An Improved Gauss-Seidel Algorithm for Signal Detection in Massive MIMO Systems

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Abstract. Massive multiple input multiple output (MIMO) is a promising technology that has been proposed to meet the requirement of the fifth generation wireless communications systems. For uplink massive MIMO systems, the typical linear detection such as minimum mean square error (MMSE) shows near-optimal performance. However, due to the direct matrix inverse, the computational complexity of the MMSE detection algorithm is too high, especially when there are a large number of users. Thus, in this paper, we propose an improved Gauss-Seidel algorithm by utilizing delayed over relaxation (DOR) scheme, which is named as delayed over relaxation Gauss-Seidel (DRGS) algorithm. The basic idea of the DOR scheme is to combine the predicted iterative step $(n + 1)$ with the iteration of step $(n - 1)$. The scheme can provide a significant improvement of the convergence speed for iterative algorithm. The theoretical analysis of DRGS algorithm shows that the proposed algorithm can reduce the computational complexity from $O(K^3)$ to $O(K^2)$, where K is the number of users. Simulation results verify that the DRGS algorithm can achieve almost the same BER performance as that of MMSE detection with a small number of iterations.

Keywords: Massive MIMO system · Delayed over relaxation Gauss-Seidel algorithm · MMSE detection · BER

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1 Introduction

Massive multiple-input multiple-output (MIMO) is widely considered as one of the most emerging technologies for future wireless communications [1–5]. Differing from the traditional MIMO (4×4, 8×8), massive MIMO systems are equipped with hundreds of antennas at the base station (BS) to serve user equipment (UE) simultaneously [6, 7]. It has been proved that massive MIMO can better meet the requirement of fast date rate, high spectral efficiency and wide coverage than conventional MIMO systems through theoretical analysis. Unfortunately, the augment of antennas increases the dimensions of matrix, which results in a significant increase in the computational complexity of signal processing [8].

The optimal detection algorithm, such as maximum likelihood (ML) detection algorithm, is highly impractical because of vast calculations [9]. To achieve near-optimal performance, several linear signal detection algorithms, such as zero-forcing (ZF) and minimum mean square error (MMSE), have been proposed [10]. Unfortunately, the algorithms mentioned above have high computational complexity due to a full matrix inversion operation, notably when the number of users is large.

Therefore, it is necessary to simplify matrix inversion calculation for massive MIMO systems. Recently, many effective researches about low complexity approximate matrix inversion have been conducted [11–16]. One typical category is to replace calculating matrix inversion directly with polynomial expansion (PE) [11, 12]. But, it is not practical for massive MIMO systems for a marginal reduction in complexity. The other category is based on iterative algorithms, such as Richardson algorithm [13], Jacobi algorithm [14] and Gauss-Seidel algorithm [15]. The Richardson algorithm [13] was proposed to avoid the high complexity caused by direct matrix inversion. Nevertheless, its convergence requires a large number of iterations which would lead to prohibitive complexity for massive MIMO systems. Therefore, in order to reduce the number of iterations, the approach based on Jacobi algorithm [14] was presented. Numerous studies have showed that the convergence rate of the conventional Jacobi algorithm is lower in comparison with the conventional Gauss-Seidel (GS) algorithm [15]. Based on the conventional GS algorithm, GS algorithm with initialization is proposed in [16], which can accelerate the convergence rate and reduce the number of iterations. But existing GS-based detectors still exhibit slow convergence rates.

In order to further improve the conventional GS detection algorithm for massive MIMO systems. The GS algorithm utilizes the delayed over-relaxation [17] approach to quicken the convergence rate, which brings about a reduction in iterations and computational complexity. According to simulation results, it is proved that the proposed algorithm can solve the matrix inversion problem in an iterative procedure with low complexity.

The remainder of this paper is organized as follows: we introduce the uplink system model in Sect. 2. Section 3 describes the proposed algorithm based on GSI algorithm and provides the analysis of computational complexity. Section 4 presents and discusses the simulation results. Finally, the conclusion is drawn in Sect. 5.

Notation: In this paper, lower-case boldface letters represent the column vectors (e.g., $\mathbf{h}$), and the upper-case boldface letters refer to the matrices (e.g., $\mathbf{H}$). $\mathbf{H}(i, j)$

represents the (i, j) element of matrix $\mathbf{H}$. For the matrix $\mathbf{H}$, $\mathbf{H}^{\mathrm{T}}$, $\mathbf{H}^{\mathrm{H}}$, and $\mathbf{H}^{-1}$ indicate the transpose, the Hermitian transpose, and the inverse of $\mathbf{H}$, respectively. In addition, $\mathbf{I}_{\mathrm{K}}$ denotes the $\mathrm{K} \times \mathrm{K}$ identify matrix.

2 System Model

We consider an uplink massive MIMO system employing N antennas at the BS to serve K single-antenna users simultaneously. Not that in such system, N is larger than K.

Let vector $\mathbf{s_c}= (s_{c,1}, s_{c,2}, \ldots, s_{c,K})^{\mathrm{T}}$ represent the complex-valued transmitted signal vector from K users. The transmitted symbol $s_{c,i}$ is modulated and mapped to one point of A which denote a complex M-QAM scheme. The matrix $\mathbf{H}_c \in \mathbb{C}^{N \times K}$ denotes the flat Rayleigh fading channel matrix, and the entries of $\mathbf{H}_c$ are independently and identically distributed (i.i.d.) with zero mean and unit variance [18, 19]. The vector $\mathbf{n_c}= (n_{c,1}, n_{c,2}, \ldots, n_{c,\mathrm{K}})^{\mathrm{T}}$ denotes the additive white Gaussian noise with mean is zero and corresponding variance is σ_{n}^2. Then, the complex received signal vector $\mathbf{y_c}= (y_{c,1}, y_{c,2}, \ldots, y_{c,\mathrm{K}})^{\mathrm{T}}$ can be expressed as:

$$\mathbf{y}_c = \mathbf{H}_c \mathbf{s}_c + \mathbf{n}_c \tag{1}$$

For ease of presentation and computation, the complex-valued system model can be translated to real-valued system model as

$$\underbrace{\begin{bmatrix} \mathrm{Re}\{\mathbf{y}_c\} \\ \mathrm{Im}\{\mathbf{y}_c\} \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \mathrm{Re}\{\mathbf{H}_c\} & -\mathrm{Im}\{\mathbf{H}_c\} \\ \mathrm{Im}\{\mathbf{H}_c\} & \mathrm{Re}\{\mathbf{H}_c\} \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} \mathrm{Re}\{\mathbf{s}_c\} \\ \mathrm{Im}\{\mathbf{s}_c\} \end{bmatrix}}_{\mathbf{s}} + \underbrace{\begin{bmatrix} \mathrm{Re}\{\mathbf{n}_c\} \\ \mathrm{Im}\{\mathbf{n}_c\} \end{bmatrix}}_{\mathbf{n}} \tag{2}$$

where Re{.} and Im{.} denote the real and imaginary parts, respectively. Note that $\mathbf{y} \in \mathbb{R}^{2K \times 1}$, $\mathbf{s} \in \mathbb{R}^{2K \times 1}$, $\mathbf{n} \in \mathbb{R}^{2K \times 1}$, accordingly $\mathbf{H} \in \mathbb{R}^{2K \times 2K}$.

2.1 MMSE Detector

The task of signal detector is to obtain the estimated value of $\mathbf{s}$ from the channel matrix $\mathbf{H}$ and the received vector $\mathbf{y}$. It is well known that the MMSE detector is proved to have near-optimal performance for massive MIMO systems. Thus, utilizing the MMSE detector, the resulting estimated symbol vector $\hat{\mathbf{s}}$ can be given by

$$\hat{\mathbf{s}} = \left(\mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{2K}\right)^{-1} \mathbf{H}^H \mathbf{y} = \mathbf{A}^{-1} \mathbf{b} \tag{2}$$

where $\mathbf{A} = \mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{2K}$ is the MMSE weighting matrix, and $\mathbf{b} = \mathbf{H}^H \mathbf{y}$ is the output of the matched filter, respectively. Note that $\mathbf{A}$ is symmetric positive definite and diagonally dominant [20]. The computational complexity of direct matrix inversion is $\mathrm{O}(K^3)$, which is too high for massive MIMO systems, especially when the number of user is large. Therefore, we can convert the MMSE algorithm into solving the following linear equation as

$$\mathbf{A}\mathbf{s} = \mathbf{b} \tag{3}$$

which can be solved in an iterative way.

2.2 Conventional Gauss-Seidel Algorithm

In order to avoid the direct matrix inversion, GS algorithm was proposed in [15]. Consider the decomposition of $\mathbf{A}$ as $\mathbf{A} = \mathbf{D} - \mathbf{L} - \mathbf{U}$, where $\mathbf{D}$ is composed of diagonal elements of $\mathbf{A}$, $\mathbf{L}$ and $\mathbf{U}$ represent the strictly lower triangular matrix and the upper triangular matrix of $\mathbf{A}$, severally. Then we can utilize GS iterative algorithm to solve (3) which can be written as:

$$\hat{\mathbf{s}}_{n+1} = (\mathbf{D} - \mathbf{L})^{-1}(\mathbf{U}\hat{\mathbf{s}}_n + \mathbf{b}) \tag{5}$$

where $\hat{\mathbf{s}}_n$ is the estimated transmitted vector at the n iteration.

3 Low-Complexity Signal Detection Algorithm

In this section, we propose an improved signal detection algorithm based on GS algorithm for massive MIMO systems. Through the theoretical analysis of convergence, the proposed DRGS algorithm can achieve good convergence rate. Then we analyze the complexity of the proposed DRGS algorithm.

3.1 DRGS Algorithm

The delayed over relaxation (DOR) method [17] is introduced briefly. The form of iterative solution method for linear equation can be cast in the following general form:

$$\mathbf{s}_{n+1} = \mathbf{G}\mathbf{s}_n + \mathbf{f} \tag{5}$$

As we all know, if the spectral radius of the matrix $\mathbf{G}$, hereinafter $\rho(\mathbf{G})$, is smaller than unity, the GS algorithm is convergent. And the smaller $\rho(\mathbf{G})$, the faster the convergence of the algorithm [21]. Therefore, the DOR method was utilized to achieve better performance of convergence. The main idea of the DOR method is combing the iteration of the predicted step $(n+1)$ with the iteration of step $(n-1)$. As described above, the modified relaxation step of system (5) can be expressed as:

$$\begin{cases} \mathbf{s}^*_{n+1} = \mathbf{G}\mathbf{s}_n + \mathbf{f} \\ \mathbf{s}_{n+1} = w\mathbf{s}^*_{n+1} + (1-w)\mathbf{s}_{n-1} \end{cases} \tag{6}$$

where w represents the relaxation parameter. The DOR method leads to a significant improvement of the convergence rate of iterative algorithm.

Next, we employ the DOR method to GS algorithm, which is named as DRGS algorithms. As mentioned before, the form of Gauss-Seidel algorithm can be denoted as

$$\hat{\mathbf{s}}^{(n+1)} = (\mathbf{D}-\mathbf{L})^{-1}\mathbf{U}\hat{\mathbf{s}}^{(n)} + (\mathbf{D}-\mathbf{L})^{-1}\mathbf{b} \tag{7}$$

The principle of DRGS algorithm is to modify the iterative step by DOR method. By substituting (7) into (6), the form of joint DOR method and GS algorithm can be described as follow:

$$\begin{cases} \hat{\mathbf{s}}^{*}_{n+1} = (\mathbf{D}-\mathbf{L})^{-1}\mathbf{U}\hat{\mathbf{s}}_{n} + (\mathbf{D}-\mathbf{L})^{-1}\mathbf{b} \\ \hat{\mathbf{s}}_{n+1} = w\hat{\mathbf{s}}^{*}_{n+1} + (1-w)\hat{\mathbf{s}}_{n-1} \end{cases} \tag{8}$$

Furthermore, as well known, the initial value of **s** plays an important role in the convergence rate and the detection accuracy of the iterative algorithm when the number of iterations is limited. Thus, we take diagonal matrix **D** and lower triangular matrix **L** into consideration to approximate $\mathbf{A}^{-1}$ and represent the initial solution as $\mathbf{s}_0 = (\mathbf{D} - \mathbf{L})^{-1}\mathbf{b}$.

3.2 Convergence Proof

The convergence of the proposed DRGS is analyzed on a theoretical basis in this section. As [17] shows, the convergence rate of DRGS method is closely related to $\rho(\mathbf{G})$. Therefore, the spectral radius of G is considered firstly. We define the $\mathbf{G} = (\mathbf{D} - \mathbf{L})^{-1}\mathbf{U}$, where $\mathbf{U} = \mathbf{L}^{T}$.

Theorem 1: Let $\mathbf{G} \in \mathbb{C}^{2K\times 2K}$ has eigenvalues $\lambda_i, i = 1:2K$. Then the spectral radius $\rho(\mathrm{G})$ is [21, 22]: $\rho(\mathbf{G}) = \max_{1\le i\le 2K}|\lambda_i|$.

At first, we set **r** as an arbitrary $2K \times 1$ non-zero real-valued vector. On the basic of the definition of eigenvalue, we have:

$$\begin{aligned} \mathbf{G}\mathbf{r} &= (\mathbf{D}-\mathbf{L})^{-1}\mathbf{L}^{T}\mathbf{r} = \lambda_i\mathbf{r} \\ \mathbf{L}^{T}\mathbf{r} &= \lambda_i(\mathbf{D}-\mathbf{L})\mathbf{r} \end{aligned} \tag{9}$$

Multiple by $\mathbf{r}^{\mathrm{T}}$ on both sides of (9), we can get

$$\mathbf{r}^{T}\mathbf{L}^{T}\mathbf{r} = \lambda_i\mathbf{r}^{T}(\mathbf{D}-\mathbf{L})\mathbf{r} \tag{10}$$

Then we transpose both sides of (11) simultaneously, where $\mathbf{D} = \mathbf{D}^{\mathrm{T}}$. Another equation can be obtained as

$$\mathbf{r}^{T}\mathbf{L}\mathbf{r} = \lambda_i\mathbf{r}^{T}\left(\mathbf{D}-\mathbf{L}^{T}\right)\mathbf{r} \tag{11}$$

Add (11) and (10) will lead to

$$\mathbf{r}^{T}\left(\mathbf{L}^{T}+\mathbf{L}\right)\mathbf{r} = \lambda_i\mathbf{r}^{T}\left(\mathbf{D}-\mathbf{L}-\mathbf{L}^{T}\right)\mathbf{r} \tag{12}$$

As shown earlier, we can depose **A** as $\mathbf{A} = \mathbf{D} - \mathbf{L} - \mathbf{L}^{\mathrm{T}}$. Combine (12) and the decomposition form of **A**, we have

$$(1-\lambda_i)\mathbf{r}^T\mathbf{D}\mathbf{r} = (1+\lambda_i)\mathbf{r}^T\mathbf{A}\mathbf{r} \tag{13}$$

Since the MMSE filtering matrix $\mathbf{A}$ is positive definite matrix, thus, the diagonal matrix $\mathbf{D}$ of $\mathbf{A}$ is positive definite. Then we can infer that $\mathbf{r}^T\mathbf{A}\mathbf{r} > 0$ and $\mathbf{r}^T\mathbf{D}\mathbf{r} > 0$. Further, we can get $(1-\lambda_i)(1+\lambda_i) > 0$, which indicates $|\lambda_i| < 1$. Combing the conclusion and Theorem 1, we can get $\rho(\mathbf{G}) < 1$.

As [17] shows, assume the iterative matrix of the DRGS algorithm is $\mathbf{M}$ and $w \in (1,2)$, when the spectral radius $\rho(\mathbf{G}) > \sqrt{w-1}$, the spectral radius of $\mathbf{M}$ can be written as

$$0 < \rho(\mathbf{M}) < \rho(\mathbf{G}) < 1 \tag{14}$$

Hence, we can get the proposed DRGS algorithm is convergent.

3.3 Computational Complexity Analysis

For massive MIMO systems, computational complexity is one of the important factors in measuring the performance of detector. The complexity is defined as the required real multiplications in solving $\mathbf{A}^{-1}$. Focus on the iterative form of DRGS algorithm, we calculate the complexity in each step. Firstly, the complexity of $\hat{\mathbf{x}}^*_{n+1}$, which is the same as the conventional GS algorithm, is equal to $4K^2$. Then, to achieve $\hat{\mathbf{x}}_{n+1}$, we require to calculate two scalar multiplication with $2K \times 1$ vector with the complexity is $4K$.

In Table 1, we compare the complexity of the proposed algorithm DRGS with the Jacobi algorithm and GS algorithm. As is shown in this table, the proposed algorithm has almost the same computational complexity as the conventional Jacobi algorithm and GS algorithm. Thus, the complexity of DRGS almost no increase in complexity and is much lower than the traditional MMSE signal detection.

Table 1. Computational complexity comparison of different algorithms

Algorithm	Complexity (iteration times T)
MMSE	$(5/3)K^3 + (3/2)K^2 + (8/3)\ K$
Jacobi [14]	$4K^2 + 10K + (4K^2 - 2K)T$
GS [16]	$8K^2 + 4K + 4K^2T$
DRGS	$4K^2 + 4K + (4K^2 + 4K)T$

4 Simulation Results

In this section, we utilize Monte-Carlo simulation to evaluate the proposed algorithm. In order to verify the performance of the DRGS algorithm, we provide bit error rate (BER) simulation result compared with the conventional GS algorithm. The BER performance of the typical MMSE is used as the benchmark for comparison. We consider the Rayleigh fading channel model as channel model. And the channel matrix

$\mathbf{H}_c$ follow the complex-valued Gaussian distribution with zero mean and unity variance. And the simulation environment is assumed to be the uplink massive MIMO system with 128×32 and 256×64, respectively. The 64QAM modulation scheme is utilized in the simulations.

Figure 1 illustrates the BER performance of the proposed DRGS algorithm against the relaxation parameter w. For comparison, we set the signal-to-noise (SNR) as 14 dB and the number of iterations is $T = 3$. The minimum BER is almost 10^{-6} for $N \times K = 128 \times 32$ and $N \times K = 256 \times 64$, when selecting the optimal relaxation parameter. Fortunately, we can find the optimal w is 1.1 for both above-mentioned systems. Moreover, according to extensive simulation results, it is found that the optimal relaxation parameter w is invariable when the result of (N/K) is fixed. Thus, the optimal w is easily to be ascertained when the result of (N/K) have been known.

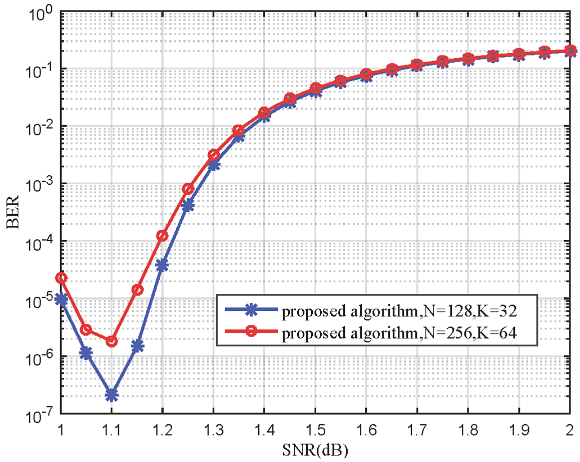

Fig. 1. BER performance of the proposed DRGS detection algorithm against the relaxation parameter w.

Figure 2 shows the comparison between the proposed DRGS algorithm with the Jacobi detection algorithm [14] and GS detection [16] algorithm, and the BER performance of the typical MMSE signal detection is utilized as the benchmark. The relaxation parameter w is considered to be set as $w = 1.1$. It is observed that the BER of both algorithms are closer to that of MMSE algorithm when the number of iterations increases. In addition, the proposed DRGS algorithm performs better in BER than the conventional algorithms with the same number of iterations. As we can observe in Fig. 2, in order to achieve the BER of 10^{-5}, the SNR required by the proposed algorithm is 12 dB, and the SNR required by the Jacobi algorithm and GS algorithm is more than 16 dB, when $T = 3$. Therefore, the convergence rate of our proposed DRGS algorithm is faster as compared with the conventional algorithm.

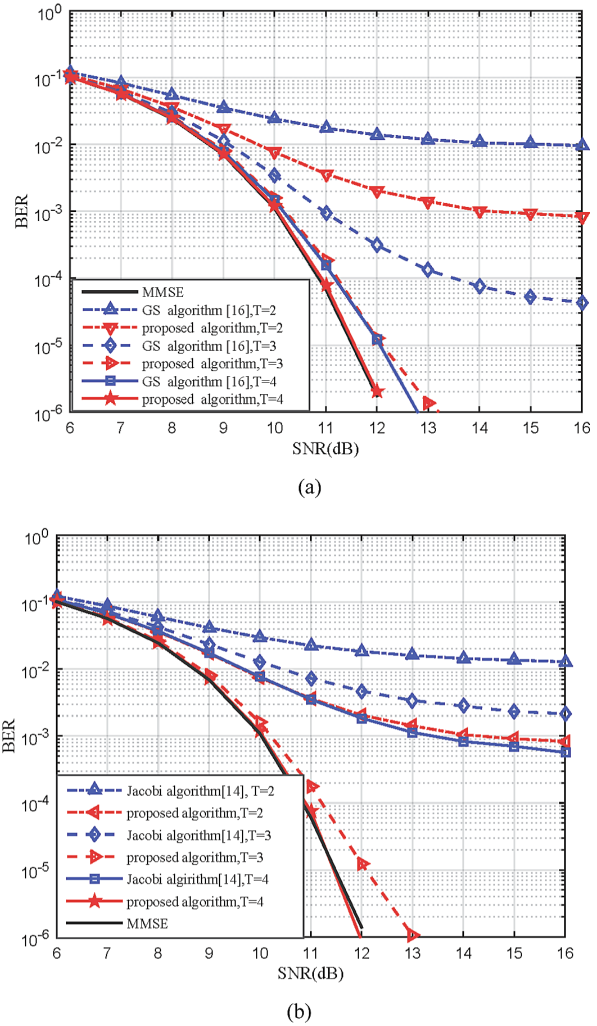

Fig. 2. BER performance comparison between the proposed algorithm and the conventional iterative algorithm with 64QAM modulation.

5 Conclusion

In this paper, we propose an improved GS algorithm that called DRGS algorithm in signal detection for massive MIMO systems. The DRGS algorithm is proved to realize MMSE solution and avoid direct matrix inversion and reduce the complexity from $O(K^3)$ to $O(K^2)$. In addition, through theoretical analysis and simulation results, it is proved that the DRGS algorithm can reach the approximate performance of typical

MMSE algorithm with small number of iterations. Generally speaking, the proposed DRGS can achieve high convergence rate with low complexity for massive MIMO systems.

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