DOA Estimation for Coherent and Incoherent Sources Based on Co-prime Array

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Abstract. In this paper, a DOA estimation method based on co-prime array is proposed to resolve the coherent and incoherent hybrid sources. Firstly, with respect to the difference co-array of co-prime array, the desired units with corresponding contiguous intervals in the correlation matrix are extracted and rearranged into an augmented correlation matrix. Then we decorrelate the augmented correlation matrix by reconstructing matrix algorithm, forward spatial smoothing and forward-backward spatial smoothing algorithm. Finally, through MUSIC spatial spectrum searching on the basic of the decorrelated correlation matrix, DOA estimation towards sources is obtained. The simulation results show that the proposed method can achieve DOA estimation of the coherent and incoherent hybrid sources with more number than physical array. Through comparison, it can be concluded that the reconstructing matrix algorithm obtains a larger number of distinguishable sources and the error performance of which is better under low SNR. However, the spatial smoothing algorithms have a better estimation error performance in the case of low snapshot.

Keywords: DOA estimation · Co-prime array · Coherent and incoherent sources · Difference co-array

1 Introduction

Direction-of-arrival (DOA) estimation is one of the main part of the array signal processing area, which is widely used in mobile communications, radar, sonar detection, wireless navigation and so on. Uniform linear array (ULA) is adopted by traditional DOA estimation algorithms in general, such as the multiple signal

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classification (MUSIC) algorithm [11]. However, the number of resolvable sources is limited by the number of physical elements. Therefore, sparse arrays are been introduced, such as co-prime array [9], nested array [8], minimum redundancy array [7].

Most of the existing DOA estimation algorithms based on co-prime array are against incoherent sources from the point of view of difference co-array [1–6, 9, 10, 12–14]. For example, making use of the one-to-one correspondence between the position of co-array elements and the correlative intervals, the units of correlation matrix are vectorized to be an equivalent single snapshot coherent signal. By spatial smoothing, spatial spectrum of incoherent DOA is obtained [9, 10, 14]. Because of the larger number of elements in difference co-array than the physical elements in co-prime array, the number of resolvable sources can be broken through the limit of physical elements of co-prime array. However, these kind of algorithms are only flexible in dealing with incoherent sources, which is invalid for coherent sources owing to the non-zero cross correlation between two coherent sources.

In this paper, we propose an algorithm to estimate coherent and incoherent sources based on co-prime array. Firstly, with respect to the difference co-array of co-prime array, we extract the corresponding units from the correlation matrix to form an augmented correlation matrix. Then the decorrelation is performed on the augmented correlation matrix by using spatial smoothing method and reconstructing matrix method. Finally, through MUSIC spatial spectrum searching on the decorrelated correlation matrix, DOA estimation towards a mix of scattered sources is implemented. The simulation results show that the proposed algorithm can estimate the coherent and incoherent sources whose number is larger than physical arrays. Furthermore, we compare the performance of reconstructing matrix method, forward spatial smoothing method, and forward and backward spatial smoothing method under the condition of eliminating coherence. The results demonstrate that reconstructing matrix method archives a larger number of DOA estimation, and the error performance of which is better under low signal-to-noise ratio (SNR). Nevertheless, the spatial smoothing algorithm has a better estimation error performance in the case of low snapshot.

2 Signal Model with Coherent and Incoherent Sources for Co-prime Array

A co-prime array consists of two sparse physical arrays as depicted in Fig. 1, where one having N sensors positioned at $\{Mnd_0,\ 0 \leq n \leq N-1\}$, and the other comprising M sensors with positions $\{Nmd_0,\ 0 \leq m \leq 2M-1\}$, M and N being co-prime integers and d_0 being the interval of two adjacent elements and equal to one-half wavelength.

Based on the model of co-prime array above, assume that a mix of scattered sources with L incoherent sources and K coherent sources are located at far field of the array. For simplification, the incoherent sources are ordered from 1 to L and the coherent sources are ordered from $L+1$ to $L+K$. The power of

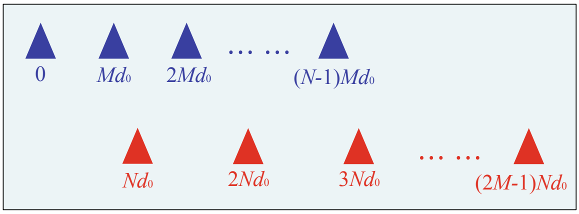

Fig. 1. The model of co-prime array.

$L+K$ sources are $\sigma_1^2(w_0), \sigma_2^2(w_0), \cdots, \sigma_L^2(w_0), \cdots \sigma_{L+K}^2(w_0)$ with directions $[\theta_1, \theta_2, \cdots \theta_L, \cdots \theta_{L+K}]$. Assuming that the sources are uncorrelated and the noise is spatially and temporally white. The power of noise is $\sigma_n^2(w_0)$. Assume that the sources and noise are independent. The received data vector at frequency w_0 can be expressed as

$$X(w_0) = A(w_0) S(w_0) + N(w_0) \tag{1}$$

where $S(w_0) = [s_1(w_0), s_2(w_0), \cdots s_L(w_0), \cdots s_{L+\mathrm{K}}(w_0)]^T$ is the source signal vector at w_0, $N(w_0)$ is the corresponding noise vector, $A(w_0)$ is the array manifold matrix at w_0, and the superscript $(\cdot)^T$ denotes matrix transpose. The $(i,j)\,th$ unit of the manifold matrix can be expressed as

$$[A(w_0)]_{i,j} = e^{jk_0 x_i \sin(\theta_j)}, i = 1, 2, \cdots, N+2M-1; j = 1, 2, \cdots, L, \ldots L+K \tag{2}$$

where x_i is the location of the ith physical sensor of the array, θ_j is the DOA of the jth source, and $k_0 = w_0/c$ is the wavenumber at w_0 with c being the speed of propagation in free space.

3 DOA Estimation for Coherent and Incoherent Sources

3.1 Calculation of Correlation Matrix

According to (1), the correlation matrix with $L+K$ sources is as follows

$$R_{\mathrm{xx}} = E\left\{X(w_0) X^H(w_0)\right\} \tag{3}$$

where the superscript $(\cdot)^H$ denotes Hermitian operation, and $E\{\cdot\}$ denotes the statistical expectation operator. In practice, (3) is replaced by sample averaging.

Here, the cross-correlation coefficient is defined as

$$\rho_{mn} = E\{s_m(w_0) s_n^*(w_0)\}, L+1 \le m, n \le L+K, m \ne n \tag{4}$$

Based on (2) and (4), the units with correlative interval p in correlation matrix can be expressed as

$$r(p) = \sum_{l=1}^{L+K} \sigma_l^2 e^{jk_0 p d_0 \sin(\theta_l)} + \sum_{m=L+1}^{L+K} \sum_{n=L+1}^{L+K} \rho_{mn} e^{jk_0(x_i \sin(\theta_m) - x_{i'} \sin(\theta_n))} + \sigma_n^2 \delta(p) \tag{5}$$

where pd_0 is the position difference of two physical array sensors located at x_i and $x_{i'}$. This indicates that the phase of desired first terms in correlation units for DOA estimation are uniquely determined by the positions of difference co-array. The second cross-terms between two coherent sources in correlation units are regarded as interferences which will be removed by the subsequent decorrelation operation.

3.2 The Model of Difference Co-array

The difference co-array of the co-prime array in Fig. 1 is shown in Fig. 2. The element positions of the difference co-array form the set as

$$S(w_0) = \{\pm (Mnd_0 - Nmd_0)\}, 0 \le n \le N-1, 0 \le m \le 2M-1 \tag{6}$$

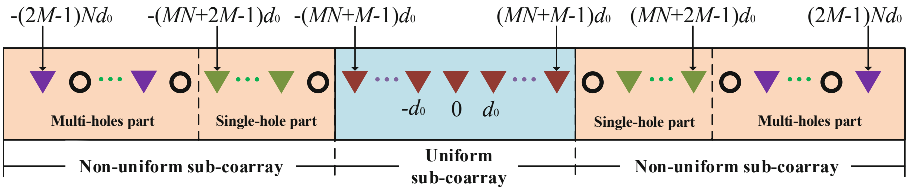

Fig. 2. The model of difference co-array.

As shown in Fig. 2, we can see that the difference co-array extends from $-N(2M-1)d_0$ to $N(2M-1)d_0$ and has a contiguous set of elements between $-(MN+M-1)d_0$ and $(MN+M-1)d_0$ with the inter-element spacing of d_0. The other positions of difference co-array are discontinuous and have holes.

3.3 Correlation Matrix Augmentation

As for the continuous elements from $-(MN+M-1)d_0$ to $(MN+M-1)d_0$ in difference co-array, the correlation units with corresponding correlative interval $-(MN+M-1)$ to $(MN+M-1)$ are extracted from correlation matrix to form a vector as

$$[r(-MN-M+1), r(-MN-M+2), \cdots, r(MN+M-1)] \tag{7}$$

Then these correlation units in (7) are rearranged into an $(MN+M) \times (MN+M)$ augmented correlation matrix as

$$R_{xx} = \begin{pmatrix} r(0) & r(1) & \cdots & r(MN+M-1) \\ r(-1) & r(0) & \cdots & r(MN+M-2) \\ \vdots & \vdots & \ddots & \vdots \\ r(-MN-M+1) & r(-MN-M+2) & \cdots & r(0) \end{pmatrix} \tag{8}$$

Obviously, the augmented correlation matrix is equivalent to that of a uniform array with $MN+M$ physical elements and inter-element spacing of d_0.

3.4 Decorrelation of Augmented Correlation Matrix

In the case of existing coherent sources, the augmented correlation matrix R_{xx} suffers from the phenomenon of rank defect, which attributes to the failure of DOA estimation with eigenvalue decomposition, such as MUSIC algorithm. The key to solve the rank defect is to perform decorrelation about the augmented correlation matrix. The general decorrelation algorithms include spatial smoothing algorithm and reconstructing matrix algorithm.

Forward spatial smoothing algorithm is an effective algorithm to decorrelate coherence of sources. The main method of which is to divide the uniform linear physical array of N_0 elements into L_0 interlaced subarrays where the number of elements in each subarray is $m_0 = N_0 - L_0 + 1$. Therefore, the decorrelation can be realized by averaging the correlation matrices of all subarrays, and then a $m_0 \times m_0$ smoothed correlation matrix can be expressed as

$$R_f = \frac{1}{L_0} \sum_{k=1}^{L_0} F_k R_{\mathrm{xx}} {F_k}^T \tag{9}$$

where $F_k = \left[0_{m_0 \times (k-1)} \,|I_{m_0}|\, 0_{m_0 \times (N_0 - k - m_0 + 1)}\right]$, and I_{m_0} is a $m_0 \times m_0$ identity matrix. When the number m_0 of subarray elements in subarray is larger than the number M_0 of coherent sources, through smoothing, the rank of correlation matrix is recovered as M_0. For N_0 physical sensors, forward spatial smoothing algorithm can achieve the decorrelation of $N_0/2$ coherent sources. The principle of backward spatial smoothing algorithm is similar to that of forward spatial smoothing. The distinction is that the dividing of physical array starting from the last element.

At the basis of rotation invariance of ULA, the forward and backward spatial smoothing algorithm deal with the correlation matrix by forward smoothing and backward smoothing at the same time. The smoothed correlation matrix is formed as

$$R_{fb} = \frac{1}{2L_0} \sum_{k=1}^{L_0} F_k \left(R_{\mathrm{xx}} + J R_{\mathrm{xx}}^* J\right) F_k^T \tag{10}$$

where J is permutation matrix with dimension N_0, whose value of units in back-diagonal is 1 and others are 0. For N_0 physical elements, forward and backward spatial smoothing algorithm can achieve a decorrelation of $3N_0/2$ coherent sources.

The decorrelating coherence of spatial smoothing algorithm is on the premise of sacrificing the degrees-of-freedom of DOA estimation, namely the reduction of resolvable sources. The reconstructing matrix algorithm is a better choice to guarantee the degrees-of-freedom and decorrelation at the same time.

Based on the reconstructing matrix algorithm, the correlation units in (7) can also be rearranged into another $(MN + M) \times (MN + M)$ augmented correlation matrix as

$$R_{Toeplitz} = \begin{pmatrix} r(0) & r(1) & \cdots & r(MN + M - 1) \\ 0 & r(0) & \cdots & r(MN + M - 2) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r(0) \end{pmatrix} \tag{11}$$

The reconstructed correlation matrix $R_{Toeplitz}$ is an upper triangular matrix which is a full rank matrix regardless of the relativity among sources. Then the matrix $R_{Toeplitz}$ is processed as

$$R'_{Toeplitz} = J_M R^*_{Toeplitz} J_M \tag{12}$$

where J_M is a $(MN + M) \times (MN + M)$ permutation matrix just as J in (10). Finally, the desired correlation matrix is generated by $R'_{xx} = (R_{Toeplitz} + R'_{Toeplitz})/2$, which can be used to realize DOA estimation of at most $MN+M-1$ coherent sources more than physical elements of co-prime array.

3.5 Utilizing the Holes

The DOA estimation algorithm above based on difference co-array of co-prime array utilizes only the contiguous co-array elements, abandoning all the discontinuous ones, namely the degrees-of-freedom is $MN + M - 1$. This unexploited discontinuous co-array elements mean the loss of degrees-of-freedom. In order to make the best of all degrees-of-freedom from the difference co-array, the correlation units corresponding to these holes in co-array can be regarded as with zero. In this case, the dimension of reconstructed correlation matrix R'_{xx} can be extended from $MN + M$ to $(2M - 1)N + 1$. Therefore, the degrees-of-freedom of DOA estimation is raised to $(2M - 1)N$. In other words, at most $(2M - 1)N$ sources can be resolved.

4 Simulation and Performance Analysis

4.1 Simulation of DOA Estimation

Based on Fig. 1, a co-prime array with $M = 2$ and $N = 3$, including $2M+N-1 = 6$ physical sensors is applied in the simulation. The inter-element spacing of d_0 is set as 1. The element positions of two sub-array is $\{0, 2, 4\}$ and $\{3, 6, 9\}$, respectively. Furthermore, based on (6), the contiguous elements in difference co-array is from $-(MN+M-1) = -7$ to $MN+M-1 = 7$. Through reconstructing matrix algorithm, a reconstructed correlation matrix of 8×8 is obtained. Based on MUSIC algorithm, 7 coherent sources can be estimated at most. A mix of 3 coherent and 4 incoherent sources are simulated with a total of 2000 snapshots and SNR of 0 dB. The result of DOA estimation is shown in Fig. 3. The vertical dashed and dotted lines in the figure indicate the true DOAs of the sources of coherent and incoherent sources. As expected, the DOAs of all 7 sources are accurately estimated.

For further study, all elements of difference co-array including discontinuous elements from $-(2M-1)N = -9$ to $(2M-1)N = 9$ are utilized by filling the correlation units corresponding to the hole in $-(MN+M) = -8$ and $MN+M = 8$ with 0. Under the condition of 2000 snapshots and 0 dB SNR, Fig. 4 provides the DOA estimation result of 4 coherent and 5 incoherent sources. Obviously, all 9 sources are resolved accurately, which demonstrates the improvement of

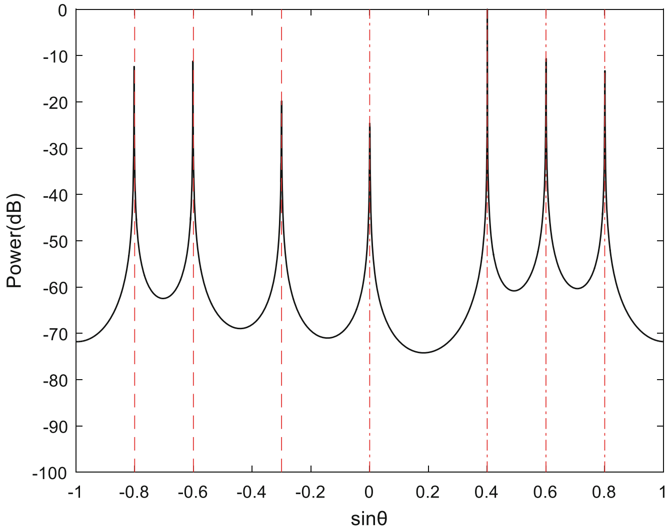

Fig. 3. DOA estimation of 3 coherent sources and 4 incoherent sources.

resolvable sources by using these discontinuous co-array elements. Considering the worst condition that all the sources are coherent, the following simulations are performed to compare the performance of forward spatial smoothing algorithm, forward and backward spatial smoothing algorithm and reconstructing matrix algorithm in maximum number of DOA estimation. More specifically, three groups of simulations are carried out under the conditions of 3, 4 and 5 coherent sources, respectively. Besides, only the contiguous co-array elements are considered in the following simulations. The simulation results are given in Figs. 5, 6 and 7, where the vertical lines indicate the true DOAs.

It can be seen from Figs. 5 and 6 that reconstructing matrix and spatial smoothing algorithm can both estimate the sources correctly. As shown in Fig. 7, when the number of coherent sources increases to 5, forward spatial smoothing algorithm has lost efficacy. Furthermore, in the case of 7 coherent sources, only the reconstructing matrix algorithm still stays correct estimation, which means the superiority in resolvable ability of reconstructing matrix algorithm (Fig. 8).

4.2 Performance Analysis

In this section, the error performance comparison of three decorrelation algorithms is provided by 500 Monte Carlo trials on the basis of 4 coherent sources. The root-mean-square error (RMSE) of DOA estimation for 4 coherent sources is calculated as

$$RMSE = \frac{1}{4}\sum_{k=1}^{4}\sqrt{\frac{1}{500}\sum_{j=1}^{500}\left(\hat{\theta}_{jk}-\theta_k\right)^2} \tag{13}$$

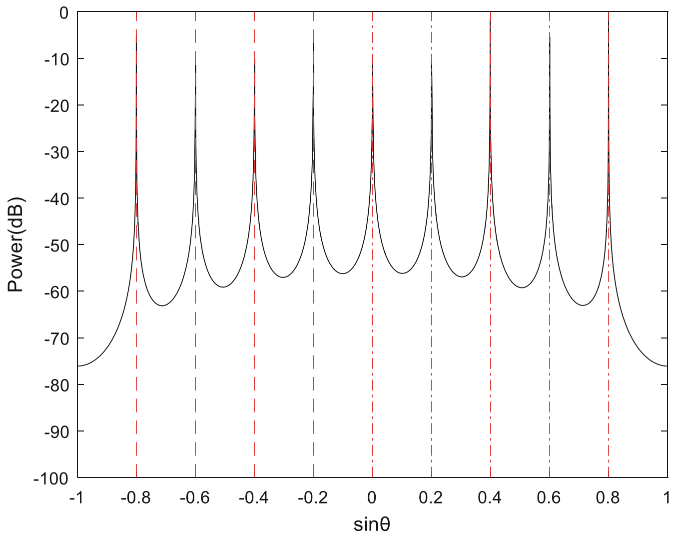

Fig. 4. DOA estimation of 4 coherent sources and 5 incoherent sources.

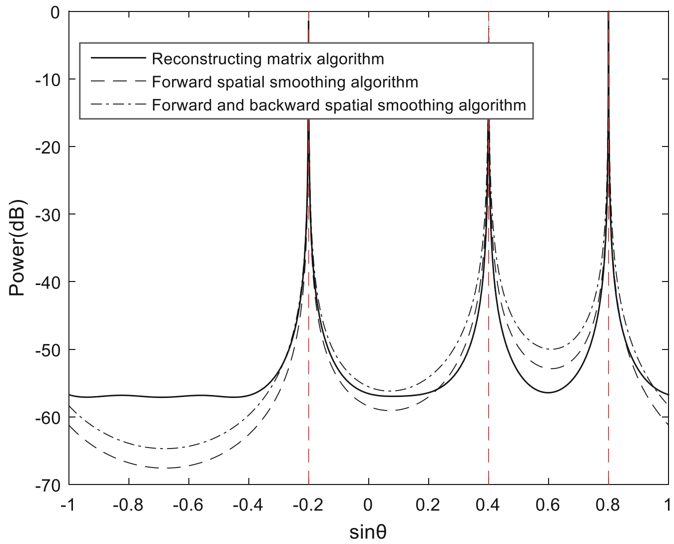

Fig. 5. DOA estimation of three algorithms under 3 coherent sources.

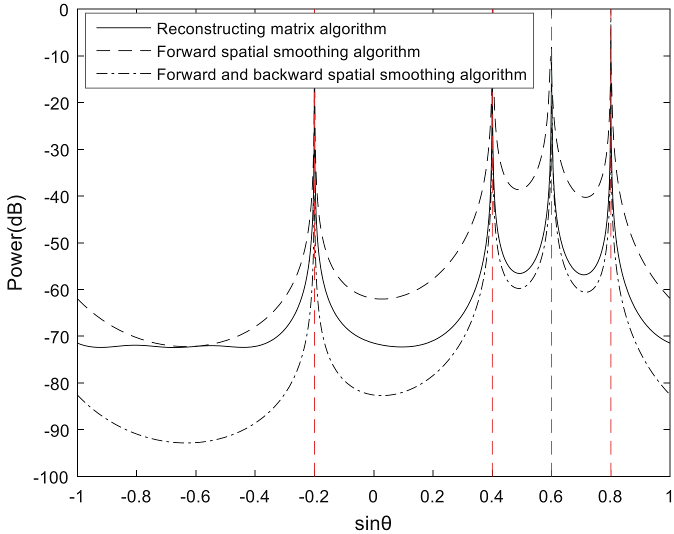

Fig. 6. DOA estimation of three algorithms under 4 coherent sources.

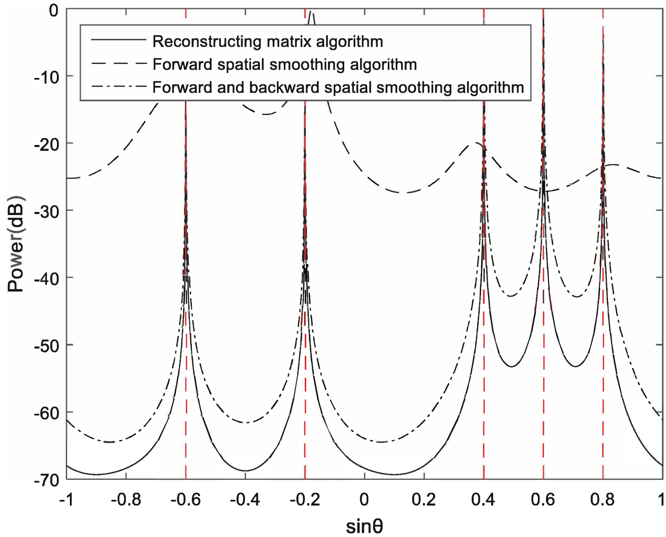

Fig. 7. DOA estimation of three algorithms under 5 coherent sources.

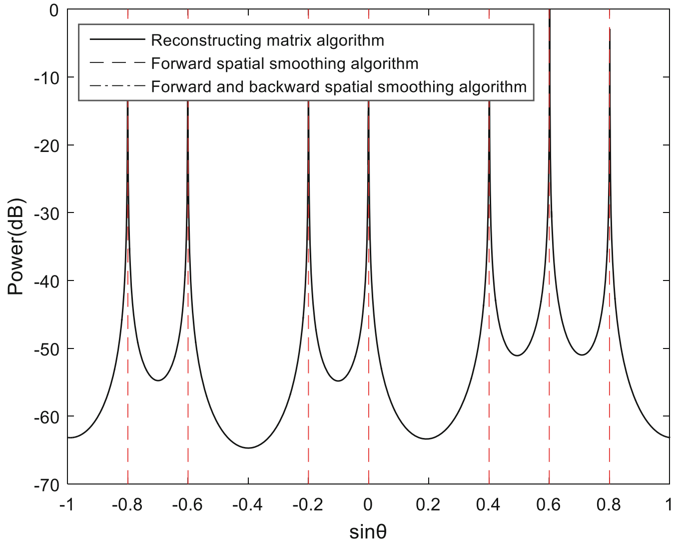

Fig. 8. DOA estimation of three algorithms under 7 coherent sources.

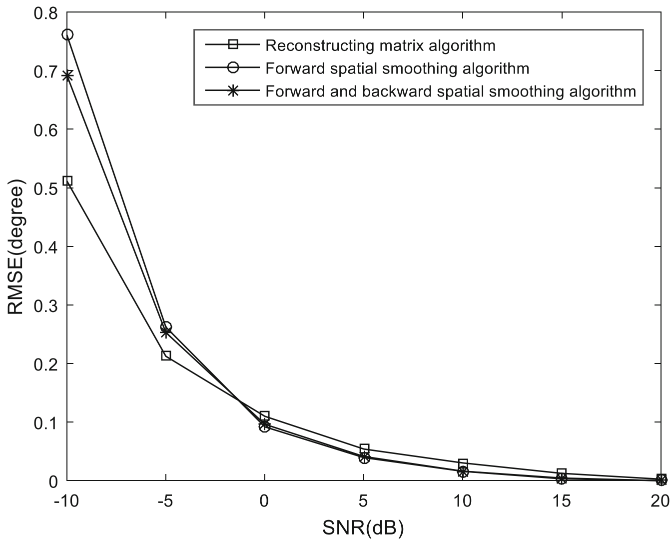

Fig. 9. Performance of three algorithms under different SNRs.

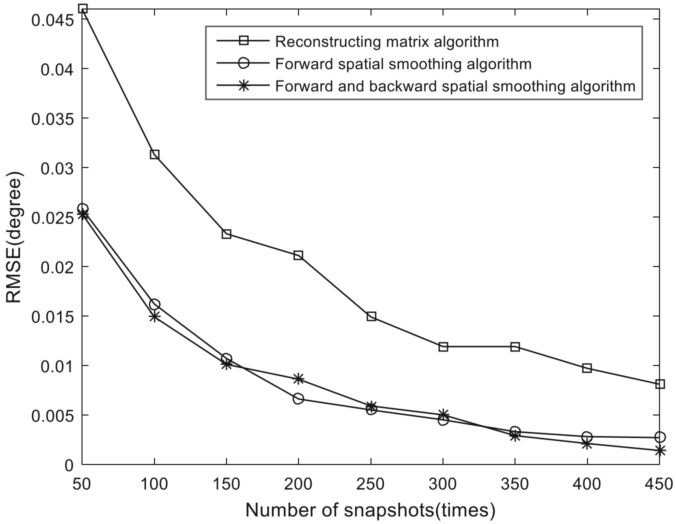

Fig. 10. Performance of three algorithms under different numbers of snapshots.

where θ_k is the true DOA and $\hat{\theta}_j k$ is the estimated DOA.

Figure 9 indicates that the RMSE of reconstructing matrix algorithm is minimum under the same SNR, and the advantage is clear under low SNR. From Fig. 10, we can make a conclusion that the RMSE of reconstructing matrix algorithm is maximum in the three algorithms, and two spatial smoothing algorithms perform similarly in RMSE.

5 Conclusion

A DOA estimation algorithm for coherent and incoherent sources is been proposed based on co-prime array in this paper. Through three decorrelation algorithms such as forward spatial smoothing, forward and backward spatial smoothing and reconstructing matrix, the augmented correlation matrix is decorrelated to serve for accurate MUSIC spectrum searching. The simulation results indicate that the reconstructing matrix algorithm can estimate more sources and have a lower error under same SNR, while the spatial smoothing algorithm has a better estimation error performance in the case of low snapshot.

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