An ESPRIT Parameter Estimation Algorithm Based on Non-circular Signal for MIMO Radar

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Abstract. In this paper, a method of parameter estimation based on Non-Circular Signal via two-dimensional ESPRIT algorithm for MIMO radar is proposed. The algorithm considers the characteristic of the maximum non-circularity signal, and utilizes not only the covariance matrix but also the pseudo-covariance matrix to obtain the data matrix of the non-circular signal. This algorithm expands the received signal data matrix by data recombination, which increases the number of effective array elements of MIMO radar and improves the utilization of echo information. Then the parameter estimates are obtained by using the two-dimensional ESPRIT algorithm. Compared with the ESPRIT algorithm developed in other references, this algorithm has higher accuracy of parameter estimation in the case of lower signal-to-noise ratio or fewer array elements. Meanwhile, the algorithm can improve the accuracy of parameter estimation when the targets are close together. Simulation results indicate that the proposed algorithm improves the performance significantly.

Keywords: Parameter estimation · Two dimensional ESPRIT algorithm · Non-circular signal · MIMO radar

1 Introduction

Multiple-input–multiple-output (MIMO) radar is a new type of system radar, which utilizes multiple antennas to simultaneously transmit orthogonal waveforms and uses multiple antennas to receive the reflected signals. It has been shown that MIMO radars have many potential advantages over conventional phased-array radars [1]. According to the antennas configuration, MIMO radars can be divided into two types. One is called distributed MIMO radar, and the second one is the collocated MIMO radar, which is studied in this paper. In recent years, different methods have been developed for estimating DOD and DOA of multiple targets in collocated MIMO radar. Capon algorithms [2] have been used for angle estimation in bistatic MIMO radar, while multiple signal classification algorithms [3] can also work well for angle estimation.

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However, the above mentioned algorithms require the spectrum searches, which lead to high computational complexity. In [4], a rotational invariance techniques (ESPRIT) algorithm is proposed for angle estimation, and the reduced-dimensional transformation is employed to reduce the complexity. Aiming at the problem that the traditional ESPRIT algorithm has poor performance under the background of Gaussian white noise, [5] improves the traditional ESPRIT algorithm. It divides the planar array into multiple subspace arrays, and applies the ESPRIT algorithm to each antenna which improves the parameter estimation accuracy and resolution. However, the ESPRIT algorithm in [5] is still not perfect in parameter estimation when the distances of several far-field targets are close. In [6], the non-circular signal used in the radar system has potential advantages for improving the performance of all aspects of the radar. Therefore, this paper further studies the MIMO radar angle estimation algorithm based on non-circular signals.

The main objective of this paper is to propose a parameter estimation method based on Non-circular (NC) signal via two-dimensional (2-D) ESPRIT algorithm (NC-2D-ESPRIT). The studied algorithm considers the characteristics of the maximum non-circularity signal, and converts the real signal into maximum non-circularity signals by phase shifting [7], which uses the covariance matrix and the pseudo covariance matrix [8, 9] of the maximum non-circular signal to obtain the data matrix of the non-circular signal. Then, this algorithm expands the received signal data matrix by data recombination [10], which achieves the purpose of increasing the number of effective array elements of MIMO radar. Compared to the ESPRIT [5], the proposed algorithm has slightly better parameter estimation performance and improves the accuracy of parameter estimation when the distance of multiple targets is relatively close.

The remainder of this paper is structured as follows. Section 2 develops the system model on NC signal, and Sect. 3 proposes the parameter estimation theory of NC-2D-ESPRIT. Section 4 analyzes the simulation results, while the conclusion is presented in Sect. 5.

2 System Model

Consider a MIMO radar system with Uniform linear array of transmitters and receivers which utilizes M transmitters to transmit orthogonal signals and N receivers to receive the reflected signals. Define $d_t = \mathrm{N}d_r$, d_r is the distance from the transmitter to transmitter, d_r is the distance from the receiver to receiver. Thus, the M + N array elements of the transmitting and receiving arrays are equivalent to M $\times$ N-dimensional virtual transceiver array.

The signal received by the virtual transceiver array after filtering can be expressed as

$$\mathrm{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} A_x D_1 (A_y) \\ A_x D_2 (A_y) \\ \vdots \\ A_x D_N (A_y) \end{bmatrix} \mathrm{S} + \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_N \end{bmatrix} \tag{1}$$

Where $A_x = [a_x(\theta_1, \varphi_1), a_x(\theta_2, \varphi_2), \ldots, a_x(\theta_K, \varphi_K)]$ is the direction matrix of M array elements along the x-axis in which $a_x(\theta_k, \varphi_k)$ represents the signal from the signal source corresponding to the array response of the virtual transceiver array in the x-axis direction. $A_y = [a_y(\theta_1, \varphi_1), a_y(\theta_2, \varphi_2), \ldots, a_y(\theta_K, \varphi_K)]$ is the direction matrix of N array elements along the y-axis in which $a_y(\theta_k, \varphi_k)$ represents the signal from the signal source corresponding to the array response of the virtual transceiver array in the y-axis direction, and $\mathbf{W} = [W_1, W_2, \ldots, W_N]$ is the received noise matrix. S is a data matrix for noncircular signals [9] as follow:

$$\mathrm{S} = \Psi S_0 = [\mathrm{S}(1), \mathrm{S}(2), \ldots, \mathrm{S}(\mathrm{P})] \tag{2}$$

Where $\Psi = \{e^{j\Psi_1}, e^{j\Psi_2}, \ldots e^{j\Psi_k}\}$, and the different values on its diagonal lines represent the non-circular phase of non-circular sources, $S_0 = \left[s_{01,}s_{02}, \ldots, s_{0k}\right]^T$, S_0 is a real valued matrix, and s_{0k} represents the zero initial phase real signal of the maximum non-circular signal obtained by phase shifting.

3 Principle of Direction of Arrival Estimation Algorithm Based on NC-2D-ESPRIT

The data matrix Y received by the virtual transceiver array established in this paper is extended [14] as follows:

$$Y' = \begin{bmatrix} Y \\ JY^* \end{bmatrix} = \mathrm{A}S_0 + W' \in C^{2MN \times P} \tag{3}$$

Where J is an MN × MN-dimensional exchange matrix with all the anti-corner elements being 1, the rest of the positions take the value 0, and $Y^* \in C^{MN \times P}$ represents the conjugate matrix of matrix Y, A is the joint direction vector matrix after extending the transceiver array, $W' = \begin{bmatrix} W \\ JW^* \end{bmatrix}$ is the restructured noise matrix.

Define two sub-matrices from A to satisfy:

$$A_2 = A_1 D_2\left(A_y\right) = A_1 \psi_1 \tag{4}$$

Where, A_1 respectively denote the first N-1 rows of matrix A and A_2 denote the last N-1 rows of matrix A. Then, the covariance matrix of data in (3) will be $R_{zz} = \mathrm{E}[Y^{'} Y^{'H}]$ [8] can be decomposed. Taking the larger k eigenvalues obtained by the decomposition to form the signal subspace as $\mathbf{E}_s$, whose K columns are the eigenvectors associated with the K largest eigenvalues of R_z. For DOD estimation, the matrix $\mathbf{E}_s$ is dividing into E_{x1} and E_{y1}, which are obtained as follows:

$$\begin{bmatrix} E_{x1} \\ E_{y1} \end{bmatrix} = \begin{bmatrix} A_1 T_4 \\ A_1 D_2(A_y) T_4 \end{bmatrix} \tag{5}$$

Where T_4 is a full rank non-singular matrix, from the rotational invariance property of the antenna array, (5) satisfies $E_{y1} = E_{x1}T_4^{-1}\Psi_y T_4 = H_y T_4$, therefore, its least-squares solution can be expressed as:

$$\widehat{\mathrm{H}}_y = E_{x1}^{+} E_{y1} \tag{6}$$

Decompose the eigenvalue of $\widehat{\mathrm{H}}_y$ to get $\widehat{\mathrm{T}}_4 = \Gamma T_4$, $\widehat{\Psi}_y = \Gamma \Psi_y \Gamma^{-1}$ and $\widehat{r}_{yk} = \cos\widehat{\theta}_k sin\widehat{\Psi}_k$. Where Γ is a permutation matrix, $\widehat{r}_{yk}$ indicates that the eigenvalue position of $\widehat{\mathrm{H}}_y$ is K.

Similarly, for DOA estimation, by reconstructing the signal subspace we can get as follows:

$$E_s' = E_s\widehat{T}_4^{-1} = \mathrm{A}T_4\widehat{T}_4^{-1} = \mathrm{A}\Gamma^{-1} \tag{7}$$

Define two signal sub-matrices E_{x2} and E_{y2} from E_s' and two sub-matrices A_3 *and* A_4 from A to satisfy:

$$E_{x2} = A_3\Gamma^{-1} \tag{8}$$

$$E_{y2} = A_4\Gamma^{-1} = E_{x2}\Gamma\Psi_x\Gamma^{-1} = E_{x2}H_x \tag{9}$$

Then the estimated value that can be solved by the least squares method as follows:

$$\widehat{\mathrm{H}}_x = E_{x2}^{+} E_{y2} \tag{10}$$

Decompose the eigenvalue of $\widehat{\mathrm{H}}_x$ to get $\widehat{\Psi}_x = \Gamma\Psi_x\Gamma^{-1}$, $\widehat{r}_{Xk} = \cos\widehat{\theta}_k sin\widehat{\Psi}_k$. Where $\widehat{r}_{Xk}$ indicates that the eigenvalue position of $\widehat{\mathrm{H}}_x$ is K. In practice, the estimation of the elevation angle and azimuth of the MIMO radar based on NC-2D-ESPRIT can be obtained by the combination of Eqs. (11) and (12) as follows.

$$\widehat{\theta}_k = cos^{-1}\sqrt{(\widehat{r}_{Xk}^2 + \widehat{r}_{yk}^2)} \tag{11}$$

$$\widehat{\Psi}_k = tan^{-1}\sqrt{\frac{\widehat{r}_{yk}}{\widehat{r}_{Xk}}} \tag{12}$$

4 Simulations

This section discusses the capability of the proposed algorithm in fields of target angle resolution, array number and signal-to-noise ratio (SNR). Assume that there are K = 3 independent targets with the angles being $(25^\circ, -20^\circ)$, $(45^\circ, 10^\circ)$, $(35^\circ, -10^\circ)$, $\mathrm{M} = 6$, $\mathrm{N} = 6$, $LC = 200$, and the number of snapshots is P = 100 times.

Table 1 shows, compared with the ESPRIT [8], the estimated performance is significantly improved regardless of the azimuth or the pitch angle differs by 0.5°. It indicates that the algorithm can achieve a difference of 0.5° between the target angles under multiple targets. Therefore, the algorithm improves the radar resolution.

Table 1. The comparison of the difference of 0.5° between the target angles

SNR	Angle condition			
	Azimuth difference between 0.5°		Elevation difference between 0.5°	
	ESPRIT [8] RMSE/º	NC-ESPRIT RMSE/º	ESPRIT [8] RMSE/º	NC-ESPRIT RMSE/º
0 dB	1.3400	0.3973	0.9712	0.3840
5 dB	0.5741	0.1705	0.4860	0.1682
10 dB	0.2351	0.0919	0.1589	0.0905
15 dB	0.1240	0.0507	0.0854	0.0487
20 dB	0.0740	0.0286	0.0583	0.0251
25 dB	0.0400	0.0154	0.0334	0.0146
30 dB	0.0287	0.0086	0.0232	0.0078

Figure 1 shows the effect of SNR when estimating the target angle using the ESPRIT [8] and the NC-2D-ESPRIT algorithm. Increase the SNR from 0 dB to 30 dB at 2 dB, while other simulation conditions remain unchanged. We take the average of the RMSE of the three targets for comparison. It can be discovered that the angle estimation performance curve of the proposed algorithm is obviously better than that of the ESPRIT algorithm [8]. Furthermore, we can get that the RMSE of the proposed algorithm is lower. And the performance estimation of the proposed algorithm is obviously better than the ESPRIT algorithm [8], especially in low SNR.

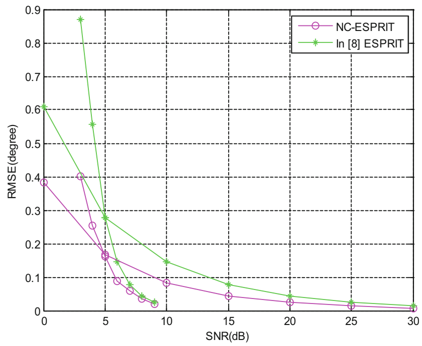

Fig. 1. Angle estimation with different SNR

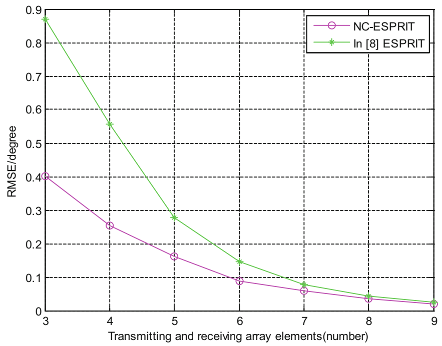

Fig. 2. Angle estimation comparison with different array elements

Figure 2 presents the effect of signal-to-array elements by using the two algorithms when change the number of transmitting and receiving elements from 3 to 9 at an interval with SNR = 10 dB.

It is shown that, the accuracy on parameter estimation of the proposed algorithm is significantly higher than the ESPRIT [8] under the same elements. When the number of elements is fewer, the RMSE of the proposed algorithm still remains small, indicating that parameters can still be accurately estimated. With the increasing number of array elements, both algorithms can estimated exactly. However, setting a large number of array elements will lead to an increase in the amount of calculation and energy, so in practice, we need to consider the actual situation and the integrated environment.

5 Conclusion

In this paper, a parameter estimation algorithm based on Non-Circular Signal via two-dimensional ESPRIT algorithm for MIMO radar has been discussed. This algorithm introduces the maximum non-circularity signal, and combines the ESPRIT algorithm to estimate MIMO radar parameters. Simulations show that the algorithm improves the accuracy of angle estimation regardless of the number of different elements or different SNR, especially in low SNR and fewer elements. Furthermore, the algorithm can achieve accurate estimation when the angle difference between multiple targets is 0.5°. In summary, the proposed algorithm provides a more convenient solution for MIMO radar parameter estimation.

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