



# The Low Complexity Multi-user Detection Algorithms for Uplink SCMA System

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**Abstract.** 5G research gradually focus on non-orthogonal multiple access technology, owing to huge traffic, more mobile terminals, and explosive growth of throughput capacity in recent years. Sparse code multiple access (SCMA) is a multi-dimensional codebook-based non-orthogonal multiplexing technique proposed to address the above requirements. This paper investigates the Message Passing Algorithm (MPA) in the receiver of SCMA and proposes two multi-user detection algorithms with low computational complexity in uplink. Improved Variable Message Passing Algorithm (IVMPA) reduces computational complexity compared to Variable Message Passing Algorithm (VMPA) by changing the users iteration order. Incomplete Iterative Message Passing Algorithm (IIMPA) is proposed to reduce the number of iteration for users with high signal-to-noise ratio (SNR) and reduces computational complexity compared to MPA.

**Keywords:** SCMA · Multi-user detection · Message Passing Algorithm · Complexity

## 1 Introduction

SCMA is a sparse code multiple access technology [1], which is similar to the Low Density Signature (LDS) system. LDS as a special case in Code Division Multiple Access (CDMA), its codewords are obtained through spreading Quadrature Amplitude Modulation (QAM) symbols with the low density characteristic sequences [2]. Compared with LDS, SCMA can also improve the spectrum efficiency under the premise of obtaining the low density spread spectrum gain and the moderate complexity of demodulation.

In SCMA uplink, the bit stream of each user is first mapped to N-dimensional complex constellation, and then the constellation is mapped into a K-dimensional sparse codeword. The codeword contains N non-zero elements and  $N < K$ . Each

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SCMA layer or user holds its own codebook and the codebook includes more than one codeword [3]. In order to improve the spectrum efficiency, multiple users are multiplexed on the same resource block [4, 5]. The size of constellation and the length of codeword of users who are overloaded on the same resource is uniform.

Although SCMA can improve the spectrum efficiency remarkably, the multi-user detection algorithm needs to be considered because of multiple users multiplexing on the same resource. SCMA adopting Message Passing Algorithm (MPA) algorithm for multi-user detection. However the detection complexity of MPA is high and the high complexity will directly affect system performance such as time delay and design complicity of receiver. In order to reduce the complexity of detection algorithm, reference [6] put forward a detection scheme that combines codebook and receiver. The scheme minimizes the complexity of SCMA uplink network detection based on codebook design and implementation of codebook distribution scheme. The technology firstly uses a two-party coordinative search to design a cost-effective algorithm, which determines the mapping matrix and the codebook allocation scheme. Based on the designed mapping matrix, this scheme utilizes the characteristics of SCMA multidimensional modulation to design the parent constellation and users' codebook to reduce the detection complexity.

A low-complexity multi-user detection algorithm based on List-Spherical Decoding (LSD) was proposed in [7]. The LSD algorithm only considers the codewords within a sphere and avoids a detailed search of all possible codebook groups. The LSD algorithm first sets up the depth-first search tree algorithm and proposes several methods for updating the access nodes to reduce the size of the search tree. The LSD is used to set the retrieved users and codeword range before MPA detection for avoiding the ergodic search detection in the receiver. The simulation results show that the scheme is nearly close to ML detection performance and greatly reduces the decoding complexity. Reference [8] proposed the Log-Message Passing Algorithm (Log-MPA) and compared Log-MPA with traditional MPA in details. The effect of message passing iterations times in Log-MPA decoding on system performance was also investigated. The simulation results show that the Log-MPA algorithm has better performance and lower hardware implementation complexity than MPA. Simulation and board-level verification results show that the performance of Log-MPA is close to MPA. Therefore the Log-MPA decoder can achieve a balance between performance and complexity.

This paper investigates the Message Passing Algorithm (MPA) in the receiver and proposes two multi-user detection algorithms with low computational complexity in uplink. The proposed Improved Variable Message Passing Algorithm (IVMPA) changes the users iteration order based on Variable Message Passing Algorithm (VMPA) and thus reducing computational complexity in the receiver. The another proposed multi-user detection algorithm is Incomplete Iterative Message Passing Algorithm (IIMPA) that reduces the number of iterations for

some users with high signal-to-noise ratio (SNR) and thus reducing computational complexity compared to MPA.

The rest of this paper is organized as follows. Section 2 is devoted to introduce the system model. In Sect. 3, the multi-user detection algorithms in the receiver are provided. The simulation results and complexity analysis are given in Sect. 4. The final conclusions are in Sect. 5.

## 2 Previous Work

In uplink, each user holds an own codebook. Input data of multiple users is mapped to multi-dimensional codewords based the codebooks of users, and then overload and send the codewords on the  $K$  OFDMA subcarriers. After receiving the codewords overlapping on the subcarriers, the receiver can demodulate the data of each user utilizing the sparsity of the SCMA codewords through multi-user detection scheme with a certain complexity.

### 2.1 Basic Model of SCMA Uplink

Assume that the codewords of  $J$  users are transmitted with  $K$  resources and reach the same receiver at a time slot, the received signal can be described as

$$y = \sum_{u=1}^J \sqrt{p_u} \text{diag}(h_u) x_u + n \quad (1)$$

where  $x_u$  is a column vector of  $K$  dimension representing the codeword of user  $u$ , and the received signal  $y$  is also a column vector of  $K$  dimension.  $p_u$  and  $h_u$  represent the received power and the channel vectors on  $K$  OFDMA resources of user  $u$ , respectively.  $\text{diag}(h_u)$  represents diagonal matrix and  $n$  is Additive White Gaussian Noise (AWGN).

Figure 1 shows the process that six users transmit data on four resource blocks to the base station for receiving and demodulating. The detailed communication process is that each 2bit binary data of each user is mapped to a high dimensional complex codeword which contains two non-zero elements, and the codewords of six users are reused on the four resource blocks by sparse codewords. The receiver adopts MPA multi-user detection algorithm to detect and demodulate received data. The idea of MPA algorithm is to update the user's message probability according to the initial codewords information, channel state and noise power, and then output codewords information when the iteration reaches a certain number of times. In uplink, the time slot synchronization needs to be guaranteed, and the slot synchronization part is similar to the LTE system.

The  $J$  codewords that are mapped from the  $J$  codebooks are overloaded and overlapped on  $K$  resources, and the overload rate is defined as

$$\lambda = \frac{J}{K} \quad (2)$$

Changing the number of codebooks  $J$  and the length of codewords  $K$  can achieve different overload rates. Normally, the value of  $\lambda$  will be greater than 1.

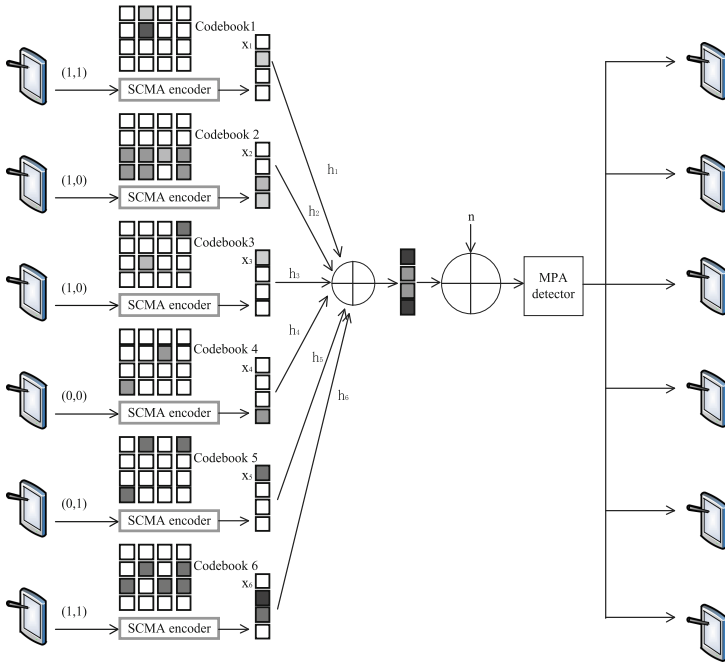


Fig. 1. The SCMA uplink model.

### 2.2 Design of SCMA Codebook

The codebook design of SCMA includes two parts, the design of factor map matrix and the generation of multidimensional constellation. The factor graph matrix represents the occupancy of each user on the resource blocks, whose sparsity determines the sparsity of SCMA codewords and also shows the users overload on the resources. The multidimensional constellation determines the mapping relationship between users data and the complex constellation. The distribution of points in the constellation determines the relationship between non-zero codewords of different users and the performance of demodulation.

## 3 The Multi-user Detection Algorithms

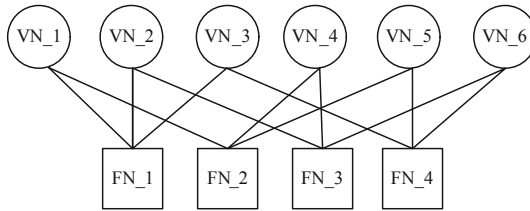
This section introduces several multi-user detection algorithms, including the existing detection algorithms and the detection algorithms proposed in this paper.

### 3.1 The Original Algorithm

In non-orthogonal transmission, the received signal at the receiver includes multiple users' symbols, which makes it difficult to separate and demodulate users'

codewords overlapped on the same resources at the receiver. In SCMA uplink, the receiver adapts Message Passing Algorithm (MPA) as the multi-user detection scheme.

The MPA algorithm performs users detection and data demodulation through factor graph matrix based on the received signal, channel state estimation, and noise power. The algorithm iterates from the initial conditional probabilities of the function node (FN) to the variable node (VN), where the function node represents the resource node and the variable node represents the user node. The received signal from the receiver, the channel estimation for each user and the noise estimation on each FN are taken as input, and then related message values of FN and VN are iterated and updated in order. For each iteration, the update of message values from FN to VN and from VN to FN is independent. After enough iterations, posterior probabilities for users' codewords are obtained. An example with 6 variable nodes and 4 function nodes is shown in Fig. 2.



**Fig. 2.** Factor graph representation of an SCMA system with VN = 6, FN = 4.

Each iteration of detector has two steps. The first step is that message values from FNs to VNs are calculated based on the received signal, channel state and noise power. All FNs to VNs message values are updated in parallel. The function node  $k$  to variable node  $j$  message values can be written as

$$M_{c_k \rightarrow u_j}^t(x_j) = \sum_{x_j} \left\{ \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{1}{2\sigma^2} \left\| y_k - \sum_{v \in \xi_k} h_{k,v} x_{k,v} \right\|^2 \right) \prod_{l \in \xi_k / \{j\}} M_{u_k \rightarrow c_l}^{t-1}(x_j) \right\} \quad (3)$$

where  $t$  is the current number of iterative,  $\xi_k$  represents the index of 1 in the  $k$ -th row of the factor graph matrix. When all FNs to VNs message values have updated, this iteration will take the second step.

The second step is the update of message values from FNs to VNs. Similarly, the VN to FN message values depend on the other connected function nodes. Utilizing the output of the first step, the variable node  $j$  to the function node  $k$

message values can be written as

$$M_{u_j \rightarrow c_k}^t(x_j) = \prod_{m \in \varsigma_j / \{k\}} M_{c_m \rightarrow u_j}^t(x_j) \tag{4}$$

where  $\varsigma_j$  is the index of 1 in the  $j$ -th column of the factor graph matrix. When the number of iterations reaches the pre-set maximum value, the output of MPA detector which is the estimation of users' codewords can be expressed as

$$Q(x_j) = \prod_{k \in \varsigma_j} M_{c_k \rightarrow u_j}^{t_{\max}}(x_j) \tag{5}$$

Through above methods, the data of multi-user can be demodulated and separated.

### 3.2 The Variable MPA Algorithm

In above algorithm, the iterative model is parallel where VNs to FNs message values can be updated until all FNs to VNs message values complete the update. This mode wastes some useful message values to a certain extent. The change in the internal iteration update structure can speed up message values convergence. After each user's FN to VN message values updating the relevant VNs to FNs message values will update. This mode ensures the real-time performance of the iteration and update, and avoids the waste of the message values. The formula can be changed as follows

$$M_{c_k \rightarrow u_j}^t(x_j) = \sum_{x_j} \left\{ \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{1}{2\sigma^2} \left\| y_k - \sum_{v \in \xi_k} h_{k,v} x_{k,v} \right\|^2 \right) \right. \tag{6}$$

$$\left. \times \prod_{\substack{if(l > j), \\ l \in \xi_k / \{j\}}} M_{u_k \rightarrow c_l}^{t-1}(x_j) \prod_{\substack{if(l < j), \\ l \in \xi_k / \{j\}}} M_{u_k \rightarrow c_l}^t(x_j) \right\}$$

$$M_{u_j \rightarrow c_k}^t(x_j) = \prod_{\substack{if(m < j), \\ m \in \varsigma_j / \{k\}}} M_{c_m \rightarrow u_j}^t(x_j) \prod_{\substack{if(m > j), \\ m \in \varsigma_j / \{k\}}} M_{c_m \rightarrow u_j}^{t-1}(x_j) \tag{7}$$

### 3.3 The Improved Variable MPA Algorithm

In uplink, the path from each user path to the base station is different and therefore users' data experience different fading conditions, which may be large-scale fading or small-scale fading. Although the transmission power of all users sharing the same resource blocks is equal, users experiencing different fading

conditions results in different receiving signal-to-noise ratios (SNR). Therefore, situations where some users' signal power is in the advantage or disadvantage of receiving signal will appear. VNs and FNs message values of users with high received signal power have obviously higher reliability, and message values of users with low signal power is less reliable. In this case, if users with high SNR can preferentially enter the iteration loop, the performance of the multi-user detection can be improved.

Before iteration, the users are sorted according to the received SNR from largest to smallest. The calculation formula can be described as

$$SNR_i = \frac{p_i |h_i|^2}{\sigma^2} \tag{8}$$

where,  $H = (h_1, \dots, h_J)$  is the channel gain matrix, and  $h_j (j = 1, \dots, J)$  represents a column vector with size  $K$  ( $K$  OFDMA subcarriers).

According to the SNR, IVMPA scheme gets the users' iteration order. Based the sorted users order, the algorithm updates the message values of FNs and VNs in order to take full advantage of more reliable information. The new formula can be described as

$$M_{c_k \rightarrow u_j}^t(x_j) = \sum_{x_j} \left\{ \frac{1}{\sqrt{2\pi\sigma}} \exp \left( -\frac{1}{2\sigma^2} \left\| y_k - \sum_{v \in \xi_k} h_{k,v} x_{k,v} \right\|^2 \right) \times \prod_{\substack{k=order(a) \\ l=order(b) \\ a < b}} M_{u_k \rightarrow c_l}^{t-1}(x_j) \prod_{\substack{k=order(a) \\ l=order(b) \\ a > b}} M_{u_k \rightarrow c_l}^t(x_j) \right\} \tag{9}$$

$$M_{u_j \rightarrow c_k}^t(x_j) = \prod_{\substack{j=order(a) \\ m=order(b) \\ a > b}} M_{c_m \rightarrow u_j}^t(x_j) \prod_{\substack{j=order(a) \\ m=order(b) \\ a < b}} M_{c_m \rightarrow u_j}^{t-1}(x_j) \tag{10}$$

According to the preset user update sequence, FN to VNs and VN to FN message values of each user are updated in turn. For each FN-VN pair currently iteration, if  $M_{c_k \rightarrow u_j}(x_j)$  is updated, the same FN-VN pair soft information  $M_{u_j \rightarrow c_k}(x_j)$  is immediately updated so that each updated soft message value can enter the next iteration update in time, thereby accelerating the convergence speed. After a user's all soft message values are updated, update the next user. When all users are updated, the detector will start next iteration same as before. The specific process is shown in Algorithm 1.

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**Algorithm 1.** The IVMPA Algorithm

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**Require:**  $F, \sigma^2, t_{\max}, t_0, H$ ;  
 1: Initialization:  $M_{c_k \rightarrow u_j}^0(x_j) = \frac{1}{M}$ ;  
 2: **if**  $t \leq t_0$  **then**  
 3:     **for** all  $u_j (j = \text{order}(i), i = 1, \dots, J)$  **do**  
 4:         conduct equation (9);  
 5:         conduct equation (10);  
 6:     **end for**  
 7: **end if**  
 8: **if**  $t = t_{\max}$  **then**  
 9:      $Q(x_j) = \prod_{k \in \zeta_j} M_{c_k \rightarrow u_j}^{t_{\max}}(x_j)$ ;  
 10:      $X_j = \max Q(x_j) = \max \prod_{k \in \zeta_j} M_{c_k \rightarrow u_j}^{t_{\max}}(x_j)$ ;  
 11: **end if**

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**3.4 The Incomplete Iterative MPA Algorithm**

In uplink, each user experiences different fading conditions, and the received SNR at the receiver is not equal. The users with high received SNR has relatively higher reliability and the iterative convergence of soft information is faster than other users. Based on the above basis, an Incomplete Iterative MPA Algorithm (IIMPA) is proposed to enable users with high received SNR to stop the iterations early, and the remaining users continue to iterate until obtaining the codewords information of all users. Because some users stop the iterations in advance, the incomplete iterative algorithm can reduce the demodulation complexity and delay to some extent.

Based on the received SNR, a set of users  $\Psi_1$  for partial iterations and a set of all iterative users  $\Psi_2$  are defined. And the number of iterations for the partial iterative users is set in advance, assuming that the size of  $\Psi_1$  is  $num$ . At the initial iteration, the algorithm iteratively updates the soft information values of FNs to VNs and VNs to FNs for all users. The process can be described as shown in (3) and (4).

When the initial number of iterations  $iter_1$  is reached, the users of  $\Psi_1$  soft information are no longer updated, and the users of  $\Psi_2$  continue to perform the remaining number of iterations until the maximum number of iterations  $t_{\max}$  is reached. The specific process is shown in Algorithm 2. The soft information of FNs to VNs and VNs to FNs can be written as

$$M_{c_k \rightarrow u_j}^t(x_j) = \sum_{x_j} \left\{ \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{1}{2\sigma^2} \left\| y_k - \sum_{v \in \xi_k} h_{k,v} x_{k,v} \right\|^2 \right) \cdot \prod_{l \in \Psi_2 / \{j\}} M_{u_k \rightarrow c_l}^{t-1}(x_j) \prod_{l \in \Psi_1} M_{u_k \rightarrow c_l}^{t_0}(x_j) \right\} \quad (11)$$

**Algorithm 2.** The IIMPA Algorithm

---

**Require:**  $F, \sigma^2, t_{\max}, iter_1, H$ ;  
 Initialization:  $M_{c_k \rightarrow u_j}^0(x_j) = \frac{1}{M}$ ;  
 2: **if**  $t \leq iter_1$  **then**  
     conduct equation (3);  
 4:     conduct equation (4);  
   **end if**  
 6: **for** all  $u_j$  ( $j = order(i), i = 1, \dots, J$ ) **do**  
     conduct equation (9);  
 8:     conduct equation (10);  
   **end for**  
 10: **if**  $t = iter_1$  **then**  
     **for**  $u_j \in \Psi_1$  **do**  
 12:          $Q(x_j) = \prod_{k \in \mathcal{S}_j} M_{c_k \rightarrow u_j}^{t_{iter_1}}(x_j)$ ;  
         $X_j = \max Q(x_j)$ ;  
 14:      **end for**  
   **end if**  
 16: **if**  $t > iter_1$  **then**  
     **for**  $u_j \in \Psi_2$  **do**  
 18:         conduct equation (11);  
        conduct equation (12);  
 20:      **end for**  
   **end if**  
 22: **for**  $u_j \in \Psi_2$  **do**  
      $X_j = \max \prod_{k \in \mathcal{S}_j} M_{c_k \rightarrow u_j}^{t_{\max}}(x_j)$ ;  
 24: **end for**

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$$M_{u_j \rightarrow c_l}^t(x_j) = \prod_{m \in \Psi_2 / \{k\}} M_{c_m \rightarrow u_j}^t(x_j) \prod_{m \in \Psi_1} M_{c_m \rightarrow u_j}^{t_0}(x_j) \quad (12)$$

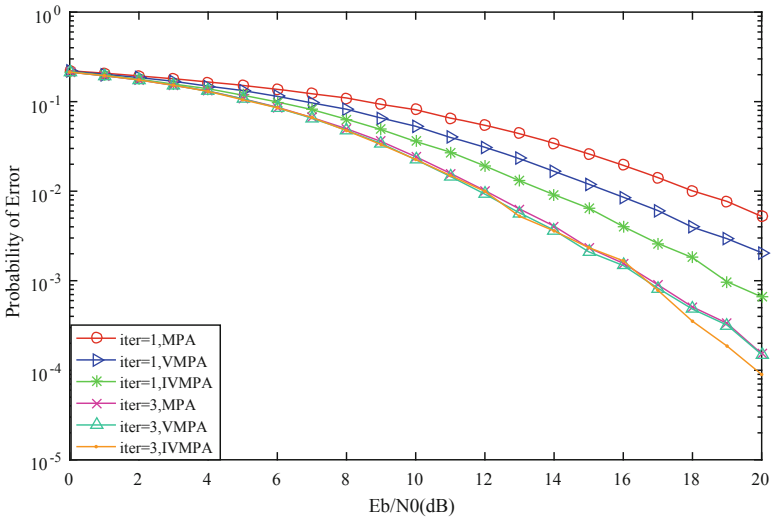
## 4 Numerical Results and Analysis

In this section, we will first simulate the bit error rate (BER) performance of the multi-user detection algorithms in SCMA uplink, and then compare the algorithm complexity and demodulation delay. The codebooks of SCMA layers are designed based on the principles reported in [9]. The dimension of SCMA codewords is 4 with 2 non-zero elements. The number of SCMA layers is 6 shared with 4 OFDMA subcarriers. The transmitted signals from SCMA layers have equal powers. All simulations are performed in Rayleigh channel.

### 4.1 BER Performances of Detection Algorithms

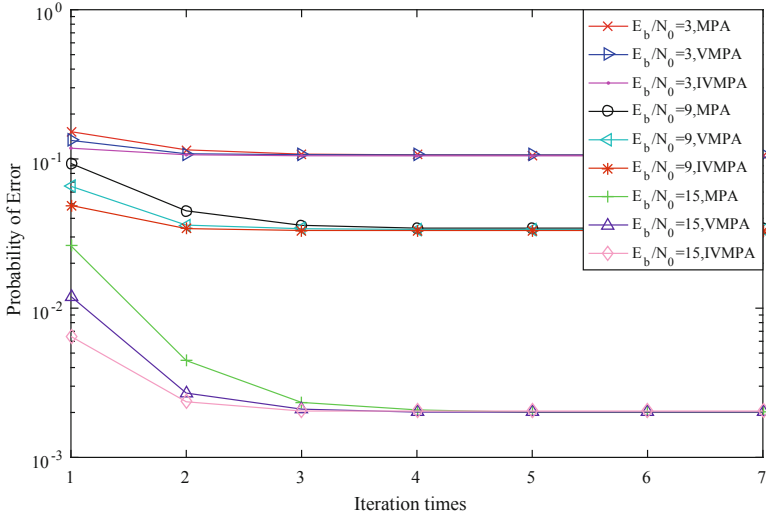
Figure 3 shows that the BER performance comparison between MPA, VMPA, and IVMPA with 1 iteration and 3 iterations in SCMA uplink with Rayleigh

channel. The horizontal axis of Fig. 5 is the per bit energy to noise energy ratio, which is equal to each layer. In Rayleigh channel, when the receiver iteration times is 1 and the ratio less than 4, three algorithms seem to have near performance. That is because the influence of the noise power on the demodulation performance is greater than the influence of the order change of the internal iteration. When the ratio more than 6, IVMPA has a significant performance gain over VMPA and MPA algorithms. When the ratio is greater than 8 dB, the IVMPA algorithm obtains about 2 dB performance gain compared to the VMPA algorithm, and the IVMPA algorithm obtains about 5 dB performance gain compared to the MPA algorithm. The reason is that the order of iteration becomes the main factors affecting system performance compared to the smaller noise power. When the number of iterations is 3, the difference in BER performance between three algorithms is very small. The conclusion is that the IVMPA algorithm obtains obvious performance gains when fewer iterations.

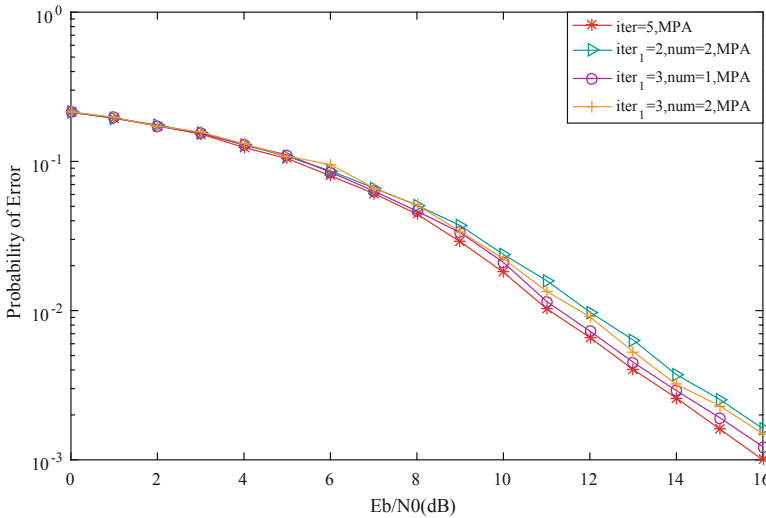


**Fig. 3.** Performance comparison between MPA, VMPPA, and IVMPA with 1 iteration and 3 iterations in uplink with Rayleigh channel.

Figure 4 shows the comparison of convergence speed of MPA, VMPPA, and IVMPA algorithms. With the increase of bit-to-noise ratio, the difference between three algorithms is more obvious. That is because the noise power at low SNR greatly affects the performance, and the influence of the iteration times on performance is submerged. When the number of iterations is 2, the BER of IVMPA is almost equal that the number of iterations is 4 of VMPPA and MPA algorithm. When the number of iterations are 5, the BER of three algorithms is basically similar. The conclusion is that the IVMPA algorithm can reduce the number of iterations at the expense of very little performance. In other words, the IVMPA algorithm can achieve convergence faster.



**Fig. 4.** Performance among MPA, VMPA, IVMPA over different ranges of detection iterations.



**Fig. 5.** Performance between IIMPA and MPA algorithm.

Figure 5 shows that the performance comparison between IIMPA and MPA algorithms. From Fig. 7, it can be seen that BER performance of IIMPA algorithm gradually decreases along with the size of users set  $iter_1$  increases under the same conditions. When the number of partially iteration users does not change, the link performance decreases along with the value decreases. The rea-

sons for this phenomenon are given as follows. Firstly, the soft information of users who have stopped iteration early has not yet converged. Obviously, the fewer the number of iterations, the worse the reliability of the message value. Secondly, the unreliable soft information will affect the convergence speed of the remaining users.

### 4.2 The Complexity of Detection Algorithms

The computational complexity of multiplication in MPA detection is described as

$$Mul_{MPA} = [(d_f - 1) M^{d_f} d_v J + (d_v - 1) M d_f K] I_T \tag{13}$$

The computational complexity of addition can be obtained from

$$Add_{MPA} = M d_f d_v J I_T \tag{14}$$

$Exp_{MPA} = M^{d_f} J I_T$  is the complexity of index in MPA detector.  $M$  is the size of SCMA codebooks.  $I_T$  is the iterations in detector.  $d_f$  is the number of layers sharing one subcarrier.  $d_v$  is the number of subcarriers used by one layer.  $J$  is the amount of layers. The computational complexity of received SNR is proportional to the number of users, defined as  $Mul_{SNR}$ . When users' transmit power is equal, the SNR of each user is proportional to the channel gain. In this case, the relative size of the received SNR can be replaced by the value of the channel gain value.  $Mul_{SNR} = 6N$  and  $Add_{SNR} = 2N - 1$ . When the SCMA codeword is sparse, the computational complexity is relatively low.

It can be seen from the above formulas that the complexity of the detection algorithms is linear with the number of iterations. And It is obvious that reducing the number of iterations can greatly reduce the computational complexity in the receiver.

The delay analysis among three algorithms are as follows.

The time required for MPA algorithm through the demodulation process can be described as

$$\tau_{MPA} = \left\{ \begin{aligned} &\tau_{Mul} \cdot [(d_f - 1) M^{d_f} d_v J + (d_v - 1) M d_f K] \\ &+ M d_f d_v J \cdot \tau_{Add} + M^{d_f} J \cdot \tau_{Exp} \end{aligned} \right\} \cdot I_T \tag{15}$$

$\tau_{Mul}$  is multiplication delay,  $\tau_{Add}$  is addition delay,  $\tau_{Exp}$  is index delay. It can be seen that the total delay is proportional to detector iteration times  $I_T$ . Then, The IVMPA algorithm compared to MPA algorithm and VMPA algorithm, the receiver processing delay reduced  $\frac{1}{2}\tau_{MPA} - \tau_{order}$  and  $\frac{1}{3}\tau_{MPA} - \tau_{order}$ , respectively.

Based on the above simulation and analysis results, the IVMPA algorithm can reduce the number of detection iterations and computational complexity and processing delay at the expense of a small amount of system performance.

The computational complexity analysis of IIMPA algorithm is as follow.

When  $num = 1$ , the multiplication can be described as

$$\begin{aligned}
 Mul_{num=1} = & \\
 & [(d_f - 1) M^{d_f} d_v J + (d_v - 1) M d_f K] (I_T - iter_1) + \\
 & \left[ \begin{array}{l} ((d_f - 1) M^{d_f} (K - d_v) d_f + (d_f - 2) M^{d_f - 1}) \\ d_v (d_f - 1) + (d_v - 1) M d_v (J - 1) \end{array} \right] iter_1
 \end{aligned} \tag{16}$$

and the addition is

$$\begin{aligned}
 Add_{num=1} = & \\
 & M^{d_f} d_v J (I_T - iter_1) + \\
 & (M^{d_f} (K - d_v) d_f + M^{d_f - 1}) d_v (d_f - 1) iter_1
 \end{aligned} \tag{17}$$

the computational complexity of index can be written as

$$\begin{aligned}
 Exp = & \\
 & M^{d_f} J \cdot iter_1 + M^{d_f - 1} (MJ - M + 1) \cdot (I_T - iter_1)
 \end{aligned} \tag{18}$$

According to above formulas, we can conclude that IIMPA can reduce the computational complexity in some extent. With the increase of partial iteration users, the complexity and delay decrease, while its performance also decreases. It can reach a balance between performance and the complexity of detector.

## 5 Conclusion

Two low complexity algorithms are proposed to reduce the detector computational complexity and demodulation delay. In SCMA uplink, it is complex for the receiver to separate and demodulate users' data with multiplexing technology. Based on the original algorithm, IVMPA adds the user sort strategy before the iteration, which can take full advantage of high reliability users information. This feature provides the IVMPA algorithm to reduce the iteration times and bring about low complexity and delay. IIMPA reduces the computational complexity by terminating some users' iteration in advance. A balance between user performance and computational complexity is achieved by selecting the appropriate number of iterations and the number of partial iterations users.

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