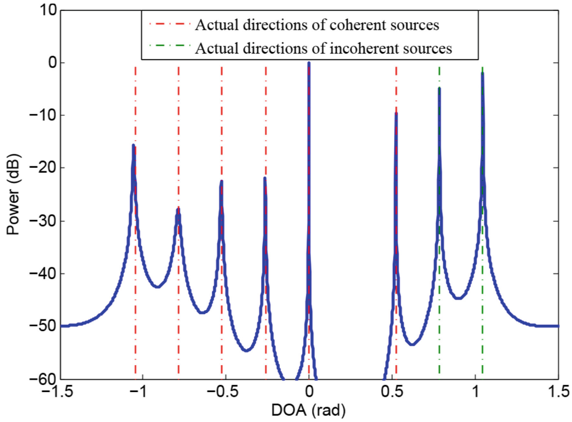


Fig. 5. MUSIC spectrums with 2000 snapshots and 10 dB SNR based on the virtual MIMO array with sum co-array equivalence.

can be resolved as shown in Fig. 7(a). On the other hand, the virtual MIMO array 4 only has two transmitters which can be used to estimate two coherent targets. In other words, the number of resolvable coherent targets reaches the lower limit. However, the most thirteen receivers brings about the maximum resolvable number of twelve for the total of coherent and incoherent targets. Figure 7(b) demonstrates twelve targets with two coherent targets are identified successfully.

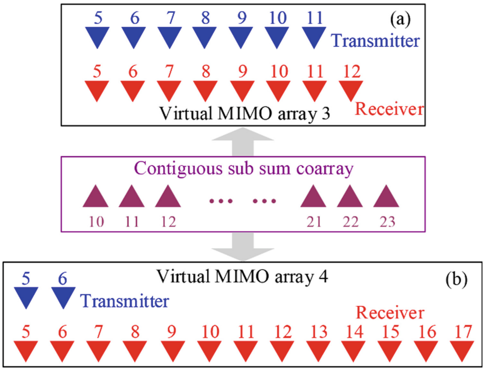
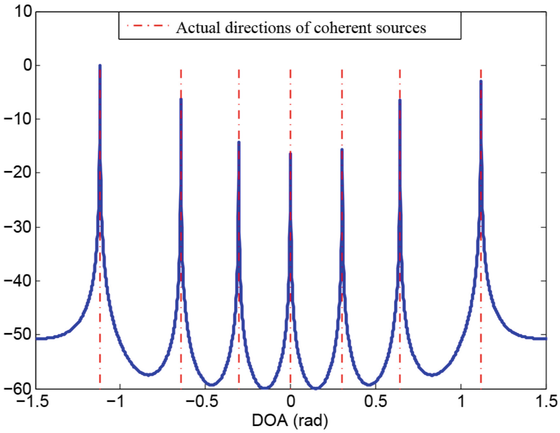
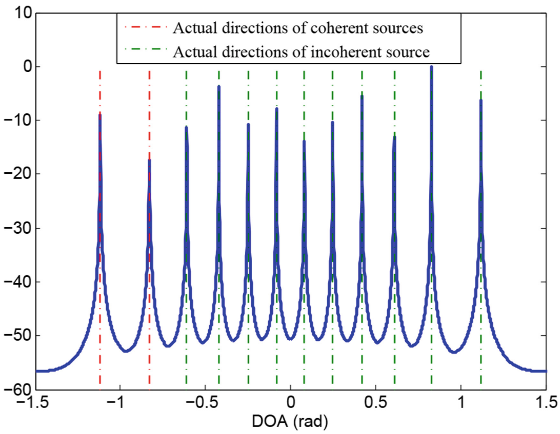


Fig. 6. Geometries of two virtual MIMO arrays based on sum co-array equivalence.



(a)



(b)

Fig. 7. MUSIC spectrums with 2000 snapshots and 10 dB SNR based on (a) the virtual MIMO array 3 and (b) the virtual MIMO array 4.

4 Conclusion

In order to increase the numbers of resolvable total targets and therein coherent targets to be more than the numbers of physical receivers and transmitters, the active DOA estimation algorithm for the monostatic co-prime MIMO array is designed in this paper based on the presented definition of sum co-array equivalence and virtual MIMO arrays. By using the determined virtual MIMO array with more virtual transmitting and receiving sensors than the physical sensors in the co-prime array, the numbers of resolvable total targets and therein coherent targets breaks through the limitation of the numbers of receiving sensors and transmitting sensors, respectively. Based on the sum co-array equivalence, multiple virtual MIMO arrays with different numbers of transmitters and receivers can be obtained. Therefore, a desired virtual MIMO array can be selected flexibly to meet one specific demand of target identification.

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